7. (β) $9^x - 8 \cdot 3^x - 9 = 0$

$(3^2)^x - 8 \cdot 3^x - 9 = 0$

$(3^x)^2 - 8 \cdot 3^x - 9 = 0$

$3^x = t$

$(a^\nu)^\mu = a^{\nu\mu}$

$t^2 - 8t - 9 = 0$

$t = 9$ $\qquad$ $t = -1$

$3^x = 9$ $\qquad$ $3^x = -1$

$3^x = 3^2$ $\qquad$ Αδύνατη

$x = 2$

(δ) $3 \cdot 2^{4x} - 5 \cdot 4^x = 2$

$3 \cdot (2^x)^4 - 5 \cdot (2^x)^2 - 2 = 0$ $\qquad$ $2^x = t$

$3 \cdot t^4 - 5t^2 - 2 = 0$

$t^2 = \lambda$

$3\lambda^2 - 5\lambda - 2 = 0.$

$\Delta = 25 + 24 = 49$

$\lambda = \frac{5 \pm 7}{6}$ → (2), $-\frac{1}{3}$

$t^2 = 2$ $\qquad$ $t^2 = -\frac{1}{3}$

$(2^x)^2 = 2$ $\qquad$ Αδύνατη

$2^{2x} = 2^1$

$2x = 1$ $\qquad$ $x = \frac{1}{2}$

6. (B) $9 \cdot 3^{x-1} + 3^{x+2} = 36$

$9 \cdot 3^{x} \cdot 3^{-1} + 3^{x} \cdot 3^{2} = 36$ $\quad a^{v+\mu} = a^{v} \cdot a^{\mu}$

$3^{x} = t$

$9 \cdot t\,\frac{1}{3} + t \cdot 9 = 36$

$3t + 9t = 36$

$12t = 36$

$t = 3$

$3^{x} = 3$

$3^{x} = 3^{1} \Rightarrow x = 1$

(Δ) $2^{x+1} + 2^{x+2} + 2^{x-1} + 2^{x-2} = 54$

$2^{x} \cdot 2^{1} + 2^{x} \cdot 2^{2} + 2^{x} \cdot 2^{-1} + 2^{x} \cdot 2^{-2} = 54$

$2t + 4t + \frac{1}{2}t + \frac{1}{4}t = 54$ $\quad 2^{x} = t$

$6t + \frac{3t}{4} = 54 \Rightarrow 24t + 3t = 4 \cdot 54$

$27t = 216 \Rightarrow t = 8 \Rightarrow 2^{x} = 2^{3}$ $\quad x = 3$

Εντυπα 23

1. (α) $5^x = 25$

$5^x = 5^2$

$x = 2$

(β) $3^x - 1 = 0$

$3^x = 1$

$3^x = 3^0$

$x = 0$

(ε) $\left(\frac{1}{3}\right)^x = \frac{1}{9} \Leftrightarrow \left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^2 \Rightarrow x = 2$

2. (α) $3^x = \frac{1}{9} \Rightarrow 3^x = \left(\frac{1}{3}\right)^2 \Rightarrow 3^x = 3^{-2}$

$x = -2$

3. (α) $49^x = 7$

$(7^2)^x = 7^1$

$7^{2x} = 7^1$

$2x = 1$

$x = \frac{1}{2}$

(β) $4^x = \frac{1}{8}$

$(2^2)^x = \left(\frac{1}{2}\right)^3$

$2^{2x} = 2^{-3}$

$2x = -3$

$x = -\frac{3}{2}$

(ε) $\sqrt{2} = 2^1 \cdot 2^x$

$2^{1/2} = 2^{1+x}$

$\frac{1}{2} = 1 + x$

$x = -\frac{1}{2}$

$\sqrt[\mu]{a^\nu} = a^{\nu/\mu}$

4. (α) $e^x - e^2 = 0$

$e^x = e^2$

$x = 2$

(γ) $e^{x-1} = \sqrt{e}$

$e^{x-1} = e^{1/2}$

$x - 1 = \frac{1}{2}$

$x = \frac{3}{2}$

5. (α) $2^x + 1 = 0$

$2^x = -1$

Αδύνατη.

(γ) $13^{x^2-4x} = 1$

$13^{x^2-4x} = 13^0$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ $x = 4$.

9. ⑧ $3^{x+2} + 3^{1-x} = 28$

$3^x \cdot 3^2 + 3^1 \cdot 3^{-x} = 28$

$9 \cdot 3^x + 3 \cdot \frac{1}{3^x} = 28$

$3^x = t$

$9t + 3\frac{1}{t} = 28$

$9t^2 + 3 = 28t$

$9t^2 - 28t + 3 = 0$

$t = \frac{28 \pm 26}{18}$

$\frac{1}{9}$

3

$t = 3$

$3^x = 3$

$x = 1$

$t = \frac{1}{9}$

$3^x = \left(\frac{1}{3}\right)^2$

$3^x = 3^{-2}$

$x = -2$

8. (β) $e^{2x} - (e+1)e^x + e = 0$

$e^{2x} - e \cdot e^x - e^x + e = 0$

$e^x(e^x - e) - (e^x - e) = 0$

$(e^x - e)(e^x - 1) = 0$

$e^x - e = 0$ ή $e^x - 1 = 0$

$e^x = e^1$ $e^x = e^0$

$x = 1$ $x = 0$

(δ) $e^{2x} + 3 = 4e^{-2x}$

$(e^x)^2 + 3 = 4\frac{1}{e^{2x}}$ $\Rightarrow (e^x)^2 + 3 = \frac{4}{(e^x)^2}$

$e^x = t$

$t^2 + 3 = \frac{4}{t^2}$ $\Rightarrow t^4 + 3t^2 - 4 = 0$

$t^2 = \lambda$

$\lambda^2 + 3\lambda - 4 = 0$

$\lambda = -4$ $\lambda = 1$

$t^2 = -4$ αδύν. $t^2 = 1$ $t = 1$ ή $t = -1$

$e^x = 1$ $e^x = -1$ Αδύν.

$x = 0$

9. ⓐ $2^{x+2} + 3 = 2^{-x}$

$a^{\nu+\mu} = a^{\nu} \cdot a^{\mu}$

$a^{-\nu} = \frac{1}{a^{\nu}}$

$2^x \cdot 2^2 + 3 = \frac{1}{2^x}$

$2^x = t$

$4t + 3 = \frac{1}{t}$

$4t^2 + 3t = 1$

$4t^2 + 3t - 1 = 0$

$\Delta = 9 + 16 = 25$

$t = \frac{-3 \pm 5}{8}$ → $\frac{1}{4}$, -1

$a^x > 0$

$t = \frac{1}{4}$ ή $t = -1$

$2^x = -1$

Αδύνατη.

$2^x = \frac{1}{4}$

$2^x = \frac{1}{2^2}$

$2^x = 2^{-2}$

$x = -2$

(B) $2^{1-x} - 2^{1+x} = 3$

$2^1 \cdot 2^{-x} - 2^1 \cdot 2^x = 3$

$2 \cdot \frac{1}{2^x} - 2 \cdot 2^x = 3$

$2^x = t$

$2 \cdot \frac{1}{t} - 2 \cdot t = 3$

$2 - 2t^2 = 3t$

$-2t^2 - 3t + 2 = 0$

$\Delta = 9 - 4(-2) \cdot 2$

$\Delta = 9 + 16 = 25$

$t = \frac{3 \pm 5}{-4}$ → -2, $\frac{1}{2}$

$t = -2$

$2^x = -2$

Άδικη

$t = \frac{1}{2}$

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

12. ① $5^{-x} < 5$

$5^{-x} < 5^1$

$-x < 1$

$x > -1$

$a^0 = 1$

ε) $5^{x^2} - 1 < 0$

$5^{x^2} < 1$

$5^{x^2} < 5^0$

$x^2 < 0$

Δύναται.

σ2) $e^{x^2} \leq \frac{1}{e^x}$

$e^{x^2} \cdot e^x \leq 1$

$e^{x^2+x} \leq e^0$

$e \approx 2,7$

$x^2 + x \leq 0$

x		-1		0
x^2+x	$+$	$\|$	$-$	$\|$ $+$

$x \in [-1, 0]$.

13. (B) $e^x > \frac{1}{e} \Rightarrow e^x > e^{-1} \Rightarrow x > -1$

(δ) $e^{-x} < \sqrt{e} \Rightarrow e^{-x} < e^{1/2}$

$\sqrt{x} = x^{1/2}$

$-x < \frac{1}{2}$

$x > \frac{1}{2}$.

(σz) $e^{-x} - e^{2x} \leq 0$

$e^{-x} \leq e^{2x}$

$-x \leq 2x$

$-3x \leq 0$

$3x \geq 0$

$x \geq 0$.

14. (1) $\left(\frac{3}{4}\right)^x \geq \frac{16}{9}$

$$\left(\frac{3}{4}\right)^x \geq \left(\frac{4}{3}\right)^2$$

$$\left(\frac{3}{4}\right)^x \geq \left(\frac{3}{4}\right)^{-2}$$

$$x \leq -2$$

15. (3) $e^{2x} - e \cdot e^x < 0$

$$(e^x)^2 - e \cdot e^x < 0$$

Θέτω $e^x = t$

$(e^x)^2 = (e^2)^x = e^{2x}$

$(a^v)^\mu = a^{v\mu}$

$$t^2 - e \cdot t < 0$$

$$t(t-e) < 0$$

$t = 0$ $\quad$ $t = e$

t		0		e	
$t^2 - et$	$+$		$-$		$+$

$0 < t < e$ $\qquad$ $t \in (0, e)$.

$0 < e^x < e$ $\qquad$ $e^x < e^1$

$x < 1$.

15.

(8) $9^x - 4 \cdot 3^x + 3 > 0$

$(3^2)^x - 4 \cdot 3^x + 3 > 0$

$(3^x)^2 - 4 \cdot 3^x + 3 > 0$

$3^x = t$

$t^2 - 4t + 3 > 0$

t		1		3	
t^2-4t+3	$+$		$-$		$+$

$t \in (-\infty, 1) \cup (3, +\infty)$

$t < 1$ ni $t > 3$

$3^x < 1$ $\qquad$ $3^x > 3$

$3^x < 3^0$ $\qquad$ $3^x > 3^1$

$x < 0$ $\qquad$ $x > 1$

$x \in (-\infty, 0) \cup (1, +\infty)$

Άσκηση SOS

Δίνεται συνάρτηση $f(x) = \left(\frac{\lambda+4}{2-\lambda}\right)^x$

Να βρεθεί λ $(\lambda \neq 2)$ ώστε η C_f

1. Να έχει πεδίο ορισμού το $\mathbb{R}$

Πρέπει $\frac{\lambda+4}{2-\lambda} > 0$

λ		-4		2
$\lambda+4$	$-$	0	$+$	$+$
$2-\lambda$	$+$	$+$	0	$-$
$\frac{\lambda+4}{2-\lambda}$	$-$	0	$+$	$-$

$\lambda \in (-4, 2)$

2. Να είναι εκθετική

Πρέπει $\frac{\lambda+4}{2-\lambda} > 0$ και $\frac{\lambda+4}{2-\lambda} \neq 1$

$\lambda \in (-4, 2)$

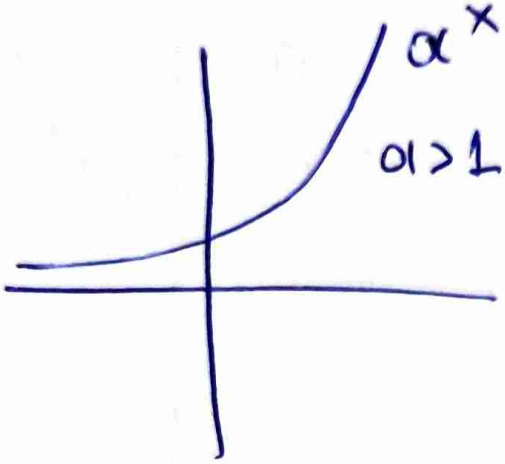
$\lambda + 4 \neq 2 - \lambda$

$2\lambda \neq -2$

$\lambda \neq -1$.

$\lambda \in (-4, -1) \cup (-1, 2)$

3. Να λυθεί η ανίσωση



πρέπει $\frac{\lambda+4}{2-\lambda} > 1$

$$\frac{\lambda+4}{2-\lambda} - 1 > 0$$

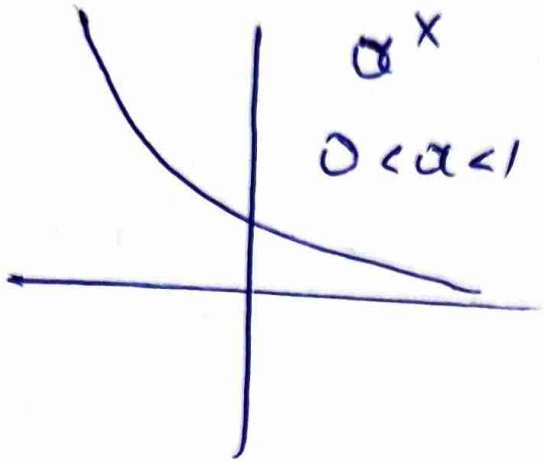
$$\frac{\lambda+4}{2-\lambda} - \frac{2-\lambda}{2-\lambda} > 0 \Rightarrow \frac{\lambda+4-2+\lambda}{2-\lambda} > 0$$

$$\frac{2\lambda+2}{2-\lambda} > 0 \Rightarrow \frac{2(\lambda+1)}{2-\lambda} > 0$$

λ		-1		2	
$\lambda+1$	$-$	0	$+$	$+$	
$2-\lambda$	$+$	$+$	0	$-$	
$\frac{2\lambda+2}{2-\lambda}$	$-$	$+$	0	$-$	

$\lambda \in (-1, 2)$

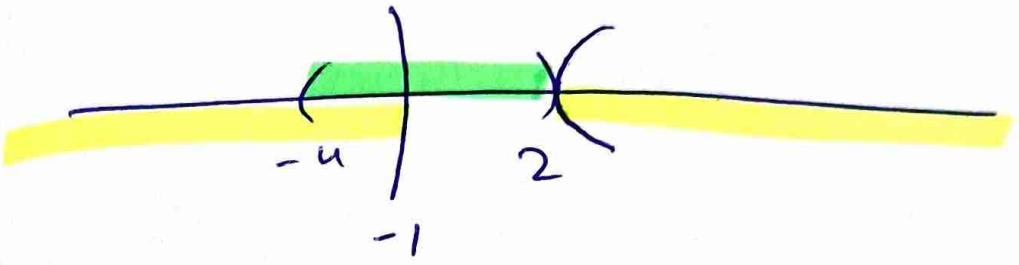
4. Να επιλυθεί η ανίσωση



πρέπει $0<\frac{\lambda+4}{2-\lambda}<1$

$\frac{\lambda+4}{2-\lambda}>0$ και $\frac{\lambda+4}{2-\lambda}<1$

$\lambda\in(-4,2)$ $\lambda\in(-\infty,-1)\cup(2,+\infty)$



$\lambda\in(-4,-1)$.

Θεμα 7

$$f(x) = \frac{\alpha x}{x^2+1} \qquad A\left(-1, \frac{1}{2}\right)$$

(α) $\frac{1}{2} = \frac{\alpha(-1)}{(-1)^2+1} \iff \frac{1}{2} = \frac{-\alpha}{2}$

$\alpha = -1$

$$\boxed{f(x) = \frac{-x}{x^2+1}}$$

(β). $f(-x) = \frac{-(-x)}{(-x)^2+1} = \frac{x}{x^2+1} = -\frac{-x}{x^2+1}$

$= -f(x)$

περιττη.

$D_f = \mathbb{R}$

$x \in D_f$ και $-x \in D_f$.

(γ) Αρκει να $f(x) \le f(1)$

$$\frac{-x}{x^2+1} \le \frac{1}{2}$$

$-2x \le x^2+1$

$0 \le x^2+2x+1$

$0 \le (x+1)^2$ ✓

Θεμα 6

(α) $f\uparrow [0,1]$

$f\downarrow [1,2]$

$f\uparrow [2,3)$

(Β) i) $\frac{1}{5} > \frac{1}{7}$

$f\uparrow$

$f(\frac{1}{5}) > f(\frac{1}{7})$

ii) Εστω $\frac{3}{2} < \frac{4}{3}$

$9 < 8$ Ο.Χ.Ι

αρα $\frac{3}{2} > \frac{4}{3}$

$f\downarrow$

$f(\frac{3}{2}) < f(\frac{4}{3})$

(Γ) $A(1,2)$ Ο.Τ. Μεγιστο

$B(2,1)$ Ο.Τ. Ελαχιστο.

(Δ) $g(x) = |f(x) - 2| + |f(x) - 1|$

(−) (+)

- $1 \le f(x) \le 2$

$-1 \le f(x) - 2 \le 0$

- $1 \le f(x) \le 2$

$0 \le f(x) - 1 \le 1$

$g(x) = -f(x) + 2 + f(x) - 1 = 1.$

Επαναληπτικό

Θέματα

Μπορλάς.

Εποχιο Μαθημ

23

(6) αγ

(7) αγ

(8) αγ