

# Evoluta 7

$$2 < x < 3$$

8.

(a)

$$x+1$$

$$\bullet \quad 2 < x < 3 \Rightarrow 2+1 < x+1 < 3+1$$

$$\underline{\underline{3 < x+1 < 4}}$$

(b)

$$2x-3$$

$$\bullet \quad 2 < x < 3 \Rightarrow 4 < 2x < 6$$

$$4-3 < 2x-3 < 6-3$$

$$\underline{\underline{1 < 2x-3 < 3}}$$

(c)

$$-3x+1$$

$$\bullet \quad 2 < x < 3 \Rightarrow -6 > -3x > -9$$

$$1-6 > 1-3x > 1-9$$

$$\underline{\underline{-5 > 1-3x > -8}}$$

⑧

$$2-x$$

$$2 < x < 3 \Rightarrow -2 > -x > -3$$

$$2-2 > 2-x > 2-3$$

$$\underline{\underline{0 > 2-x > -1}}$$

⑨

$$\frac{x}{3} - 1$$

$$2 < x < 3 \Rightarrow \frac{2}{3} < \frac{x}{3} < 1$$

$$\frac{2}{3} - 1 < \frac{x}{3} - 1 < 1 - 1$$

$$\underline{\underline{-\frac{1}{3} < \frac{x}{3} - 1 < 0}}$$

⑩

$$\frac{2}{x}$$

$$2 < x < 3$$

$$\Rightarrow \frac{1}{2} > \frac{1}{x} > \frac{1}{3}$$

$$\frac{2}{2} > \frac{2}{x} > \frac{2}{3}$$

$$1 > \frac{2}{x} > \frac{2}{3}$$

11.  $x+y$

$$\begin{cases} \textcircled{a} \quad 1 < x < 3 \\ 2 < y < 5 \end{cases} \quad \textcircled{+} \quad \underline{\underline{3 < x+y < 8}}$$

⑬  $xy$

$$\begin{cases} 1 < x < 3 \\ 2 < y < 5 \end{cases} \quad \textcircled{\cdot} \quad \underline{\underline{2 < xy < 15}}$$

⑭  $2x+y$

$$\begin{cases} 1 < x < 3 \Rightarrow 2 < 2x < 6 \\ 2 < y < 5 \Rightarrow 2 < y < 5 \end{cases} \quad \textcircled{+} \quad \underline{\underline{4 < 2x+y < 11}}$$

⑮  $x-3y$

$$\begin{aligned} 1 < x < 3 &\Rightarrow 1 < x < 3 && \Rightarrow 3 > x > 1 && \textcircled{+} \\ 2 < y < 5 &\Rightarrow -6 > -3y > -15 && \Rightarrow -6 > -3y > -15 \\ &&&&&& \underline{\underline{-3 > x-3y > -14}} \end{aligned}$$

⑧

$x-y$

$$\begin{aligned} \bullet 1 < x < 3 &\Rightarrow 3 > x > 1 \\ \bullet 2 < y < 5 &\Rightarrow -2 > -y > -5 \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < x < 3 \\ \bullet 2 < y < 5 \end{aligned}} \right\} \textcircled{+}$$

$$\underline{\underline{1 > x-y > -4}}$$

⑨

$\frac{x}{y}$

$$\begin{aligned} \bullet 1 < x < 3 &\Rightarrow 3 > x > 1 \\ \bullet 2 < y < 5 &\Rightarrow \frac{1}{2} > \frac{1}{y} > \frac{1}{5} \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < x < 3 \\ \bullet 2 < y < 5 \end{aligned}} \right\} \textcircled{\cdot}$$

$$\underline{\underline{\frac{3}{2} > \frac{x}{y} > \frac{1}{5}}}$$

12.  $1 < x < 2$

(A)  $x^2 + 1$

$\bullet 1 < x < 2 \Rightarrow 1^2 < x^2 < 2^2 \Rightarrow 1 < x^2 < 4$

$2 < x^2 + 1 < 5$

(B)  $2x^3 - 1$

$\bullet 1 < x < 2 \Rightarrow 1 < x^3 < 8$

$\Rightarrow -2 < 2x^3 < 16$

$1 < 2x^3 - 1 < 15$

(C)  $1 - \frac{4}{x^2}$

$\bullet 1 < x < 2$

$\Rightarrow 1 < x^2 < 4$

$\Rightarrow 1 > \frac{1}{x^2} > \frac{1}{4}$

$\Rightarrow -4 < -\frac{4}{x^2} < -1$

$-3 < 1 - \frac{4}{x^2} < 0$

$$\textcircled{d} \quad \underline{(x+1)^2 - 2}$$

$$\bullet \quad 1 < x < 2 \quad \Rightarrow \quad 2 < x+1 < 3$$

$$4 < (x+1)^2 < 9$$

$$\underline{\underline{2 < (x+1)^2 - 2 < 7}}$$

13.

(a)  $x+y$

$$\begin{array}{l} \bullet 2 \leq x \leq 3 \\ \bullet 1 \leq y \leq 2 \end{array} \left\{ \begin{array}{l} \textcircled{+} \\ \hline \hline \end{array} \right. \underline{\underline{3 \leq x+y \leq 5}}$$

(b)  $x \cdot y$

$$\begin{array}{l} \bullet 2 \leq x \leq 3 \\ \bullet 1 \leq y \leq 2 \end{array} \left\{ \begin{array}{l} \textcircled{\cdot} \\ \hline \hline \end{array} \right. \underline{\underline{2 \leq xy \leq 6}}$$

(c)  $2x-3y+2$

$$\begin{array}{l} \bullet 2 \leq x \leq 3 \Rightarrow 4 \leq 2x \leq 6 \Rightarrow 6 \geq 2x \geq 4 \\ \bullet 1 \leq y \leq 2 \Rightarrow -3 \geq -3y \geq -6 \end{array} \quad \begin{array}{l} \hline \hline \end{array} \quad \begin{array}{l} \textcircled{+} \\ \hline \hline \end{array}$$

$$\underline{\underline{5 \geq 2x-3y+2 \geq 0}}$$

⑧  $x-y$

•  $2 \leq x \leq 3 \Rightarrow 3 \geq x \geq 2$

•  $1 \leq y \leq 2 \Rightarrow -1 \geq -y \geq -2$

$\left. \begin{array}{l} 3 \geq x \geq 2 \\ -1 \geq -y \geq -2 \end{array} \right\} \textcircled{+} \underline{\underline{2 \geq x-y \geq 0}}$

⑨  $\frac{x}{y}$

•  $2 \leq x \leq 3 \Rightarrow 3 \geq x \geq 2$

•  $1 \leq y \leq 2 \Rightarrow 1 \geq \frac{1}{y} \geq \frac{1}{2}$

$\left. \begin{array}{l} 3 \geq x \geq 2 \\ 1 \geq \frac{1}{y} \geq \frac{1}{2} \end{array} \right\} \textcircled{+} \underline{\underline{3 \geq \frac{x}{y} \geq 1}}$

⑩  $x^2+y^3$

•  $2 \leq x \leq 3 \Rightarrow 4 \leq x^2 \leq 9$

•  $1 \leq y \leq 2 \Rightarrow 1 \leq y^3 \leq 8$

$\left. \begin{array}{l} 4 \leq x^2 \leq 9 \\ 1 \leq y^3 \leq 8 \end{array} \right\} \textcircled{+} \underline{\underline{5 \leq x^2+y^3 \leq 17}}$



14. ①  $2 \leq x \leq 3 \Rightarrow 2 \geq -x \geq -3$   
 $4 \leq y \leq 5 \Rightarrow 5 \geq y \geq 4$  ②

$$7 \geq y - x \geq 1$$


---



---

③.  $y^2 - xy$

•  $2 \leq x \leq 3$

•  $4 \leq y \leq 5 \Rightarrow 16 \leq y^2 \leq 25 \Rightarrow 25 \geq y^2 \geq 16$

+

•  $2 \leq x \leq 3$  } ④  $8 \leq x \cdot y \leq 15 \Rightarrow -8 \geq -xy \geq -15$   
 •  $4 \leq y \leq 5$  }

$$17 \geq y^2 - xy \geq 1$$


---

④  $x - \frac{1}{y}$

⊕

•  $2 \leq x \leq 3$

•  $4 \leq y \leq 5 \Rightarrow \frac{1}{4} \geq \frac{1}{y} \geq \frac{1}{5} \Rightarrow -\frac{1}{4} \leq -\frac{1}{y} \leq -\frac{1}{5}$

$$2 - \frac{1}{4} \leq x - \frac{1}{y} \leq 3 - \frac{1}{5}$$

$$\frac{7}{4} \leq x - \frac{1}{y} \leq \frac{14}{5}$$


---

⑤  $x^2 - y^3$

•  $2 < x < 3 \Rightarrow 4 < x^2 < 9 \Rightarrow 9 > x^2 > 4$

•  $4 < y < 5 \Rightarrow 64 < y^3 < 125$

⊕

$$-64 > -y^3 > -125$$

$$-55 > x^2 - y^3 > -121$$


---

33. (a) xy

$$\begin{cases} \cdot -3 < x < -2 \\ \cdot -2 < y < -1 \end{cases} \Rightarrow \underline{\underline{6 > xy > 2}}$$

(b)  $\frac{x}{y}$

$$\begin{cases} \cdot -3 < x < -2 \Rightarrow -2 > x > -3 \\ \cdot -2 < y < -1 \Rightarrow -\frac{1}{2} > \frac{1}{y} > -1 \end{cases} \Rightarrow$$

$$\underline{\underline{1 < \frac{x}{y} < 3}}$$

(c)  $x^2 + y^3$

$$\begin{cases} \cdot -3 < x < -2 \Rightarrow 9 > x^2 > 4 \Rightarrow 4 < x^2 < 9 \\ \cdot -2 < y < -1 \Rightarrow -8 < y^3 < -1 \end{cases} \Rightarrow$$

$$\underline{\underline{-4 < x^2 + y^3 < 8}}$$

34.

(A)  $\alpha - \beta$

$$\begin{aligned} \bullet 1 < \alpha < 3 &\Rightarrow 3 > \alpha > 1 \\ \bullet -4 < \beta < -2 &\Rightarrow 4 > -\beta > 2 \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < \alpha < 3 \\ \bullet -4 < \beta < -2 \end{aligned}} \right\} \textcircled{+} 7 > \alpha - \beta > 3$$


---

(B)  $\alpha \beta$

$$\begin{aligned} \bullet 1 < \alpha < 3 &\Rightarrow -1 > -\alpha > -3 \\ \bullet -4 < \beta < -2 &\Rightarrow -2 > \beta > -4 \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < \alpha < 3 \\ \bullet -4 < \beta < -2 \end{aligned}} \right\} \textcircled{\cdot}$$

$$2 < -\alpha \beta < 12$$

$$-2 > \alpha \beta > -12$$


---

(C)  $\frac{\alpha}{\beta}$

$$\begin{aligned} \bullet 1 < \alpha < 3 &\Rightarrow 3 > \alpha > 1 \Rightarrow -3 < -\alpha < -1 \\ \bullet -4 < \beta < -2 &\Rightarrow -\frac{1}{4} > \frac{1}{\beta} > -\frac{1}{2} \Rightarrow -\frac{1}{2} < \frac{1}{\beta} < -\frac{1}{4} \end{aligned} \quad \left. \vphantom{\begin{aligned} \bullet 1 < \alpha < 3 \\ \bullet -4 < \beta < -2 \end{aligned}} \right\} \textcircled{\cdot}$$

$$\frac{3}{2} < -\frac{\alpha}{\beta} < \frac{1}{4} \Rightarrow -\frac{3}{2} > \frac{\alpha}{\beta} > -\frac{1}{4}$$

$$(f) \quad a^2 + B^2$$

$$\bullet \quad 1 < a < 3 \Rightarrow 1 < a^2 < 9 \Rightarrow 9 > a^2 > 1$$

$$\bullet \quad -4 < B < -2 \Rightarrow 16 > B^2 > 4 \quad \text{---} \oplus$$

$$25 > a^2 + B^2 > 5$$


---



---

$$(g) \quad \frac{a}{a+1}$$

$$\bullet \quad 1 < a < 3 \Rightarrow 2 < a+1 < 4 \Rightarrow \frac{1}{2} > \frac{1}{a+1} > \frac{1}{4}$$

$$\bullet \quad 1 < a < 3 \Rightarrow 3 > a > 1 \quad \checkmark \oplus$$

$$\frac{3}{2} > \frac{a}{a+1} > \frac{1}{4}$$


---



---

15.

$$a \leq x \leq b$$

$$x \in [a, b]$$

$$a < x < b$$

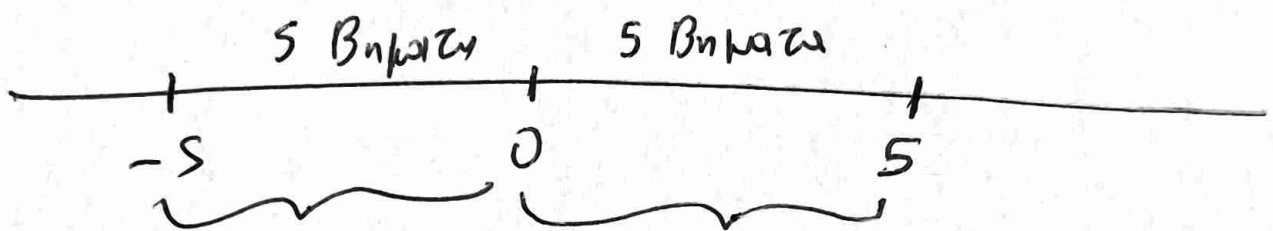
$$x \in (a, b)$$

$$a \leq x < b$$

$$\Rightarrow x \in [a, b)$$

# Απόλυτη Τιμή

Τι ονομάζουμε απόλυτη τιμή ενός αριθμού  $x$ ;



$$|5| = 5$$

$$|-5| = 5.$$

## Γαλκων

$$1. |x| = x \quad \text{αν} \quad x \geq 0$$

$$2. |x| = -x \quad \text{αν} \quad x < 0$$

$$3. |x| = |-x|$$

$$4. |x \cdot y| = |x| \cdot |y|$$

$$5. \left| \frac{x}{y} \right| = \frac{|x|}{|y|}$$

$$6. |x+y| \leq |x| + |y|$$

$$7. |x|^2 = |x^2| = x^2$$

$$8. |x| \geq x \quad \text{και} \quad |x| \geq -x$$



# Επορω Μαθημα

Παρασκευα

7

(35)

$$2 \leq x \leq 3$$

$$1 \leq y \leq 2$$

(36)

$$3 \leq x \leq 5$$

$$-2 \leq y \leq -1$$

(43)

(38)

(39)