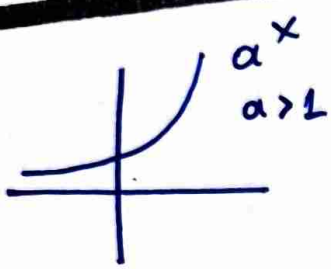
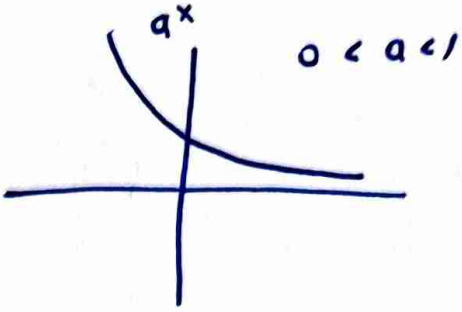
Συνάρτηση

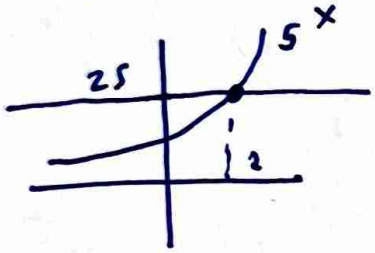
1. 

εκθετική συνάρτηση



2. $a^x = \theta$ τότε $x = \log_a \theta$

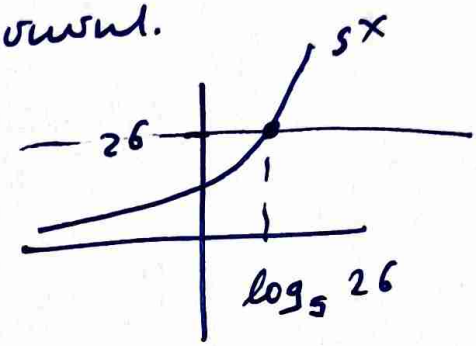
ο λογάριθμος είναι αριθμός και είναι λύση μιας εκθετικής εξίσωσης.



$5^x = 25$

$5^x = 5^?$

$x = ?$



3. $\log_a a^x = x$ $\qquad$ $\log_a 1 = 0$

$a^{\log_a x} = x$ $\qquad$ $\log_a a = 1$

4. $\log_a(\theta_1 \theta_2) = \log_a \theta_1 + \log_a \theta_2$

$\log_a\left(\frac{\theta_1}{\theta_2}\right) = \log_a \theta_1 - \log_a \theta_2$

$\log_a \theta^k = k \log_a \theta$

5. Δεκαδικοί λογάριθμοι

$\log_{10} x = \log x$

$\log 10^x = x$

$10^{\log x} = x$

$\log 1 = 0$

$\log 10 = 1$

$\log \vartheta_1 \vartheta_2 = \log \vartheta_1 + \log \vartheta_2$

$\log \frac{\vartheta_1}{\vartheta_2} = \log \vartheta_1 - \log \vartheta_2$

$\log \vartheta^k = k \log \vartheta.$

$\log_e x = \ln x$

$\ln e^x = x$

$e^{\ln x} = x$

$\ln 1 = 0$

$\ln e = 1$

$\ln \vartheta_1 \vartheta_2 = \ln \vartheta_1 + \ln \vartheta_2$

$\ln \frac{\vartheta_1}{\vartheta_2} = \ln \vartheta_1 - \ln \vartheta_2$

$\ln \vartheta^k = k \ln \vartheta$

Exercici 24

1. (B) $\log_3 81 =$

$= \log_3 3^4 = 4$

(E) $\log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2} = -2$

(52) $\log_5 \frac{1}{\sqrt{5}} = \log_5 \frac{1}{5^{1/2}} = \log_5 5^{-\frac{1}{2}} = -\frac{1}{2}$

2. (B) $\ln x = 3$

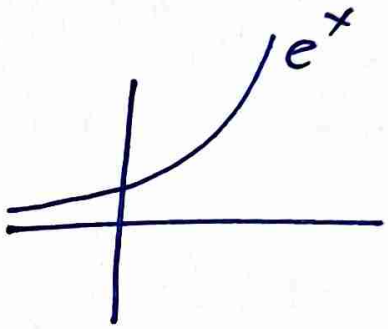
$e^{\ln x} = e^3$

$\boxed{x = e^3}$

(8) $\ln x = -3$

$e^{\ln x} = e^{-3}$

$\underline{x = e^{-3}}$



(52) $2\ln x + 1 = 0$

$2\ln x = -1$

$\ln x = -\frac{1}{2}$

$e^{\ln x} = e^{-\frac{1}{2}}$

$x = \frac{1}{e^{1/2}} = \frac{1}{\sqrt{e}}$

3. (B) $\ln \frac{1}{e} = \ln 1 - \ln e = 0 - 1 = -1$

~~(Δ) $\ln x = -3$~~

(Δ) $\ln e \cdot \sqrt{e} = \ln e + \ln \sqrt{e} = 1 + \ln e^{\frac{1}{2}}$

$= 1 + \frac{1}{2} = \frac{3}{2}$

(E) $\ln \frac{1}{\sqrt[3]{e^2}} = \ln 1 - \ln \sqrt[3]{e^1} =$

$= -\ln e^{\frac{2}{3}} = -\frac{2}{3}$

(Σ) $\ln^2 \frac{1}{e^2} = \left(\ln \frac{1}{e^2}\right)^2 = (\ln 1 - \ln e^2)^2$

$= (-2)^2 = 4$

Ιδιοτικά

$\ln e^x = x$

$e^{\ln x} = x$

$\ln 1 = 0$

$\ln e = 1$

$\ln \frac{\alpha}{\beta} = \ln \alpha - \ln \beta$

$\ln \alpha\beta = \ln \alpha + \ln \beta$

Θέμα 1ο

$P(x) = x^3 - (a+1)x^2 + (a-1)x + 2$

έχει παράγοντα το $x-2$

(α) Για $a = 2$

$P(2) = 2^3 - (a+1)\cdot 2^2 + 2(a-1) + 2 = 0$

$8 - 4(a+1) + 2a - 2 + 2 = 0$

$8 - 4a - 4 + 2a = 0$

$-2a + 4 = 0 \Rightarrow -2a = -4$

$\boxed{a = 2}$

(Β)

$$\begin{array}{r|l} x^3 - 3x^2 + x + 2 & x+3 \\ -(x^3 + 3x^2) & \hline x^2 - 6x + 19 \\ \hline -6x^2 + x + 2 & \\ -(-6x^2 - 18x) & \\ \hline 19x + 2 & \\ -(19x + 57) & \\ \hline -55 & \end{array}$$

(8) $P(x) \leq 17x + 14$

$$x^3 - 3x^2 + x + 2 \leq 17x + 14$$

$$x^3 - 3x^2 + x + 2 - 17x - 14 \leq 0$$

$$x^3 - 3x^2 - 16x - 12 \leq 0$$

1	−3	−16	−12	(−1)
↓	−1	4	12	
1	−4	−12	0	

$$(x+1)(x^2 - 4x - 12) \leq 0$$

(−1) (6) (−2)

x		−2		−1		6	
x+1	−		−	0	+		+
$x^2-4x-12$	+	0	−		−	0	+
P(x)	−		+		−		+

$$x \in (-\infty, -2] \cup [-1, 6].$$

Θεμα 20

$P(1) = 2$

$P(-1) = 8$

(a) $P(x) = (x^2-1)\,M(x) + U(x)$

$\downarrow$ 200 $\qquad \downarrow$ 100

$$\boxed{P(x) = (x^2-1)\,M(x) + \alpha x + \beta}$$

$P(1) = \alpha + \beta = 2$

$P(-1) = -\alpha + \beta = 8$

$\oplus \quad 2\beta = 10 \qquad \underline{\underline{\beta = 5}}$

$\alpha = -3$.

$\underline{\underline{U(x) = -3x+5}}$

(B) $\sqrt{U(x)} = 3x+1$

$\sqrt{-3x+5} = 3x+1$

$-3x+5 \geq 0$

$3x+1 \geq 0$

$-3x+5 = (3x+1)^2$

$-3x+5 = 9x^2+6x+1 \qquad \Delta = 81 + 36 \cdot 4$

$0 = 9x^2+9x-4 \qquad \Delta = 225 \qquad x = \frac{-9 \pm 15}{18}$ $\frac{1}{3}$ $\cancel{-\frac{4}{3}}$

Θεμα 3ο

$P(x) = 2x^4 + x^3 + \alpha x^2 + \beta x + 2$ εχει παραγοντα το $x^2 - 2x + 1$

$\Rightarrow (x-1)^2$

	2	1	α	β	2	(1)
		2	3	$\alpha+3$	$\alpha+\beta+3$	
↓		2	3	$\alpha+3$	$\alpha+\beta+3$	$\alpha+\beta+5=0$
	2					

	2	3	$\alpha+3$	$\alpha+\beta+3$	(2)
↓		2	5	$\alpha+8$	
	2	5	$\alpha+8$	$2\alpha+\beta+11=0$	

$$\begin{cases} \alpha+\beta+5=0 \\ 2\alpha+\beta+11=0 \end{cases}$$

$-\alpha - 6 = 0$

$\alpha = -6$

$-1+\beta = 0$

$\beta = 1$

(B) $2x^4 + x^3 - 6x^2 + x + 2 = 0$

					(1)
	2	1	−6	1	2
↓		2	3	−3	−2
	2	3	−3	−2	0

$(x-1)(2x^3 + 3x^2 - 3x - 2) = 0$

				(1)
	2	3	−3	−2
↓		2	5	2
	2	5	2	0

$(x-1)(x-1)(2x^2 + 5x + 2) = 0$

$(x-1)^2(2x^2 + 5x + 2) = 0$

$\Delta = 25 - 16 = 9$

$x = 1$

$x = \frac{-5 \pm 3}{4}$ → $-\frac{1}{2}$, -2

① $\frac{3x-1}{x+2} \geq 2$

$\frac{3x-1}{x+2} - 2 \geq 0$

$\frac{3x-1}{x+2} - \frac{2(x+2)}{x+2} \geq 0$

$\frac{3x-1-2x-4}{x+2} \geq 0$

$\frac{x-5}{x+2} \geq 0$

x		-2		5
$x-5$	$-$	$-$	$-$	$0+$
$x+2$	$-$	$0+$	$+$	$+$
$P(x)$	$+$	$-$	$+$	

$x \in (-\infty, -2) \cup [5, +\infty)$

Επομενο Μαθημα

24

1. α γ ε
2. α γ ε
3. α γ σε η
   θ ια
5.
6.
7.
8.
9.
10.