Τριγωνομετρικές εξισώσεις

1. $\eta\mu x = \eta\mu\theta \Leftrightarrow x = 2k\pi + \theta$
$x = 2k\pi + \pi - \theta$

2. $\sigma\upsilon\nu x = \sigma\upsilon\nu\theta \Leftrightarrow x = 2k\pi + \theta$
$x = 2k\pi - \theta$

3. $\varepsilon\varphi x = \varepsilon\varphi\theta \Leftrightarrow x = k\pi + \theta$

4. $\sigma\varphi x = \sigma\varphi\theta \Leftrightarrow x = k\pi + \theta.$

Εφαρμογή 13

1. (δ) $\varepsilon\varphi x = \varepsilon\varphi \frac{2\pi}{11}$

$$x = k\pi + \frac{2\pi}{11}$$

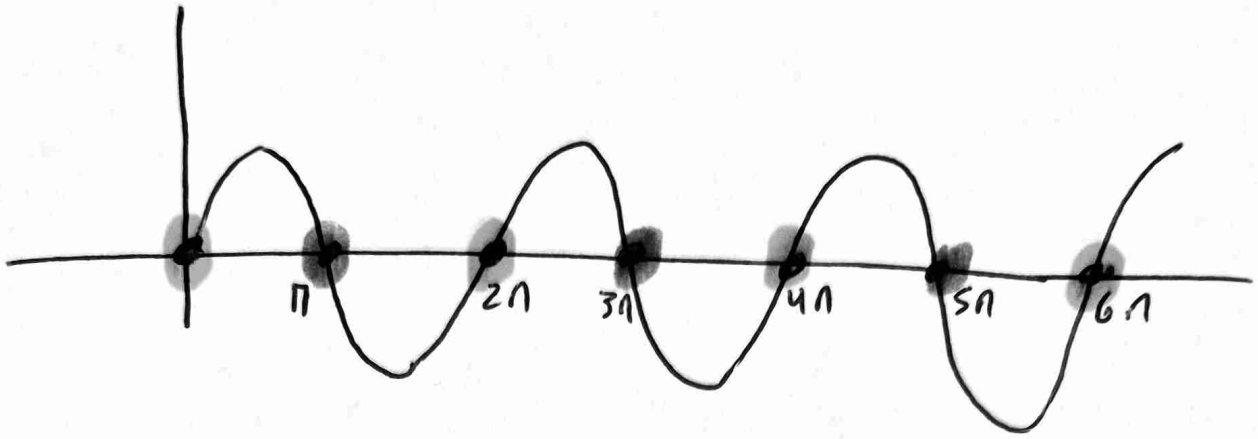
2. (β) $2\sigma\upsilon x = \sqrt{3}$

$$\sigma\upsilon x = \frac{\sqrt{3}}{2}$$

$$\sigma\upsilon x = \sigma\upsilon \frac{\pi}{6}$$

$$x = 2k\pi + \frac{\pi}{6} \quad \text{ή} \quad x = 2k\pi - \frac{\pi}{6}$$

$$\sin x = 0$$



$\sin x = 0 \quad \Leftrightarrow \quad \sin x = \sin 0$

$x = 2k\pi + 0$

$x = 2k\pi$

ou

$x = 2k\pi + \pi - 0$

$x = 2k\pi + \pi$

$k=0 \Rightarrow x=0$

$k=1 \Rightarrow x=2\pi$

$k=2 \Rightarrow x=4\pi$

$k=3 \Rightarrow x=6\pi$

$k=0 \quad x=\pi$

$k=1 \quad x=3\pi$

$k=2 \quad x=5\pi$

3. (01) $\sqrt{2}\,\text{нрx} - 1 = 0$

$$\sqrt{2}\,\text{нрx} = 1$$

$$\text{нрx} = \frac{1}{\sqrt{2}}$$

$$\text{нрx} = \frac{\sqrt{2}}{2}$$

$$\text{нрx} = \text{н}\mu\,\frac{\pi}{4}$$

$x = 2k\pi + \frac{\pi}{4}$ и $x = 2k\pi + \pi - \frac{\pi}{4}$

$x = 2k\pi + \frac{3\cdot\pi}{4}$

4. (B) $\cos x = -\cos\frac{\pi}{9}$

$\cos x = \cos\left(\pi - \frac{\pi}{9}\right)$

$x = 2k\pi + \pi - \frac{\pi}{9}$ ή $x = 2k\pi - \pi + \frac{\pi}{9}$

$x = 2k\pi + \frac{8\pi}{9}$

$x = 2k\pi - \frac{8\pi}{9}$

(δ) $\sigma\varphi x + \sigma\varphi\frac{\pi}{8} = 0$

$\sigma\varphi x = -\sigma\varphi\frac{\pi}{8}$

$\sigma\varphi x = \sigma\varphi\left(-\frac{\pi}{8}\right)$

$x = k\pi - \frac{\pi}{8}$

5. (в) $\cos x = -\frac{\sqrt{3}}{2}$

$$\cos x = -\cos\frac{\pi}{6}$$

$$\cos x = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$\cos x = \cos\frac{5\pi}{6}$$

$x = 2k\pi + \frac{5\pi}{6}$ и $x = 2k\pi - \frac{5\pi}{6}$,

(г) $\operatorname{tg} x = -1$

$$\operatorname{tg} x = -\operatorname{tg}\frac{\pi}{4}$$

$$\operatorname{tg} x = \operatorname{tg}\left(-\frac{\pi}{4}\right)$$

$$x = k\pi - \frac{\pi}{4}.$$

6. (B) $(2\sin x+1)\cdot(\operatorname{ctg} x+1)=0$

$2\sin x+1=0$ и $\operatorname{ctg} x+1=0$

$\sin x=-\frac{1}{2}$

$\operatorname{ctg} x=-1$

$\sin x=-\sin\frac{\pi}{3}$

$\operatorname{ctg} x=-\operatorname{ctg}\frac{\pi}{4}$

$\sin x=\sin\left(\pi-\frac{\pi}{3}\right)$

$\operatorname{ctg} x=\operatorname{ctg}\left(-\frac{\pi}{4}\right)$

$\sin x=\sin\frac{2\pi}{3}$

$x=k\pi-\frac{\pi}{4}$

$x=2k\pi+\frac{2\pi}{3}$

$x=2k\pi-\frac{2\pi}{3}$

7. Ⓑ $\cos x \cdot (4\sin x - 1) = 0$

$\cos x = 0$ и $4\sin x - 1 = 0$

$\cos x = \cos \frac{\pi}{2}$

$x = 2k\pi + \frac{\pi}{2}$

$x = 2k\pi - \frac{\pi}{2}$

$4\sin x = 1$

$\sin x = \sin \frac{\pi}{2}$

$x = 2k\pi + \frac{\pi}{2}$

$x = 2k\pi + \pi - \frac{\pi}{2}$

$x = 2k\pi + \frac{\pi}{2}$

8. Ⓓ $\sin x \cdot \cos x + \sin x = 1 + \cos x$

$\sin x (\cos x + 1) - (1 + \cos x) = 0$

$(\cos x + 1)(\sin x - 1) = 0$

$\cos x + 1 = 0$ и $\sin x - 1 = 0$

$\cos x = -1$

$\cos x = \cos \pi$

$x = 2k\pi + \pi$ $x = 2k\pi - \pi$

$\sin x = 1$

$\sin x = \sin \frac{\pi}{2}$

$x = 2k\pi + \frac{\pi}{2}$

12. (B) $\cos 3x + \cos\left(3x - \frac{\pi}{4}\right) = 0$

$$\cos 3x = -\cos\left(3x - \frac{\pi}{4}\right)$$

$$\cos 3x = \cos\left(\pi - 3x + \frac{\pi}{4}\right)$$

$3x = 2k\pi + \pi - 3x + \frac{\pi}{4}$ ή $3x = 2k\pi - \pi + 3x - \frac{\pi}{4}$

$6x = 2k\pi + \frac{5\pi}{4}$ Αδύνατη.

$$x = \frac{k\pi}{3} + \frac{5\pi}{24}$$

13. ⓐ $\eta\mu^2 x = \frac{3}{4}$

$\eta\mu x = \frac{\sqrt{3}}{2}$ ή $\eta\mu x = -\frac{\sqrt{3}}{2}$

$\eta\mu x = \eta\mu \frac{\pi}{3}$

$\eta\mu x = -\eta\mu \frac{\pi}{3}$

$\eta\mu x = \eta\mu\left(-\frac{\pi}{3}\right)$

$x = 2k\pi + \frac{\pi}{3}$

$x = 2k\pi - \frac{\pi}{3}$

$x = 2k\pi + \pi - \frac{\pi}{3}$

$x = 2k\pi + \frac{2\pi}{3}$

$x = 2k\pi + \pi + \frac{\pi}{3}$

$x = 2k\pi + \frac{4\pi}{3}$

11. ⑤ $\sqrt{3}\,\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)+1=0$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=\frac{-1}{\sqrt{3}}$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{3}$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=-\operatorname{ctg}\frac{\pi}{3}$$

$$\operatorname{ctg}\left(\pi x+\frac{\pi}{4}\right)=\operatorname{ctg}\left(-\frac{\pi}{3}\right)$$

$$\pi x+\frac{\pi}{4}=k\pi-\frac{\pi}{3}$$

$$\pi x=k\pi-\frac{\pi}{3}-\frac{\pi}{4}$$

$$x=k-\frac{1}{3}-\frac{1}{4}$$

$$x=k-\frac{7}{12}$$

22. (B) $2\eta\mu x + 1 = 0$

$k \in \mathbb{Z}$

$(-\pi, 2\pi]$

$$\eta\mu x = -\frac{1}{2}$$

$$\eta\mu x = -\eta\mu\frac{\pi}{6}$$

$$\eta\mu x = \eta\mu\left(-\frac{\pi}{6}\right)$$

$$x = 2k\pi - \frac{\pi}{6}$$

$$-\pi < x \leq 2\pi$$

$$-\pi < 2k\pi - \frac{\pi}{6} \leq 2\pi$$

$$-1 < 2k - \frac{1}{6} \leq 2$$

$$-6 < 12k - 1 \leq 12$$

$$-5 < 12k \leq 13$$

$$-\frac{5}{12} < k \leq \frac{13}{12}$$

$k=0$: $x = -\frac{\pi}{6}$ $\quad x = \frac{11\pi}{6}$

ή

$$x = 2k\pi + \pi + \frac{\pi}{6}$$

$$x = 2k\pi + \frac{7\pi}{6}$$

$$-\pi < x \leq 2\pi$$

$$-\pi < 2k\pi + \frac{7\pi}{6} \leq 2\pi$$

$$-1 < 2k + \frac{7}{6} \leq 2$$

$$-6 < 12k + 7 \leq 12$$

$$-13 < 12k \leq 5$$

$$-\frac{13}{12} < k \leq \frac{5}{12}$$

$x = -\frac{5\pi}{6}$ $\quad x = \frac{7\pi}{6}$

17.

ΕΝΟΤΗΤΑ 12

$\omega = \frac{1}{2}$

$\rho = 2$

$c = 1$

(01) $f(x) = 1 + 2\eta\mu\frac{x}{2}$

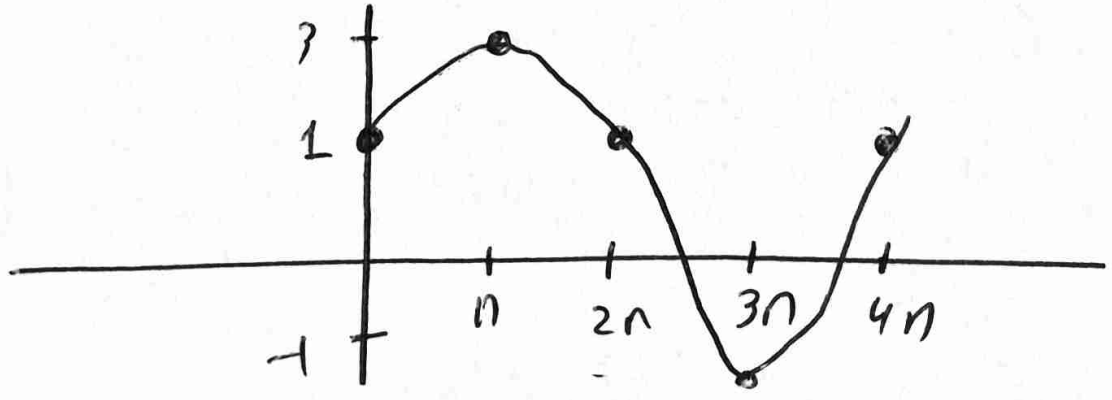
$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$Max\, f: |\rho| + c = |2| + 1 = 3$

$Min\, f: -|\rho| + c = -2 + 1 = -1$

$\frac{x}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	π	2π	3π	4π
$\eta\mu\frac{x}{2}$	0	1	0	−1	0
$2\eta\mu\frac{x}{2}$	0	2	0	−2	0
f	1	3	1	−1	1

→ $T = 4\pi$



Спорсо Майнс

13

(1) α β γ

(2) α γ δ

(3) β γ

(4) α γ

(5) α δ

(6) α

(7) α γ δ

(8) α β.

(11) α β γ

(12) α γ δ.