67. $f(x) = \varepsilon\varphi x - x + \ln(\sigma\upsilon\nu x)$

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(α) $f'(x) = \dfrac{1}{\sigma\upsilon\nu^2 x} - 1 + \dfrac{-\eta\mu x}{\sigma\upsilon\nu x}$

$$f'(x) = \frac{1 - \sigma\upsilon\nu^2 x - \eta\mu x \, \sigma\upsilon\nu x}{\sigma\upsilon\nu^2 x}$$

$$f'(x) = \frac{\eta\mu^2 x - \eta\mu x \, \sigma\upsilon\nu x}{\sigma\upsilon\nu^2 x}$$

$$f'(x) = \frac{\eta\mu x(\eta\mu x - \sigma\upsilon\nu x)}{\sigma\upsilon\nu^2 x}$$

x	$-\frac{\pi}{2}$		0		$\frac{\pi}{4}$		$\frac{\pi}{2}$
$\eta\mu x$		$-$	0	$+$		$+$	
$\eta\mu x - \sigma\upsilon\nu x$		$-$		$-$	0	$+$	
f'		$+$		$-$		$+$	
f		$\nearrow$		$\searrow$		$\nearrow$	

13 $\quad f\left(\frac{\pi}{2}-2x\right) > f\left(-\frac{\pi}{3}\right) \qquad \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

• $\frac{\pi}{4} < x < \frac{\pi}{2}$

$-2\frac{\pi}{4} > -2x > -2\frac{\pi}{2}$

$-\frac{\pi}{2} > -2x > -\pi$

$0 > \frac{\pi}{2} - 2x > -\frac{\pi}{2}$

$\forall x \in \left(-\frac{\pi}{2}, 0\right)$

$f \nearrow$

• $-\frac{\pi}{2} < -\frac{\pi}{3} < 0$

$\frac{\pi}{2} - 2x > -\frac{\pi}{3}$

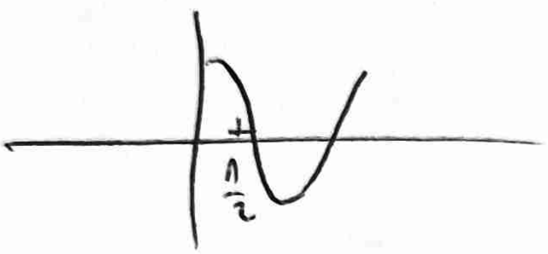
$\frac{\pi}{2} + \frac{\pi}{3} > 2x$

$\frac{\pi}{4} + \frac{\pi}{6} > x$

$x < \frac{5\pi}{6}$.

(γ) $\lim\limits_{x \to \frac{\pi}{2}^-} f(x) = \lim\limits_{x \to \frac{\pi}{2}^-} \left(\overset{+\infty}{εφx} - x + \underset{-\infty}{\ln συνx} \right)$

$\to \lim\limits_{x \to \frac{\pi}{2}^-} ημx \, \dfrac{1}{συνx} = 1 \cdot +\infty = +\infty$



$\lim\limits_{x \to \frac{\pi}{2}^-} εφx \left(1 - \cancel{\dfrac{x}{εφx}} + \cancel{\dfrac{\ln συνx}{εφx}} \right) = +\infty$

$\to \lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{x}{εφx} = 0$

$\to \lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{\ln συνx}{εφx} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\frac{-ημx}{συνx}}{\frac{1}{συν^2x}} = \lim\limits_{x \to \frac{\pi}{2}} -ημx\,συνx$ ... 0

DLH

49 $\quad \forall x>0 \quad \text{νδο} \quad 3\text{ημ}x < x(2+\text{συν}x)$

$$\underbrace{\frac{3\text{ημ}x}{2+\text{συν}x} - x}_{f(x)} < 0 \Rightarrow \quad f(x)<0$$

$$f(x)<f(0)$$

$$f \downarrow \checkmark$$

$$x>0$$

$$f'(x) = \frac{3\text{συν}x(2+\text{συν}x)+3\text{ημ}^2x}{(2+\text{συν}x)^2} - 1$$

$$f'(x) = \frac{6\text{συν}x+3\text{συν}^2x+3\text{ημ}^2x-(2+\text{συν}x)^2}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{6\text{συν}x+3(\text{ημ}^2x+\text{συν}^2x)-(4+4\text{συν}x+\text{συν}^2x)}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{6\text{συν}x+3-4-4\text{συν}x-\text{συν}^2x}{(2+\text{συν}x)^2}$$

$$f'(x) = \frac{-\cos^2 x + 2\cos x - 1}{(2+\cos x)^2}$$

$$f'(x) = -\frac{\cos^2 x - 2\cos x + 1}{(2+\cos x)^2}$$

$$f'(x) = \frac{-(\cos x - 1)^2}{(2+\cos x)^2} \le 0$$

f [illegible]

66. $f(x) = e^x - \frac{x^2}{2} + \sigma\upsilon\nu x$

α1 $f'(x) = e^x - \frac{1}{2}2x - \eta\mu x$

$f'(x) = e^x - x - \eta\mu x$.

$f'(x) = \underbrace{e^x - x - 1}_{\oplus} + \underbrace{1 - \eta\mu x}_{\oplus} > 0$ $\quad f \nearrow$

- $e^x \geq x + 1$

$e^x - x - 1 \geq 0$

- $-1 \leq \eta\mu x \leq 1$

$1 \geq -\eta\mu x \geq -1$

$2 \geq 1 - \eta\mu x \geq 0$

β $x^2 + 2f(\varepsilon\varphi x) > 2(e^x + \sigma\upsilon\nu x)$ $\quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\frac{x^2}{2} + f(\varepsilon\varphi x) > e^x + \sigma\upsilon\nu x$

$f(\varepsilon\varphi x) > e^x + \sigma\upsilon\nu x - \frac{x^2}{2}$

$f(\varepsilon\varphi x) > f(x)$.

$f \nearrow$

$\varepsilon\varphi x > x$.

$$\underbrace{\varepsilon\varphi x - x}_{g(x)} > 0 \Rightarrow g(x) > 0$$

$$g'(x) = \frac{1}{\sigma\upsilon\nu^2 x} - 1 = \frac{1-\sigma\upsilon\nu^2 x}{\sigma\upsilon\nu^2 x} = \frac{\eta\mu^2 x}{\sigma\upsilon\nu^2 x} \geq 0$$

$g \nearrow$

$g(x) > g(0)$

$g \nearrow$

$\frac{\pi}{2} > x > 0$

(8) $$\lim_{x\to 1} \frac{\ln x}{(x-1)\left[f(\ln x) - f(x-1)\right]}$$

$$= \lim_{x\to 1} \frac{\ln x}{x-1} \cdot \frac{1}{f(\ln x) - f(x-1)} = 1\cdot(-\infty) = -\infty$$

$$\rightarrow \lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x\to 1} \frac{\frac{1}{x}}{1} = 1$$

- $\ln x \leq x-1$

$f(\ln x) \leq f(x-1)$

$f(\ln x) - f(x-1) \leq 0$

59. $f(x) = x^3 - 3\ln x \quad , x>0$

(α) $f'(x) = 3x^2 - \frac{3}{x} = \frac{3x^3-3}{x} = \frac{3(x^3-1)}{x}$

$\rightarrow x^3-1=0 \Rightarrow x^3=1 \quad x=1.$

x	0		1	
f'		$-$	0	$+$
f		$\searrow$		$\nearrow$

$f(x) \geq f(1)$

$f(x) \geq 1$

(β) $\ln σφ^3 x < συν^3 x - ημ^3 x \qquad \left(0, \frac{\pi}{2}\right)$

$\ln \frac{συν^3 x}{ημ^3 x} < συν^3 x - ημ^3 x$

$\ln συν^3 x - \ln ημ^3 x < συν^3 x - ημ^3 x$

$\ln (συν x)^3 - \ln (ημ x)^3 < συν^3 x - ημ^3 x$

$3\ln συν x - 3\ln ημ x < συν^3 x - ημ^3 x$

$ημ^3 x - 3\ln ημ x < συν^3 x - 3\ln συν x$

$f(ημ x) < f(συν x)$

Το $ημx \in (-1,1)$ και $συνx \in (-1,1)$

$\forall x<1$ η $f \downarrow$ $\quad x \in \left(0, \frac{\pi}{2}\right)$

$ημx > συνx$

$\frac{ημx}{συνx} > 1 \Rightarrow εφx > 1$

$εφx > εφ\frac{\pi}{4}$

$εφx \nearrow$

$\frac{\pi}{2} > x > \frac{\pi}{4}$

(γ) $f(x^2+3) + 3\ln(x^4+1) = (x^4+1)^3$

$f(x^2+3) = (x^4+1)^3 - 3\ln(x^4+1)$

$f(x^2+3) = f(x^4+1)$

$\left.\begin{array}{l} \cdot\ x^2+3>1 \\ \cdot\ x^4+1>1 \end{array}\right\} \forall x>1$ η $f \nearrow$

$x^2+3 = x^4+1 \Rightarrow x^4 - x^2 - 2 = 0$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

$x^2 = -1$ Αδύν

Ενοτητα 24

58. $f(x) = (x-1)\ln x \quad , x>0$

(α) $f'(x) = \ln x + \dfrac{x-1}{x}$

$$\boxed{f'(x) = \ln x + 1 - \frac{1}{x}}$$

$f'(1) = 0$

$f''(x) = \dfrac{1}{x} + \dfrac{1}{x^2} > 0$

x		1	
f''	+		+
f'	↗ −	0	↗ +
f	↘		↗

$f(x) \geq f(1)$

$\underline{f(x) \geq 0}$

(B) Νδο $(a+1)^{a} > a^{a-1}$ $\quad \forall a>1$

$$a\ln(a+1) > (a-1)\ln a$$

$$f(a+1) > f(a)$$

$$\left.\begin{array}{l} \bullet\ a>1 \Rightarrow a+1>2 \\ \bullet\ a>1 \end{array}\right\} \forall x>1 \ \eta\ f \nearrow$$

$\bullet\ f \nearrow$

$$a+1>a$$

$$1>0 \ \checkmark$$

(Γ) $\dfrac{(\eta\mu x)^{\eta\mu x}}{(\sigma\upsilon\nu x)^{\sigma\upsilon\nu x}} = \sigma\varphi x$ $\quad \left(0, \frac{\pi}{2}\right)$

$$\ln(\eta\mu x)^{\eta\mu x} - \ln(\sigma\upsilon\nu x)^{\sigma\upsilon\nu x} = \ln\frac{\eta\mu x}{\sigma\upsilon\nu x}$$

$$\eta\mu x \ln \eta\mu x - \sigma\upsilon\nu x \ln \sigma\upsilon\nu x = \ln \eta\mu x - \ln \sigma\upsilon\nu x$$

$$(\eta\mu x - 1)\ln \eta\mu x = (\sigma\upsilon\nu x - 1)\ln \sigma\upsilon\nu x$$

$f(ημx) = f(συνx)$

- $ημx \in (-1, 1)$
- $συνx \in (-1, 1)$.

$\Big\}$ $\forall x < 1$ κ $f\downarrow$

$ημx = συνx$ στο $\left(0, \frac{\pi}{2}\right)$

$x = \frac{\pi}{4}$

Επομενο Μαθημα

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