Συναρτησεις

Η πρωτη μας γνωριμία με τις συναρτησεις εγινε στο γυμνασιο.

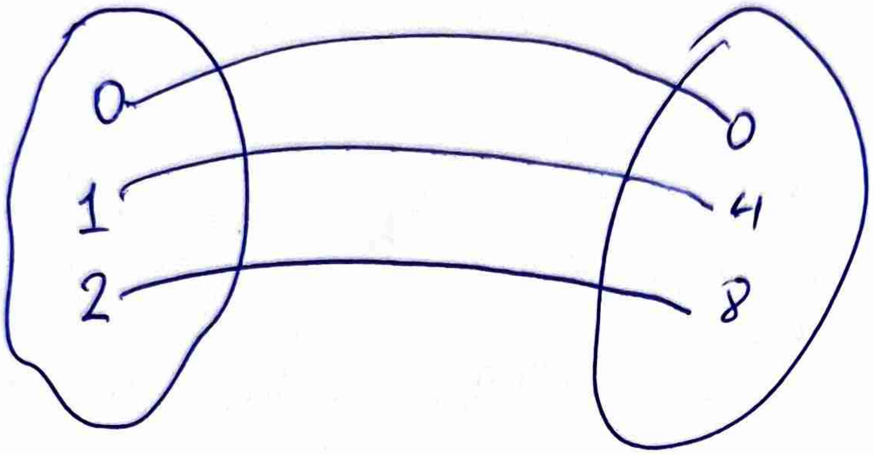
$y = \alpha x$ $\quad$ $y = \alpha x + \beta$ $\quad$ $y = \frac{\alpha}{x}$

Στο λυκειο στις συναρτησεις δινουμε ονοματα. π.χ

$f(x) = \alpha x$ $\quad$ $g(x) = \alpha x + \beta$

$h(x) = \frac{\alpha}{x}$.

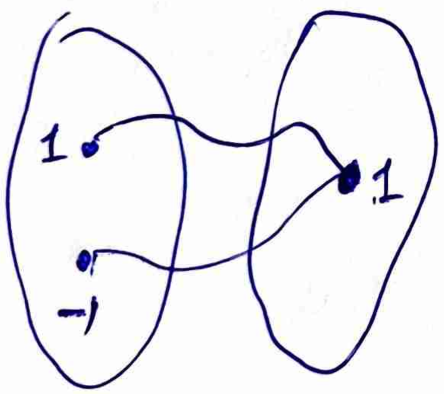
$$y = 4x$$



Πεδιο ορισμου.

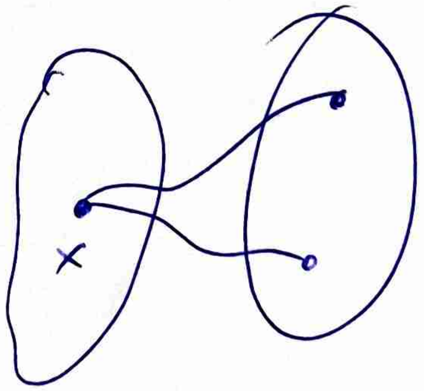
Συνολο Τιμων.

Επιτρεπεται



$f(x) = x^2$

Δεν επιτρεπεται



Δεν ειναι συναρτηση.

$$\boxed{f(x) = \frac{x^2-4}{2}}$$

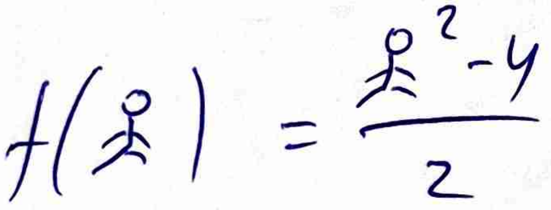
$$f(1) = \frac{1^2-4}{2} = \frac{-3}{2} \qquad f(1) = -\frac{3}{2}.$$

$$f(-1) = \frac{(-1)^2-4}{2} = -\frac{3}{2} \qquad f(-1) = -\frac{3}{2}$$

$$f(2) = \frac{2^2-4}{2} = 0 \qquad f(2) = 0$$

$$f(0) = \frac{0^2-4}{2} = -2 \qquad f(0) = -2$$

$$f(k) = \frac{k^2-4}{2}$$



$$f(a-1) = \frac{(a-1)^2-4}{2} = \frac{a^2-2a+1-4}{2} = \frac{a^2-2a-3}{2}$$

$$f(x) = \frac{x}{x^2-1}$$

$$f(1) = \frac{1}{1^2-1} = \frac{1}{0}$$

Прспи $x^2-1 \neq 0$

$\rightarrow x^2-1=0$

$x=1$ $x=-1$

$$ПО_f = A_f = D_f = R - \{1,-1\}$$

$$\boxed{f(x) = \sqrt{x^2 - 4}}$$

Прати $x^2 - 4 \geq 0$

x		-2		2	
x^2-4	$+$		$-$		$+$

$x \in (-\infty, -2] \cup [2, +\infty)$

$D_f = (-\infty, -2] \cup [2, +\infty)$.

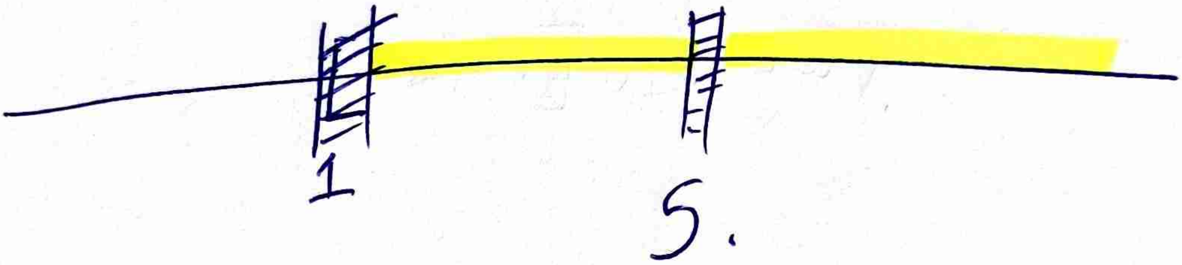
$$f(x) = \sqrt{x-1} - \frac{1}{x^2-6x+5}$$

πρεπει $x-1 \geq 0$ και $x^2-6x+5 \neq 0$

$x \neq 1$ $x \neq 5$

$x \geq 1$

$$D_f = (1,5) \cup (5,+\infty)$$



2 $f(x) = x^2 - 3\alpha x$

$f(2) = -2$

Ενότητα 23

ⓐ Νδο $\alpha = 1$

$f(2) = 2^2 - 3\cdot\alpha\cdot 2$

$-2 = 4 - 6\alpha$

$6\alpha = 4 + 2$

$6\alpha = 6$

$\alpha = 1$

$f(x) = x^2 - 3x$

ⓑ i) $f(-1) = \lambda^2 - 1$ $\Rightarrow 4 = \lambda^2 - 1$

$\lambda^2 = 5$

$\lambda = \pm\sqrt{5}$

• $f(-1) = (-1)^2 - 3(-1)$

$f(-1) = 1 + 3$

$f(-1) = 4$

ii) $f(\lambda - 1) = -2$

$(\lambda-1)^2 - 3(\lambda-1) = -2$

$\lambda^2 - 2\lambda + 1 - 3\lambda + 3 = -2$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda = 2$ $\lambda = 3$

13. $f(x) = x^2 - x - 6$, $D_f = \mathbb{R}$.

(α) Εξίσωση $f(x) = -6$

$$x^2 - x - \cancel{6} = -\cancel{6}$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$x = 0$ $\quad$ $x = 1$

(β) Ανίσωση $f(x-1) - f(3x) + 9x^2 - x < 4$

- $f(x-1) = (x-1)^2 - (x-1) - 6 = x^2 - 2x + 1 - x + 1 - 6$
$\qquad = x^2 - 3x - 4$

- $f(3x) = (3x)^2 - 3x - 6 = 9x^2 - 3x - 6$

$$x^2 - 3x - 4 - (9x^2 - 3x - 6) + 9x^2 - x < 4$$

$$x^2 - \cancel{3x} - 4 - \cancel{9x^2} + \cancel{3x} + 6 + \cancel{9x^2} - x < 4$$

$$x^2 - x - 2 < 0$$

$x \in (-1, 2)$.

x		-1		2	
$x^2 - x - 2$	$+$		$-$		$+$

8. (α) $f(x) = 2x^3 - x + 3$ $\quad x \in \mathbb{R}$

$D_f = \mathbb{R}$.

(στ) $f(x) = \dfrac{x}{x^2+x+1}$

$D_f = \mathbb{R}$.

πρέπει $x^2+x+1 \neq 0$

$\Delta = -3 < 0$

9. (γ) $f(x) = \dfrac{x}{|x|+1}$

πρέπει $|x|+1 \neq 0$

• $|x|+1 = 0$

$|x| = -1$ $\quad D_f = \mathbb{R}$.

Αδύνατη.

(δ) $f(x) = \dfrac{1}{x^2-|x|}$ $\quad D_f = \mathbb{R} - \{0, 1, -1\}$.

• πρέπει $x^2 - |x| \neq 0$

$\rightarrow x^2 - |x| = 0$

$|x|^2 - |x| = 0$

$\rightarrow |x|(|x|-1) = 0$

$|x| = 0$ ή $|x|-1 = 0$

$x = 0$ $\quad |x| = 1$

$x = 1$ $\quad x = -1$

10. δ) $f(x)=\sqrt{x^2-x+1}$

πρέπει $x^2-x+1 \geq 0$

$\Delta < 0$

x	
x^2-x+1	$+$

$D_f=\mathbb{R}$.

στ) $f(x)=\sqrt{|x+2|-1}$

πρέπει $|x+2|-1 \geq 0$

$|x+2| \geq 1$.

$x+2 \geq 1$ ή $x+2 \leq -1$

$x \geq -1$ $\qquad$ $x \leq -3$

$D_f=(-\infty,-3]\cup[-1,+\infty)$.

11. (β) $f(x) = \sqrt{x-3} + \dfrac{1}{\sqrt[3]{5-x}}$

Πρέπει $x-3 \geq 0$ και $5-x > 0$

$x \geq 3$ $\quad$ $x < 5$

$x \in [3, 5)$.

$D_f = [3, 5)$.

(δ) $f(x) = \dfrac{1}{\sqrt{x}-3}$

πρέπει $\sqrt{x}-3 \neq 0$ και $x \geq 0$

$\sqrt{x} \neq 3$

$\underline{\underline{x \neq 9}}$

$D_f = [0, 9) \cup (9, +\infty)$.

στ) $f(x) = \sqrt{\sqrt{x} - 1}$

πρέπει $x \geq 0$ και $\sqrt{x} - 1 \geq 0$

$\sqrt{x} \geq 1$

$x \geq 1$

$D_f = [1, +\infty)$

Επομενο Μαθημα

23

1

3

8 β γ δ ε

9 α β

10 α β γ ε

11 α γ ε.