7.

Ενότητα 19

ⓐ $(x-1)(x-2)(x-3)>0$

x		1		2		3	
$x-1$	−	0	+		+		+
$x-2$	−		−	0	+		+
$x-3$	−		−		−	0	+
$P(x)$	−		+		−		+

$x \in (1,2) \cup (3,+\infty)$

ⓖ $(3x-1)(x^2-4)<0$

x		-2		$\frac{1}{3}$		2	
$3x-1$	−		−	0	+		+
x^2-4	+	0	−		−	0	+
$P(x)$	−		+		−		+

$x \in (-\infty,-2) \cup (\frac{1}{3},2)$

ⓔ $(x+1)(1-x)^2 \leq 0$

x		-1		1	
$x+1$	−	0	+		+
$(1-x)^2$	+		+	0	+
$P(x)$	−		+		+

$x \in (-\infty,-1]$.

8. (B) $(x^2-4)(x^2-5x+6) \leq 0$

x		-2		2	3
x^2-4	+	0	−	0 +	+
x^2-5x+6	+	+	+	0 −	0 +
P(x)	+		−	−	+

$x \in (-2, 2) \cup (2, 3)$

9. (a) $x^3 > 1$

$x^3 > 1^3$

$\underline{\underline{x > 1}}$

(1) $x^3 < 3x^2 - 2x$

$x^3 - 3x^2 + 2x < 0$

$x(x^2 - 3x + 2) < 0$

x		0	1	2
x	−	0 +	+	+
x^2-3x+2	+	+	0 −	0 +
P(x)	−	+	−	+

$x \in (-\infty, 0) \cup (1, 2)$

23. $P(x) = x^4 + ax + \beta$

(α) $P(1) = 0$ και $P(-1) = -2$

$$\boxed{P(x) = x^4 + x - 2}$$

(β) $P(x) = 2x - 2$

$$x^4 + x - 2 = 2x - 2$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$x = 0$ ή $x^3 - 1 = 0$

$x^3 = 1$

$x = 1$

(γ) $P(x) < 2x - 2$

$x^4 + x - 2 < 2x - 2$

$x^4 - x < 0$

$x(x^3 - 1) < 0$

x		0		1
x	$-$	0	$+$	$+$
$x^3 - 1$	$-$	$-$	0	$+$
$Q(x)$	$+$	$-$	$+$	

$x \in (0, 1)$.

47. $P(x) = x^3 + \alpha x + \beta$

Ενότητα 17

διαιρείται από το $x^2 - 2x + 1 = (x-1)^2$

Βρ. α, β.

1	0	α	β	(1)
↓	1	1	$\alpha+1$	
1	1	$\alpha+1$	$\alpha+\beta+1=0$	

1	1	$\alpha+1$	(1)
↓	1	2	
1	2	$\alpha+3=0$	

$\Rightarrow \alpha = -3$

$-3+\beta+1=0$

$\beta - 2 = 0$

$\beta = 2$

Ενότητα 20

1. ⓐ $\frac{x^2}{x+1} + \frac{4}{1-x} = \frac{2}{x^2-1}$

$$\frac{x^2}{x+1} - \frac{4}{x-1} = \frac{2}{(x-1)(x+1)}$$

Πεδίο $x+1 \neq 0$ $\quad x \neq -1$

$x-1 \neq 0$ $\quad x \neq 1$

$(x-1)(x+1) \neq 0$

$$x^2(x-1) - 4(x+1) = 2$$

$$x^3 - x^2 - 4x - 4 = 2$$

$$x^3 - x^2 - 4x - 6 = 0$$

1	-1	-4	-6	3
↓	3	6	6	
1	2	2	0	

$$(x-3)(x^2+2x+2) = 0$$

$\Delta < 0$

$x = 3$

2. (a) $4\sin^3 x + 6\cos^2 x + 4\sin x + 2 = 0$

$4\sin^3 x + 4 - 4\sin^2 x + 4\sin x + 2 = 0$

$4\sin^3 x - 4\sin^2 x + 4\sin x + 3 = 0$

$\boxed{\sin x = t}$

$t^3 - t^2 + t + 3 = 0$

1	−1	1	3	(−1)
↓	−1	2	−3	
1	−2	3	0	

$(t+1)(t^2 - 2t + 3) = 0$

$\Delta < 0$

$t = -1$

$\sin x = -1$

$\sin x = \sin\frac{3\pi}{2}$

$x = 2k\pi + \frac{3\pi}{2}$ ή $x = 2k\pi + \pi - \frac{3\pi}{2}$,

(в) $\operatorname{ctg}^6 x - 4\operatorname{ctg}^3 x + 3 = 0$

$\operatorname{ctg} x = t$

$t^6 - 4t^3 + 3 = 0$

$t^3 = k$

$k^2 - 4k + 3 = 0$

$k = 3$ $\qquad$ $k = 1$

$t^3 = 3$ $\qquad$ $t^3 = 1$

$t = \sqrt[3]{3}$ $\qquad$ $t = 1$

$\operatorname{ctg} x = \sqrt[3]{3}$ $\qquad$ $\operatorname{ctg} x = 1$

Абыгем! $\qquad$ $\operatorname{ctg} x = \operatorname{ctg} \frac{\pi}{2}$

$x = 2\pi n + \frac{\pi}{2}$

7. ⑨ $\frac{5x-2}{2x-1} > 3$

$$\frac{5x-2}{2x-1} - 3 > 0$$

$$\frac{5x-2}{2x-1} - \frac{3(2x-1)}{2x-1} > 0$$

$$\frac{5x-2-6x+3}{2x-1} > 0$$

$$\frac{-x+1}{2x-1} > 0$$

x		$\frac{1}{2}$		1
$-x+1$	$+$	$+$	0	$-$
$2x-1$	$-$	0	$+$	$+$
$P(x)$	$-$	$+$	$-$	

$x \in (\frac{1}{2}, 1)$

10. (a) $\frac{x^2}{x+2} > 1$

$$\frac{x^2}{x+2} - 1 > 0$$

$$\frac{x^2}{x+2} - \frac{x+2}{x+2} > 0$$

$$\frac{x^2-x-2}{x+2} > 0$$

x		-2		-1		2	
x^2-x-2	$+$	$+$	0	$-$	0	$+$	
$x+2$	$-$	0	$+$	$+$		$+$	
$P(x)$	$-$		$+$	$-$		$+$	

$x \in (-2, -1) \cup (2, +\infty)$.

17. (B) $\frac{x^2}{x-1} > \frac{2}{x^2-1}$

$$\frac{x^2}{x-1} - \frac{2}{(x-1)(x+1)} > 0$$

$$\frac{x^2(x+1)}{(x-1)(x+1)} - \frac{2}{(x-1)(x+1)} > 0$$

$$\frac{x^3+x^2-2}{(x-1)(x+1)} > 0$$

$$\frac{x^3+x^2-2}{x^2-1} > 0$$

	1	1	0	-2	①
1		1	2	2	
	1	2	2	0	

$$\frac{(x-1)(x^2+2x+2)}{(x-1)(x+1)} > 0$$

$$\frac{x^2+2x+2}{x+1} > 0 \quad (+)$$

$x \in (-1, +\infty)$

↓

$x \in (-1, 1) \cup (1, +\infty)$

x	-1	
x^2+2x+2	+	+
$x+1$	−	+
$P(x)$	−	+

Ενότητα 21

1. β) $\sqrt{x-2} - 3 = 0$

$\sqrt{x-2} = 3$ πρέπει $x - 2 \geq 0$

$x - 2 = 9$

$x = 11$ ✓

δ) $\sqrt{x^2 - 9} = 4$ πρέπει $x^2 - 9 \geq 0$

$x^2 - 9 = 16$

$x^2 = 16 - 9$

$x^2 = 7$

~~$x = \sqrt{7}$~~ ~~$x = -\sqrt{7}$~~

Αδύνατη!

2. (B) $\sqrt{3-2x} - \sqrt{x+1} = 0$

$3-2x \geq 0$

$x+1 \geq 0$

$\sqrt{3-2x} = \sqrt{x+1}$

$3-2x = x+1$

$-2x-x = 1-3$

$-3x = -2$

$x = \frac{2}{3}$ ✓

(B) $\sqrt{25-x^2} = \sqrt{2x+1}$

$25-x^2 \geq 0$

$2x+1 \geq 0$

$25-x^2 = 2x+1$

$-x^2-2x+24 = 0$

$x^2+2x-24 = 0$

~~$x = -6$~~

$x = 4$ ✓

3. (B) $x+\sqrt{x}-2=0$

$x \geq 0$

$2-x \geq 0$

$\sqrt{x} = 2-x$

$x = (2-x)^2$

$x = 4-4x+x^2$

$x^2-5x+4=0$

~~$x=4$~~ $x=1$ ✓

(δ) $\sqrt{x} = x-2$

$x \geq 0$

$x-2 \geq 0$

$x = (x-2)^2$

$x^2-4x+4-x=0$

$x^2-5x+4=0$

$x=4$ ✓ ~~$x=1$~~

6. ① $\sqrt{5x^2+4} - x = x+1$

$\sqrt{5x^2+4} = 2x+1$

$5x^2+4 \geq 0$

$2x+1 \geq 0$

$5x^2+4 = (2x+1)^2$

$5x^2+4 = 4x^2+4x+1$

$x^2-4x+3 = 0$

$x=1$ ✓ $x=3$ ✓

8. ⑤ $\sqrt{x-1} = \sqrt{x+2} - \sqrt{3x-5}$

$\sqrt{x-1} + \sqrt{3x-5} = \sqrt{x+2}$

$\left(\sqrt{x-1} + \sqrt{3x-5}\right)^2 = \sqrt{x+2}^2$

$x-1+2\sqrt{x-1}\sqrt{3x-5}+3x-5 = x+2$

$4x-6+2\sqrt{x-1}\sqrt{3x-5} = x+2$

$2\sqrt{x-1}\sqrt{3x-5} = -3x+8$

$4(x-1)(3x-5) = (8-3x)^2$

$x-1 \geq 0$

$x+2 \geq 0$

$3x-5 \geq 0$

$\sqrt{x+2} - \sqrt{3x-5} \geq 0$

$8-3x \geq 0$

$4(x-1)(3x-5) = 64 - 48x + 9x^2$

$4(3x^2 - 5x - 3x + 5) = 9x^2 - 48x + 64$

$12x^2 - 32x + 20 = 9x^2 - 48x + 64$

$3x^2 + 16x - 44 = 0$ ✓

$x = \frac{-16 \pm 28}{6}$ ②

$-\frac{44}{6} = -\frac{22}{3}$