

Τετραγωνική Ρίζα

Τετραγωνική ρίζα ενός μη αρνητικού αριθμού α , είναι ο μη αρνητικός αριθμός x ώστε $x^2 = \alpha$.

$$\Rightarrow x^2 = \alpha \quad \Rightarrow x = \sqrt{\alpha}$$

π.χ $x^2 = 36 \quad \Rightarrow x = \sqrt{36}$

$$\underline{\underline{x = 6}}$$

Διασπορά Ρίζα

$$\sqrt{0} = 0$$

$$\sqrt{36} = 6$$

$$\sqrt{144} = 12$$

$$\sqrt{324} = 18$$

$$\sqrt{1} = 1$$

$$\sqrt{49} = 7$$

$$\sqrt{169} = 13$$

$$\sqrt{361} = 19$$

$$\sqrt{4} = 2$$

$$\sqrt{64} = 8$$

$$\sqrt{196} = 14$$

$$\sqrt{400} = 20$$

$$\sqrt{9} = 3$$

$$\sqrt{81} = 9$$

$$\sqrt{225} = 15$$

$$\sqrt{441} = 21$$

$$\sqrt{16} = 4$$

$$\sqrt{100} = 10$$

$$\sqrt{256} = 16$$

$$\sqrt{484} = 22$$

$$\sqrt{121} = 11$$

$$\sqrt{289} = 17$$

$$\sqrt{529} = 23$$

$$\sqrt{25} = 5$$

$$\sqrt{576} = 24$$

$$\sqrt{625} = 25$$

Ιδιότητες Ριζών

$$1. \sqrt{x}^2 = x$$

$$2. \sqrt{x^2} = |x|$$

$$3. \sqrt{a} \sqrt{b} = \sqrt{a \cdot b}$$

$$4. \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$5. \sqrt{x} \quad , \quad x \geq 0$$

$$6. \sqrt{x} \geq 0$$

Евклид 9

$$2. \textcircled{8} \sqrt{8 \sqrt{1 + \sqrt{2 + \sqrt{49}}}}} =$$

$$= \sqrt{8 \sqrt{1 + \sqrt{2 + 7}}}}$$

$$= \sqrt{8 \sqrt{1 + \sqrt{9}}}}$$

$$= \sqrt{8 \sqrt{1 + 3}}$$

$$= \sqrt{8 \sqrt{4}}$$

$$= \sqrt{8 \cdot 2}$$

$$= \sqrt{16}$$

$$= 4.$$

$$3. \textcircled{B} B = \sqrt{(\sqrt{2}-1)^2} + \sqrt{(\sqrt{2}-2)^2}$$

$$B = |\sqrt{2}^{\oplus} - 1| + |\sqrt{2}^{\ominus} - 2|$$

$$B = \sqrt{2} - 1 - \sqrt{2} + 2 = 1$$

$$B = 1.$$

$$\textcircled{8} \sqrt{\frac{x^2}{25}} + \sqrt{x^2 - 4x + 4} = \sqrt{\left(\frac{x}{5}\right)^2} + \sqrt{(x-2)^2}$$

$$= \left|\frac{x}{5}\right| + |x-2|$$

$$7. \text{ Nds } \left(\sqrt{x^2+3} - \sqrt{x^2+1} \right) \left(\sqrt{x^2+3} + \sqrt{x^2+1} \right) = 2$$

$$\cancel{\sqrt{x^2+3}}^2 - \cancel{\sqrt{x^2+1}}^2 = 2$$

$$x^2+3 - (x^2+1) = 2$$

$$\cancel{x^2}+3 - \cancel{x^2} - 1 = 2$$

$$3-1=2$$

$$2=2 \quad \checkmark$$

$$8. \text{ (1) } \frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$$

$$\text{(52)} \quad \frac{\sqrt{8} \sqrt{75}}{\sqrt{150}} = \frac{\cancel{\sqrt{2}} \sqrt{4} \sqrt{3} \sqrt{25}}{\sqrt{2} \sqrt{75}} =$$

$$= \frac{2 \cdot \cancel{\sqrt{3}} \cdot 5}{\cancel{\sqrt{3}} \sqrt{25}} = \frac{2 \cdot \cancel{5}}{5} = 2$$

$$16. A = \frac{\sqrt{20} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{45} - 2\sqrt{18} + 3\sqrt{27}}$$

$$A = \frac{\sqrt{4}\sqrt{5} - 2\sqrt{2}\sqrt{4} + 3\sqrt{3}\sqrt{4}}{\sqrt{5}\sqrt{9} - 2\sqrt{2}\sqrt{9} + 3\sqrt{3}\sqrt{9}}$$

$$A = \frac{2\sqrt{5} - 4\sqrt{2} + 6\sqrt{3}}{3\sqrt{5} - 6\sqrt{2} + 9\sqrt{3}}$$

$$A = \frac{2(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}{3(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}$$

$$A = \frac{2}{3},$$

12. (B) $\sqrt{7} \cdot \sqrt{3-\sqrt{2}} \cdot \sqrt{3+\sqrt{2}} = 7$

$$\sqrt{7 \cdot (3-\sqrt{2})(3+\sqrt{2})} = 7$$

$$\sqrt{7(3^2 - \sqrt{2}^2)} = 7$$

$$\sqrt{7 \cdot (9-2)} = 7$$

$$\sqrt{7 \cdot 7} = 7$$

$$\sqrt{49} = 7$$

$$7 = 7.$$

$$\textcircled{8} \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \sqrt{2-\sqrt{2+\sqrt{3}}} = 1$$

$$\sqrt{(2+\sqrt{3})(2+\sqrt{2+\sqrt{3}})(2-\sqrt{2+\sqrt{3}})} = 1$$

$$\sqrt{(2+\sqrt{3})(2^2 - \sqrt{2+\sqrt{3}}^2)} = 1$$

$$\sqrt{(2+\sqrt{3})(4 - (2+\sqrt{3}))} = 1$$

$$\sqrt{(2+\sqrt{3})(4-2-\sqrt{3})} = 1$$

$$\sqrt{(2+\sqrt{3})(2-\sqrt{3})} = 1$$

$$\sqrt{2^2 - \sqrt{3}^2} = 1$$

$$\sqrt{1} = 1 \checkmark$$

$$17. \textcircled{B} \frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4$$

$$\frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = 4$$

$$\frac{5+\sqrt{15}}{\sqrt{5}^2-\sqrt{3}^2} - \frac{\sqrt{15}-\sqrt{9}}{\sqrt{5}^2-\sqrt{3}^2} = 4$$

$$\frac{5+\sqrt{15}}{2} - \frac{\sqrt{15}-3}{2} = 4$$

$$\frac{5+\cancel{\sqrt{15}}-\cancel{\sqrt{15}}+3}{2} = 4$$

$$\frac{8}{2} = 4 \quad \checkmark$$

$$8) \quad \frac{3}{3-2\sqrt{2}} - \frac{2\sqrt{2}}{3+2\sqrt{2}} = 17.$$

$$\frac{3(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} - \frac{2\sqrt{2}(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})} = 17$$

$$\frac{9+6\sqrt{2}}{3^2-(2\sqrt{2})^2} - \frac{6\sqrt{2}-4\sqrt{2}^2}{3^2-(2\sqrt{2})^2} = 17$$

$$\frac{9+6\sqrt{2}}{9-8} - \frac{6\sqrt{2}-8}{9-8} = 17$$

$$9+6\sqrt{2} - (6\sqrt{2}-8) = 17$$

$$9 + \cancel{6\sqrt{2}} - \cancel{6\sqrt{2}} + 8 = 17$$

$$17 = 17$$

$$13. \textcircled{8} \quad \frac{3}{\sqrt{8}} = \frac{3\sqrt{8}}{\sqrt{8}^2} = \frac{3\sqrt{8}}{8}$$

$$14. \textcircled{8} \quad \frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{(\sqrt{5}-2)(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$= \frac{(\sqrt{5}-2)^2}{\cancel{\sqrt{5}^2-2^2}} = \frac{\sqrt{5}^2 - 2 \cdot \sqrt{5} \cdot 2 + 2^2}{5-4}$$

$$= \frac{5 - 4\sqrt{5} + 4}{1} = 9 - 4\sqrt{5}$$

$$15. \textcircled{8} \quad \frac{\sqrt{2}}{\sqrt{50}-\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{25}-\sqrt{3}\sqrt{4}}$$

$$= \frac{\sqrt{2}}{5\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}(\sqrt{2}+2\sqrt{3})}{(\sqrt{2}-2\sqrt{3})(\sqrt{2}+2\sqrt{3})}$$

$$= \frac{5 \cdot \sqrt{4} + 2\sqrt{6}}{(\sqrt{2})^2 - (2\sqrt{3})^2} = \frac{10+2\sqrt{6}}{50-12} = \frac{10+2\sqrt{6}}{38}$$

4. Av $x = 2\sqrt{3} - 1$
 $y = 2\sqrt{3} + 1$

$$\rightarrow x^2 - xy = (2\sqrt{3} - 1)^2 - (2\sqrt{3} - 1)(2\sqrt{3} + 1)$$

$$= (2\sqrt{3})^2 - 2 \cdot 2\sqrt{3} \cdot 1 + 1^2 - ((2\sqrt{3})^2 - 1^2)$$

$$= 12 - 4\sqrt{3} + 1 - (4 \cdot 3 - 1)$$

$$= 12 - 4\sqrt{3} + 1 - 12 + 1$$

$$= 2 - 4\sqrt{3},$$

$$35. \textcircled{B} B = \frac{\sqrt{x^2 - 12x + 36}}{2x - 12}$$

Προϋ $2x - 12 \neq 0$ και $x^2 - 12x + 36 \geq 0$

$$2x \neq 12$$

$$\underline{\underline{x \neq 6}}$$

$$(x - 6)^2 \geq 0$$

Ορίεται $\forall x \neq 6$.

$$\textcircled{\delta} \Delta = \frac{x^4 - 1}{\sqrt{x^4 + 2x^2 + 1}}$$

προϋ $x^4 + 2x^2 + 1 \neq 0$

$$(x^2 + 1)^2 \neq 0$$

✓

και $x^4 + 2x^2 + 1 > 0$

$$(x^2 + 1)^2 > 0$$

Ορίεται $\forall x \in \mathbb{R}$.

✓

36. $\underline{\underline{1 < x < 2}}$

$$A = \frac{\sqrt{x^2 - 2x + 1}}{x - 1} - \frac{\sqrt{x^2 - 4x + 4}}{x - 2}$$

$$A = \frac{\sqrt{(x-1)^2}}{x-1} - \frac{\sqrt{(x-2)^2}}{x-2}$$

$$A = \frac{|x-1|^{\oplus}}{x-1} - \frac{|x-2|^{\ominus}}{x-2}$$

• $1 < x < 2 \Rightarrow 0 < x-1 < 1$

• $1 < x < 2 \Rightarrow -1 < x-2 < 0$

$$A = \frac{\cancel{x-1}}{\cancel{x-1}} - \frac{-x+2}{x-2}$$

$$A = 1 + \frac{\cancel{x-2}}{\cancel{x-2}}$$

$$A = 1 + 1 = 2'$$

$$A = |x+1|^{\oplus} - |2x-1|^{\ominus}$$

$$\bullet -1 < x < \frac{1}{3} \Rightarrow 0 < x+1 < \frac{4}{3}$$

$$\bullet -1 < x < \frac{1}{3} \Rightarrow -2 < 2x < \frac{2}{3}$$

$$-3 < 2x-1 < -\frac{1}{3}$$

$$A = x+1 - (-2x+1)$$

$$A = x+1 + 2x-1$$

$$\underline{\underline{A = 3x}}$$

$$37. \quad d(3x, -1) < 2$$

$$(1) \quad A = \sqrt{x^2 + 2x + 1} - \sqrt{2 - 4x + 4x^2}$$

$$|3x - (-1)| < 2$$

$$|3x + 1| < 2$$

$$-2 < 3x + 1 < 2$$

$$-3 < 3x < 1$$

$$\boxed{-1 < x < \frac{1}{3}}$$

$$A = \sqrt{(x+1)^2} - \sqrt{(2x-1)^2}$$

$$A = |x+1| - |2x-1|$$

⑥

$$\underline{-1 < x < \frac{1}{3}}$$

$$B = \frac{\sqrt{x^2 + 4x + 4}}{x+2} - \frac{9x^2 - 1}{\sqrt{1 - 6x + 9x^2}}$$

$$B = \frac{\sqrt{(x+2)^2}}{x+2} - \frac{9x^2 - 1}{\sqrt{(3x-1)^2}}$$

$$B = \frac{\overset{+}{|x+2|}}{x+2} - \frac{(3x-1)(3x+1)}{\underset{-}{|3x-1|}}$$

$$\bullet -1 < x < \frac{1}{3} \Rightarrow 1 < 2+x < 2+\frac{1}{3}$$

$$\bullet -1 < x < \frac{1}{3} \Rightarrow -3 < 3x < 1 \Rightarrow -4 < 3x-1 < 0$$

$$B = \frac{\cancel{x+2}}{\cancel{x+2}} - \frac{(\cancel{3x-1})(3x+1)}{-\cancel{(3x-1)}} = 1 + 3x + 1 = 2 + 3x$$

Ασκήσιμ

Ευοτντα 1

①

② $\alpha \gamma \beta$

③ α

④

⑤

⑧ $\alpha \beta \delta \epsilon$

⑨

⑫ $\alpha \gamma$

⑬ $\alpha \beta$

⑭ $\alpha \beta \gamma$

⑮ $\alpha \beta$

⑯ $\alpha \gamma$

⑰ $\alpha \gamma$

⑳ $\alpha \gamma$