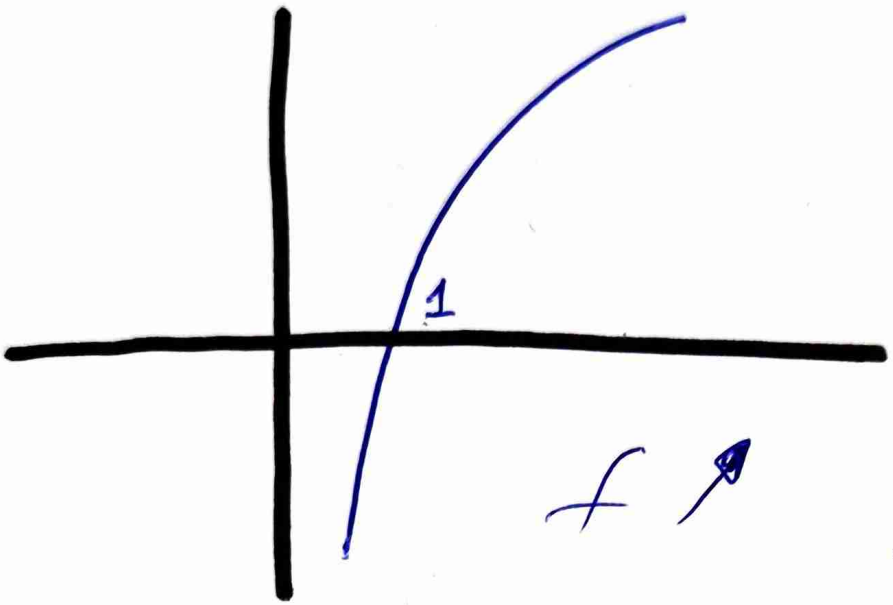
Λογαριθμική Συνάρτηση



$f(x) = \ln x$

ή $f(x) = \log_e x$

$f \nearrow$ $D_f = (0, +\infty)$.

Ενότητα 2.5

1. Ⓑ $f(x) = \ln(x-2) - \ln(3-x)$

πρέπει $x-2>0$ και $3-x>0$

$x>2$ $x<3$

$x \in (2,3)$.

⑤ $f(x) = \ln(x^2 - x + 1)$

прили $x^2 - x + 1 > 0$

$\Delta = 1 - 4 = -3$

$\Delta < 0$

$D_f = \mathbb{R}$.

x	
$x^2 - x + 1$	$+$

⑥2 $f(x) = \ln|x - 1|$

прили $|x - 1| > 0$

$D_f = \mathbb{R} - \{1\}$.

2. ⓑ $f(x) = \ln(1 - e^{-x})$

пресп $1 - e^{-x} > 0$

$1 > e^{-x}$

$\ln 1 > \ln e^{-x}$

$0 > -x$ $\quad D_f = (0, +\infty)$.

$0 < x$

3. $f(x) = \ln\left(x - \frac{1}{x}\right)$

пресп $x - \frac{1}{x} > 0$ $\quad$ кои $\quad x \neq 0$

$\frac{x^2 - 1}{x} > 0$

x		-1		0		1
x^2-1	$+$	$-$	$-$		$+$	
x	$-$		$-$	$+$		$+$
$P(x)$	$-$	$+$	$-$		$+$	

$x \in (-1, 0) \cup (1, +\infty)$

$D_f = (-1, 0) \cup (1, +\infty)$.

4. (8) $f(x) = \dfrac{\ln(1-|x|)}{x}$

при $1-|x|>0$ и $x \neq 0$

$|x|<1$

$-1<x<1$

$D_f = (-1,0) \cup (0,1)$

5. (13) $f(x) = \ln(1+\cos x)$

при $1+\cos x > 0$

$\cos x > -1$ | $\cos x \in [-1;1]$!

$D_f = \mathbb{R}$.

(δ) $f(x)=\ln(e^{2x}+e^{x}-2)$

πρεπει $e^{2x}+e^{x}-2>0$

$(e^{x})^{2}+e^{x}-2>0$

$e^{x}=t$

$t^{2}+t-2>0$

t		-2		1	
t^2+t-2	$+$	0	$-$	0	$+$

$t\in(-\infty,-2)\cup(1,+\infty)$

$t<-2$ ή $t>1$

$e^{x}<-2$ $e^{x}>1$

Αδυνατη $e^{x}>e^{0}$

$x>0$

$D_f=(0,+\infty)$.

6. (B) $f(x) = \ln \dfrac{e^x - 1}{x - 2}$

прим $x - 2 \neq 0$ $\quad$ $\ln \dfrac{e^x - 1}{x - 2} > 0$

$x \neq 2$

x		0		2	
$e^x - 1$	$-$	0	$+$		$+$
$x - 2$	$-$		$-$	0	$+$
$P(x)$	$+$		$-$		$+$

$D_f = (-\infty, 0) \cup (2, +\infty)$.

46. $f(x) = \frac{a}{\ln x}$

(a) Df.

πρέπει $x > 0$

και $\ln x \neq 0$

$e^{\ln x} \neq e^0$

$x \neq 1$

$x \in (0,1) \cup (1,+\infty)$

(β) $M(e,1)$ ανήκει $a = 1$.

$f(e) = 1 \Rightarrow \frac{a}{\ln e} = 1 \Rightarrow \frac{a}{1} = 1 \Rightarrow a = 1$

(γ) $f(x) > \frac{1}{2}$.

$\frac{1}{\ln x} > \frac{1}{2}$

$\frac{1}{\ln x} - \frac{1}{2} > 0$

$\frac{2 - \ln x}{2 \ln x} > 0$

$x \in (1, e^2)$.

x	0	1	e^2	
$2-\ln x$	$+$	$+$	0	$-$
$2\ln x$	$-$	0	$+$	$+$
$P(x)$	$-$	$\|\|$	$+$	$-$

$\ln x = 0$

$x = 1$

$2 - \ln x = e$

$\ln x = 2 \quad x = e^2$

45. $f(x) = a + \ln \frac{x}{1-x}$

(a) преди $1-x \neq 0$ или $\frac{x}{1-x} > 0$

$x \neq 1$.

$D_f = (0,1)$.

x		0		1
x	−	+	+	
1−x	+	+	−	
P(x)	−	+	−	

$x \in (0,1)$

(B) $A\left(\frac{1}{2}, 0\right)$.

$f\left(\frac{1}{2}\right) = 0 \Rightarrow f\left(\frac{1}{2}\right) = a + \ln \frac{\frac{1}{2}}{1-\frac{1}{2}} = 0$

$a + \ln \frac{\frac{1}{2}}{\frac{1}{2}} = 0 \Rightarrow a + \ln 1 = 0$

$a = 0$

$f(x) = \ln \frac{x}{1-x}$

(в) $f(x) > 0 \Rightarrow \ln \frac{x}{1-x} > 0$

$\frac{x}{1-x} > 1 \Rightarrow \frac{x}{1-x} - 1 > 0 \Rightarrow$

$$\frac{x}{1-x} - 1 > 0 \quad \Rightarrow \frac{x}{1-x} - \frac{1-x}{1-x} > 0$$

$$\frac{x-1+x}{1-x} > 0 \quad \Rightarrow \frac{2x-1}{1-x} > 0$$

x		$1/2$	1
$2x-1$	$-$	0 $+$	$+$
$1-x$	$+$	$+$	$-$
$Q(x)$	$-$	$+$	$-$

$x \in (\frac{1}{2}, 1)$.

οpw $D_f = (0,1)$

43. $f(x) = \ln(2e^x - 1)$

(α) Πρεπει $2e^x - 1 > 0$

$2e^x > 1$

$e^x > \frac{1}{2}$

$\ln e^x > \ln \frac{1}{2}$

$x > \ln 1 - \ln 2$

$x > -\ln 2$

$D_f = (-\ln 2, +\infty)$.

(β) Σημεια τομη με αξονες

$x'x$	$y'y$
$f(x) = 0$	$f(0) = \ln(2e^0 - 1)$
$\ln(2e^x - 1) = 0$	$f(0) = \ln 1$
$e^{\ln(2e^x-1)} = e^0$	$f(0) = 0$
$2e^x - 1 = 1$	$A(0, 0)$
$2e^x = 2$	
$e^x = 1$	$\ln e^x = \ln 1$
	$x = 0$

) $f(x) > 0$

$\ln(2e^x - 1) > 0$

$e^{\ln(2e^x - 1)} > e^0$

$2e^x - 1 > 1$

$2e^x > 2$

$e^x > 1$

$\ln e^x > \ln 1$

$x > 0$

44. $f(x) = \ln(\ln(x-1))$

(α) $x-1>0$ και $\ln(x-1)>0$

$x>1$

$e^{\ln(x-1)} > e^0$

$x-1>1$

$x>2$

$D_f = (2, +\infty)$.

(β) $x'x$

$f(x)=0$

$\ln(\ln(x-1))=0$

$\ln(x-1)=1$

$x-1=e$

$x=e+1$

(γ) $f(x)<0$

$x<e+1$

49. $P(x) = 10x^3 - 27x^2 + \alpha$.

Το $x-3$ παραγοντα1

(α) γιά $\alpha = -27$

$P(3) = 0$

$10 \cdot 3^3 - 27 \cdot 3^2 + \alpha = P(3) \Rightarrow$

$\alpha = -27.$

$$P(x) = 10x^3 - 27x^2 - 27$$

(β) Εξίσωση $P(x) = 0$ αφού $(x-3)$ παρ.

10	-27	0	-27	3
↓	30	9	27	
10	3	9	0	

$(x-3)(10x^2+3x+9) = 0$ μόνο για $x=3$.

$\boxed{x=3}$ (+)

(γ) Ανίσωση $\dfrac{(e^x-2)^3}{(e^x-2)^2+1} < \dfrac{27}{10} \Rightarrow$ $e^x - 2 = t$

$$\frac{t^3}{t^2+1} < \frac{27}{10} \Rightarrow 10t^3 < 27t^2 + 27 \Rightarrow$$

$$10t^3 - 27t^2 - 27 < 0$$

$t = 3$

$e^x - 2 = 3$

$e^x = 5$ $\quad x = \ln 5$

47. $f(x) = a\sqrt{1-\ln x}$

(α) D_f

Πρέπει $\sqrt{1-\ln x} \geq 0 \Rightarrow 1-\ln x \geq 0 \Rightarrow \ln x \leq 1 \Rightarrow$

(β) $M(1,1)$ υδο $a=1$.

$f(1)=1 \Rightarrow f(1) = a\sqrt{1-\ln 1} = a\sqrt{1} = a \Rightarrow a=1$

(γ) $f(x) > \sqrt{2}$.

$\sqrt{1-\ln x} > \sqrt{2} \Rightarrow 1-\ln x > 2 \Rightarrow -\ln x > 1 \Rightarrow \ln x < -1$

$e^{\ln x} < e^{-1}$

$x < \frac{1}{e}$

48. $f(x) = \ln \dfrac{e^{2x}-1}{e^x+5}$.

(α) D_f. $e^x+5 \neq 0$ που ισχύει

$\dfrac{e^{2x}-1}{e^x+5} \geq 0$

$e^{2x}-1 > 0$

$e^{2x} > 1$

$2x > 0$

$x > 0$

$x \in (0, +\infty)$

(β) Είδωση $f(x) = 2\ln 2$.

$f(x) = 2\ln 2$

$\ln \dfrac{e^{2x}-1}{e^x+5} = 2\ln 2^2$

$\dfrac{e^{2x}-1}{e^x+5} = 4$

$4e^x + 20 = e^{2x} - 1$

$4e^x + 20 = e^{2x} \cdot e^{-1}$

$4e^x + 21 = e^{2x}$

$e^x = t$

$(e^x)^2 - 4e^x + 21 = 0$

$t^2 - 4t + 21 = 0$

$\Delta = 16 + 84 = 100$

$x = \dfrac{4 \pm 10}{2} < -3$

$e^x = 7$

$x = \ln 7$

$e^x = -3$

Εισαγωγή στη Μαθηματική Λογική

Ενότητα 25

(1) $\alpha \gamma \varepsilon$

(2) $\alpha \beta$

(3) $\alpha \beta$

(4) $\alpha \beta$

(50)

(51)

(52)

(53)

(54)

(55)

(56)

(57)