9. ⓐ $\int_0^1 3x^2-2x-1 \, dx =$

$= \int_0^1 3x^2 \, dx - \int_0^1 2x \, dx - \int_0^1 1 \, dx$

$= (x^3)_0^1 - (x^2)_0^1 - (x)_0^1$

$= 1-0-(1-0)-(1-0) = 1-1-1=-1$

ⓑ $\int_{-1}^0 (x^3-3x+5) \, dx =$

$= \int_{-1}^0 x^3 \, dx - \int_{-1}^0 3x \, dx + \int_{-1}^0 5 \, dx$

$= \frac{1}{4}(x^4)_{-1}^0 - \frac{3}{2}(x^2)_{-1}^0 + 5(x)_{-1}^0$

$= -\frac{1}{4} + \frac{3}{2} + 5 = -\frac{1}{4} + \frac{6}{4} + \frac{20}{4} = \frac{25}{4}$

ⓖ $\int_1^2 4x^3 - x^2 - 1 \, dx = \int_1^2 4x^3 \, dx - \int_1^2 x^2 dx - \int_1^2 dx$

$= (x^4)_1^2 - \frac{1}{3}(x^3)_1^2 - (x)_1^2 =$

$= 15 - \frac{7}{3} - 1 = 14 - \frac{7}{3} = \frac{42}{3} - \frac{7}{3} = \frac{35}{3}.$

(στ) $\int_0^1 e^{3x-1}\,dx = \left(\frac{e^{3x-1}}{3}\right)_0^1 =$

$= \frac{1}{3}\left(e^{3x-1}\right)_0^1 = \frac{1}{3}\left(e^2 - \frac{1}{e}\right)$

(ζ) $\int_2^3 \frac{1}{x-1}\,dx = \left(\ln|x-1|\right)_2^3 =$

$= \ln 2 - \ln 1 = \ln 2$

(η) $\int_{-1}^0 \frac{1}{2x+3}\,dx = \left(\frac{\ln|2x+3|}{2}\right)_{-1}^0 =$

$= \frac{1}{2}\left(\ln|2x+3|\right)_{-1}^0 = \frac{1}{2}\left(\ln 3 - \ln 1\right) = \frac{\ln 3}{2}$

(θ) $\int_0^1 (x+1)^3\,dx \overset{x+1=t}{\underset{dx=dt}{=\!=}} \int_1^2 t^3\,dt = \frac{1}{4}\left(t^4\right)_1^2$

$= \frac{15}{4}$

(i) $\displaystyle\int_2^3 \frac{1}{(x-1)^2}\,dx \;\xrightarrow[dx=dt]{x-1=t}\; \int_1^2 \frac{1}{t^2}\,dt$

$\displaystyle = \left(-\frac{1}{t}\right)\Big|_1^2 = -\left(\frac{1}{2}-1\right) = \frac{1}{2}$

(ia) $\displaystyle\int_0^3 \frac{1}{\sqrt{x+1}}\,dx \;\xrightarrow[x+1=t^2]{\sqrt{x+1}=t}\; \int_1^2 \frac{1}{\cancel{t}}\,2\cancel{t}\,dt$

$dx = 2t\,dt$

$\displaystyle = \int_1^2 2\,dt = 2\,(t)\Big|_1^2 = 2.$

8. (α) $\int_0^1 e^x + 3x \, dx =$

$= \int_0^1 e^x \, dx + \int_0^1 3x \, dx$

$= (e^x)_0^1 + 3\int_0^1 x \, dx$

$= e^1 - e^0 + 3\,\frac{1}{2}\,(x^2)_0^1 = e - 1 + \frac{3}{2} = e + \frac{1}{2}$

(β) $\int_1^2 \frac{2}{x} + 3x^2 \, dx = \int_1^2 \frac{2}{x}\, dx + \int_1^2 3x^2 \, dx$

$= 2\int_1^2 \frac{1}{x}\, dx + (x^3)_1^2 =$

$= 2\,(\ln x)_1^2 + 7 = 2(\ln 2 - \ln 1) + 7$

$= 2\ln 2 + 7.$

(γ) $\int_0^\pi 2\,\text{συν}\,x - 3\,\text{ημ}\,x \, dx =$

$= \int_0^\pi 2\,\text{συν}\,x \, dx - \int_0^\pi 3\,\text{ημ}\,x \, dx$

$= 2\int_0^\pi \text{συν}\,x \, dx - 3\int_0^\pi \text{ημ}\,x \, dx$

$= 2\cancel{(\text{ημ}\,x)_0^\pi} - 3(-\text{συν}\,x)_0^\pi = 3(-1-1) = -6.$

7. (α) $\int_0^1 x^3\,dx = \left(\frac{x^4}{4}\right)_0^1 = \frac{1}{4}(x^4)_0^1 = \frac{1}{4}$

(β) $\int_1^2 x\,dx = \frac{1}{2}(x^2)_1^2 = \frac{3}{2}$

(γ) $\int_1^2 3\,dx = 3(x)_1^2 = 3$

(δ) $\int_1^2 \frac{1}{x^3}\,dx = \int_1^2 x^{-3}\,dx = \left(\frac{x^{-2}}{-2}\right)_1^2$

$= -\frac{1}{2}(x^{-2})_1^2 = -\frac{1}{2}\left(\frac{1}{x^2}\right)_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right)$

$= -\frac{1}{2}\left(-\frac{3}{4}\right) = \frac{3}{8}.$

(ε) $\int_1^4 3\sqrt{x}\,dx \xrightarrow[\substack{x=t^2 \\ dx=2t\,dt}]{\sqrt{x}=t} \int_1^2 3t\,2t\,dt$

$= 6\int_1^2 t^2\,dt = \frac{6}{3}(t^3)_1^2 = \frac{6}{3}\cdot 7 = \frac{42}{3}$

$= \underline{\underline{14}}$

Β' τρόπος

$$\int_1^4 3\sqrt{x}\,dx = 3\int_1^4 x^{1/2}\,dx = 3\left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)_1^4$$

$$= 3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^4 = \frac{3}{\frac{3}{2}}\cdot\left(x^{\frac{3}{2}}\right)_1^4 =$$

$$= 2\cdot\left(\sqrt{x^3}\right)_1^4 = 2\left(\sqrt{x^2}\sqrt{x}\right)_1^4 = 2\left(x\sqrt{x}\right)_1^4$$

$$= 14.$$

52 $$\int_1^2 \frac{1}{\sqrt[3]{x}}\,dx = \int_1^2 \frac{1}{x^{1/3}}\,dx =$$

$$= \int_1^2 x^{-\frac{1}{3}}\,dx = \left(\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}\right)_1^2 = \left(\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\right)_1^2$$

$$= \frac{3}{2}\cdot\left(\sqrt[3]{x^2}\right)_1^2 = \frac{3}{2}\left(\sqrt[3]{4}-1\right).$$

(3) $\int_0^1 x\sqrt{x}\,dx \quad \xrightarrow[\substack{x=t^2 \\ dx=2t\,dt}]{\sqrt{x}=t} \quad \int_0^1 t^2\, t\, 2t\,dt$

$= 2\int_0^1 t^4\,dt = \frac{2}{5}\left(t^5\right)\Big|_0^1 = \frac{2}{5}$

(4) $\int_1^4 \frac{x^2}{\sqrt{x}}\,dx \quad \xrightarrow[\substack{x=t^2 \\ dx=2t\,dt}]{\sqrt{x}=t} \quad \int_1^2 \frac{t^4}{\cancel{t}}\,2\cancel{t}\,dt$

$= 2\int_1^2 t^4\,dt = \frac{2}{5}\left(t^5\right)\Big|_1^2 = \frac{2\cdot 31}{5} = \frac{62}{5}$

(9) $\int_1^2 \frac{\sqrt{x}}{\sqrt[3]{x}}\,dx \quad \xrightarrow[\substack{\sqrt[6]{x}^3 = t^3 \\ \sqrt{x}=t^3 \\ \sqrt[6]{x}^2 = t^2 \\ \sqrt[3]{x}=t^2 \\ x=t^6 \\ dx=6t^5\,dt}]{\sqrt[6]{x}=t} \quad \int_1^{\sqrt[6]{2}} \frac{t^3}{t^2}\,6t^5\,dt$

$\int_1^{\sqrt[6]{2}} 6t^6\,dt$

$\frac{6}{7}\left(t^7\right)\Big|_1^{\sqrt[6]{2}}$

Ενότητα 36

2. (γ) $f(x) = \frac{e^x}{1+e^x}$

$E: C_f, x'x, x=0, x=\ln 2$

$E = \int_0^{\ln 2} |f(x)|\,dx = \int_0^{\ln 2} \left| \frac{e^x}{e^x+1} \right| dx =$ $\oplus$

$= \int_0^{\ln 2} \frac{e^x}{e^x+1}\,dx \qquad \frac{e^x+1=t}{e^x dx = dt}$

$= \int_2^3 \frac{1}{t}\,dt = \left(\ln|t|\right)_2^3 = \ln 3 - \ln 2$
$= \ln\frac{3}{2}.$

3. (γ) $E: C_f, x'x, x=0, x=1$

$E = \int_0^1 |f(x)|\,dx = \int_0^1 |3x^2-2x+1|\,dx$ $\oplus$ $\Delta<0$

$= \int_0^1 3x^2-2x+1\,dx = \int_0^1 3x^2 dx - \int_0^1 2x\,dx + \int_0^1 1\,dx$

$= (x^3)_0^1 - (x^2)_0^1 + (x)_0^1 =$

$= 1 - 1 + 1 = 1$

4. ⑧ E: C_f, $x'x$, $x=-1$, $x=2$

$$E = \int_{-1}^{2} |f(x)|\,dx = \int_{-1}^{2} |x^2-3x|\,dx =$$

x	-1	0	2	3	
x^2-3x	$+$	0	$-$	0	$+$

$$= \int_{-1}^{0} |\overset{+}{f(x)}|\,dx + \int_{0}^{2} |\overset{-}{f(x)}|\,dx$$

$$= \int_{-1}^{0} x^2-3x\,dx + \int_{0}^{2} -x^2+3x\,dx$$

$$= \int_{-1}^{0} x^2\,dx - \int_{-1}^{0} 3x\,dx - \int_{0}^{2} x^2\,dx + \int_{0}^{2} 3x\,dx$$

$$= \frac{1}{3}\left(x^3\right)_{-1}^{0} - 3\,\frac{1}{2}\left(x^2\right)_{-1}^{0} + \frac{1}{3}\left(x^3\right)_{0}^{2} + \frac{3}{2}\left(x^2\right)_{0}^{2}$$

$$= \frac{1}{3} + \frac{3}{2} + \frac{8}{3} + 6$$

Ενοτητα 34

6. ⓐ $\int_0^{\pi} \eta\mu x\,dx = (-\sigma\upsilon\nu x)_0^{\pi} =$

$= -(\sigma\upsilon\nu x)_0^{\pi} = -(\sigma\upsilon\nu\pi - \sigma\upsilon\nu 0) = -(-1-1) = 2$

ⓑ $\int_0^1 e^x\,dx = (e^x)_0^1 = e^1 - e^0 = e-1$

ⓒ $\int_0^{\pi} \sigma\upsilon\nu x\,dx = (\eta\mu x)_0^{\pi} = \eta\mu\pi - \eta\mu 0 = 0$

ⓓ $\int_1^e \frac{1}{x}\,dx = (\ln|x|)_1^e = \ln e - \ln 1 = 1$

ⓔ $\int_1^2 \frac{1}{x^2}\,dx = \left(-\frac{1}{x}\right)_1^2 = -\left(\frac{1}{x}\right)_1^2 = -\left(\frac{1}{2}-1\right)$
$= 1 - \frac{1}{2} = \frac{1}{2}$

ⓢⓣ $\int_0^{\frac{\pi}{4}} \frac{1}{\sigma\upsilon\nu^2 x}\,dx = (\varepsilon\varphi x)_0^{\frac{\pi}{4}} = \varepsilon\varphi\frac{\pi}{4} - \varepsilon\varphi 0 = 1$

ⓩ $\int_1^4 \frac{1}{2\sqrt{x}}\,dx = (\sqrt{x})_1^4 = \sqrt{4} - \sqrt{1} = 2-1 = 1$

ⓗ $\int_0^1 2^x\,dx = \left(\frac{2^x}{\ln 2}\right)_0^1 = \frac{1}{\ln 2}(2^x)_0^1 = \frac{1}{\ln 2}.$

ⓠ $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\eta\mu^2 x}\,dx = (-\sigma\varphi x)_{\pi/4}^{\pi/2} = -\left(\sigma\varphi\frac{\pi}{2} - \sigma\varphi\frac{\pi}{4}\right) = 1$

10. 8 $\int_1^2 \frac{(x-2)^2}{x^2}\,dx = \int_1^2 \frac{x^2-4x+4}{x^2}\,dx$

$$= \int_1^2 \frac{x^2}{x} - \frac{4x}{x^2} + \frac{4}{x^2}\,dx =$$

$$= \int_1^2 x - \frac{4}{x} + \frac{4}{x^2}\,dx =$$

$$= \int_1^2 x\,dx - \int_1^2 \frac{4}{x}\,dx + \int_1^2 \frac{4}{x^2}\,dy$$

$$= \frac{1}{2}(x^2)_1^2 - 4\,(\ln|x|)_1^2 + 4\left(-\frac{1}{x}\right)_1^2$$

$$= \frac{3}{2} - 4\ln 2 - 4\left(\frac{1}{2} - 1\right) = \frac{3}{2} - 4\ln 2 - 2 + 4$$

$$= 3 - 4\ln 2$$

13 $\int_1^2 \left(x - \frac{1}{x}\right)^2 dx = \int_1^2 x^2 - 2\cdot x\cdot\frac{1}{x} + \left(\frac{1}{x}\right)^2 dx$

$$= \int_1^2 x^2 - 2 + \frac{1}{x^2}\,dv = \int_1^2 x^2\,dx - \int_1^2 2\,dx + \int_1^2 \frac{1}{x^2}\,dx$$

$$= \frac{1}{3}(x^3)_1^2 - 2(x)_1^2 + \left(-\frac{1}{x}\right)_1^2$$

$$= \frac{7}{3} - 2 - \left(\frac{1}{2} - 1\right) = \frac{7}{3} - 2 - \frac{1}{2} + 1 = \frac{7}{3} - \frac{3}{2} = \frac{5}{6}.$$

11. α) $\int_0^{\pi} συν3x\,dx = \left(\frac{ημ3x}{3}\right)_0^{\pi} =$

$= \frac{1}{3}\left(ημ3x\right)_0^{\pi} = 0.$

β) $\int_0^{\pi} ημ2x\,dx = \left(-\frac{συν2x}{2}\right)_0^{\pi} = -\frac{1}{2}\left(συν2x\right)_0^{\pi}$

$= -\frac{1}{2}\left(συν2\pi - συν0\right) = 0$

γ) $\int_0^1 e^{3x}\,dx = \left(\frac{e^{3x}}{3}\right)_0^1 = \frac{1}{3}\left(e^{3x}\right)_0^1$

$= \frac{1}{3}\left(e^3 - 1\right) = \frac{1}{3}e^3 - \frac{1}{3}$

δ) $\int_0^1 \frac{1}{e^x}\,dx = \int_0^1 e^{-x}\,dx = \left(\frac{e^{-x}}{-1}\right)_0^1$

$= -\left(e^{-1} - 1\right) = -\frac{1}{e} + 1$

ε) $\int_0^1 \frac{1}{e^{2x}}\,dx = \int_0^1 e^{-2x}\,dx = \left(\frac{e^{-2x}}{-2}\right)_0^1$

$= -\frac{1}{2}\left(e^{-2x}\right)_0^1 = -\frac{1}{2}\left(e^{-2} - 1\right) = -\frac{1}{2}\frac{1}{e^2} + \frac{1}{2}$

Επομαο Μαθημα

Παλια

35

2

3

4

5

6

15 α β γ

16 α β δ

17 α

18 α β δ

20

21

22

23