# Εντολή 23

**6** ① $2^{x-1} + 2^{x-2} - 2^{x-3} = 10$

$2^x \cdot 2^{-1} + 2^x \cdot 2^{-2} - 2^x \cdot 2^{-3} = 10$

$2^x = t$

$t \cdot \frac{1}{2} + t \cdot \frac{1}{4} - t \cdot \frac{1}{8} = 10$

$4t + 2t - t = 80$

$5t = 80$

$t = 16$

$2^x = 2^4$

$x = 4$

**7.** ② $4^x - 5 \cdot 2^{x+1} + 16 = 0$

$2^x = t$

$(2^x)^2 - 5 \cdot 2^x \cdot 2 + 16 = 0$

$t^2 - 10t + 16 = 0$

$t = 2$ $\quad$ $t = 8$

$2^x = 2$ $\quad$ $2^x = 8$

$x = 1$ $\quad$ $x = 3$

8. ⓕ $e^{3x} + 5e^{2x} = 6e^x$

$(e^x)^3 + 5(e^x)^2 = 6e^x$

$e^x = t$

$t^3 + 5t^2 = 6t$

$t^3 + 5t^2 - 6t = 0$

$t(t^2 + 5t - 6) = 0$

$t=0$ $\quad$ $t=-6$ $\quad$ $t=1$

$e^x = 0$ $\quad$ $e^x = -6$ $\quad$ $e^x = 1$

Αδύνατη $\quad$ Αδύνατη $\quad$ $x=0$

# Άσκηση 12

$A\left(\frac{\pi}{2}, 1\right)$

$f(x) = \alpha \eta\mu x - 1$

(α). Ισο $\alpha = 2$

$f\left(\frac{\pi}{2}\right) = \alpha \cdot \eta\mu\frac{\pi}{2} - 1 = \alpha - 1$

$1 = \alpha - 1$ $\quad (\alpha = 2)$

$f(x) = 2\eta\mu x - 1$

(β) Ναο $f(5\pi + x) - f\left(\frac{11\pi}{2} + x\right) = 2(\sigma\upsilon x - \eta\mu x)$

- $f(5\pi + x) = 2\eta\mu(5\pi + x) - 1 = 2\eta\mu(\cancel{4\pi} + \pi + x) - 1 =$

$= 2\eta\mu(\pi + x) - 1 =$

$= -2\eta\mu x - 1$

- $f\left(\frac{11\pi}{2} + x\right) = 2\eta\mu\left(\frac{11\pi}{2} + x\right) - 1 =$

$= 2\eta\mu\left(\frac{10\pi}{2} + \frac{\pi}{2} + x\right) = 2\eta\mu\left(5\pi + \frac{\pi}{2} + x\right) - 1$

$= 2\eta\mu(\cancel{4\pi} + \pi + \frac{\pi}{2} + x) - 1 =$

$= 2\eta\mu\left(\pi + \frac{\pi}{2} + x\right) - 1 = -2\eta\mu\left(\frac{\pi}{2} + x\right) - 1$

$= -2\eta\mu\left(\frac{\pi}{2} - (-x)\right) - 1 =$

$= -2\sigma\upsilon(-x) - 1 = -2\sigma\upsilon x - 1$

Αρα $-2\eta\mu x - 1 - (-2\sigma\upsilon x - 1) = 2(\sigma\upsilon x - \eta\mu x)$

$-2\eta\mu x - 1 + 2\sigma\upsilon x + 1 = 2\sigma\upsilon x - 2\eta\mu x$

$0 = 0$

⑧ i) $f(x) = 0$

$2\eta\mu x - 1 = 0 \Rightarrow \eta\mu x = \frac{1}{2}$

$\eta\mu x = \eta\mu\frac{\pi}{6}$

$x = 2k\pi + \frac{\pi}{6}$ $x = 2k\pi + \pi - \frac{\pi}{6}$

$x = 2k\pi + \frac{5\pi}{6}$.

$x = 2k\pi + \frac{\pi}{6}$

$0 < x < \frac{\pi}{2}$

$0 < 2k\pi + \frac{\pi}{6} < \frac{\pi}{2}$

$0 < 12k\pi + \pi < 3\pi$

$0 < 12k + 1 < 3$

$-1 < 12k < 2$

$-\frac{1}{12} < k < \frac{1}{6}$

Για $k=0$ $\quad x = \frac{\pi}{6}$

$x = 2k\pi + \frac{5\pi}{6}$.

$0 < x < \frac{\pi}{2}$

$0 < 2k\pi + \frac{5\pi}{6} < \frac{\pi}{2}$

$0 < 12k\pi + 5\pi < 3\pi$

$0 < 12k + 5 < 3$

$-5 < 12k < -2$

$-\frac{5}{12} < k < -\frac{1}{6}$

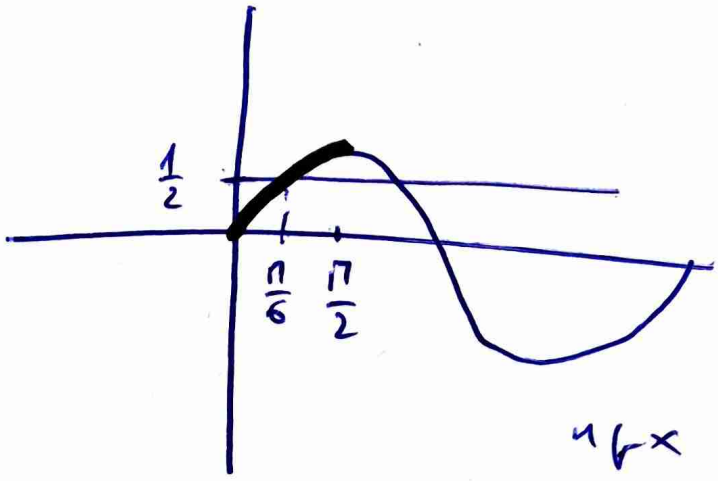
Τίποτα.

iii). $f(x) > 0$

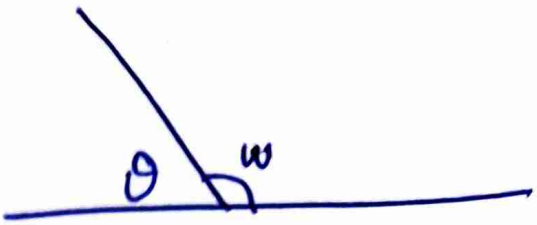
$2\eta\mu x - 1 > 0$

$\eta\mu x > \frac{1}{2}$

$x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$.



# Θεμα 10




$\theta + w = 180$

$\theta = 180 - w$

$\theta = \pi - w$

$\eta\mu w = \frac{4}{5}$.

(α) $\eta\mu^2 w + \sigma\upsilon\nu^2 w = 1$

$\left(\frac{4}{5}\right)^2 + \sigma\upsilon\nu^2 w = 1$

$\frac{16}{25} + \sigma\upsilon\nu^2 w = 1$

$\sigma\upsilon\nu^2 w = 1 - \frac{16}{25}$

$\sigma\upsilon\nu^2 w = \frac{9}{25}$

$\sigma\upsilon\nu w = \frac{3}{5}$ ή $\sigma\upsilon\nu w = -\frac{3}{5}$

$w \in 2o$

$\varepsilon\varphi w = \frac{\eta\mu w}{\sigma\upsilon\nu w} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$

$\varepsilon\varphi w = -\frac{4}{3}$

$\sigma\varphi w = -\frac{3}{4}$

(B) $\eta\mu\theta = \eta\mu(\pi-\omega) = \eta\mu\omega = \frac{4}{5}$

$\sigma\upsilon\theta = \sigma\upsilon(\pi-\omega) = -\sigma\upsilon\omega = \frac{3}{5}$

$\varepsilon\varphi\theta = \varepsilon\varphi(\pi-\omega) = -\varepsilon\varphi\omega = \frac{4}{3}$

$\sigma\varphi\theta = \sigma\varphi(\pi-\omega) = -\sigma\varphi\omega = -\frac{3}{4}$

(Γ) $8\eta\mu x = 5\sqrt{3}\,\eta\mu\omega$

$8\eta\mu x = 5\sqrt{3}\,\frac{4}{5}$

$8\eta\mu x = 4\sqrt{3}$

$2\eta\mu x = \sqrt{3}$

$\eta\mu x = \frac{\sqrt{3}}{2}$

$\eta\mu x = \eta\mu\frac{\pi}{3}$

$x = 2\kappa\pi + \frac{\pi}{3}$ ή $x = 2\kappa\pi + \pi - \frac{\pi}{3}$

$x = 2\kappa\pi + \frac{2\pi}{3}$

# Εφαρμοσμένα Μαθηματικά

## 23

(12) α β δ

(13) α γ c

(14) α β

(15) α γ .