

Ενότητα 16

3. $P(x) = x^3 + a^2 x^2 + ax - 1$

Είναι πηλ. ω 1

$$\Rightarrow P(1) = 0$$

$$P(1) = 1^3 + a^2 1^2 + a \cdot 1 - 1 = 0$$

$$\cancel{1} + a^2 + a - \cancel{1} = 0$$

$$a^2 + a = 0$$

$$a(a+1) = 0$$

$$a = 0$$

$$a = -1$$

$$4. \quad f(x) = \alpha x^3 - \beta x^2 + 1$$

Exu pita w 1

ka si pxcen A(2,1)

Apz

$$f(1) = 0$$

ka

$$f(2) = 1$$

$$\alpha \cdot 1^3 - \beta \cdot 1^2 + 1 = 0$$

$$\alpha - \beta + 1 = 0$$

$$\boxed{\alpha - \beta = -1}$$

$$\alpha \cdot 2^3 - \beta \cdot 2^2 + 1 = 1$$

$$8\alpha - 4\beta = 0$$

$$\boxed{2\alpha - \beta = 0}$$

$$\begin{cases} \alpha - \beta = -1 \\ 2\alpha - \beta = 0 \end{cases} \Rightarrow -\alpha + \beta = 1$$

$$\begin{array}{r} \\ \\ \hline \end{array} \oplus$$

$$\alpha = 1$$

$$\underline{\underline{\beta = 2}}$$

$$6. \quad P(x) = x^3 + a^2x^2 + 4ax + B^2 + 5$$

$$\text{Exu pitu to } -1$$

$$P(-1) = 0$$

$$(-1)^3 + a^2(-1)^2 + 4a(-1) + B^2 + 5 = 0$$

$$-1 + a^2 - 4a + B^2 + 5 = 0$$

$$a^2 - 4a + 4 + B^2 = 0$$

$$(a-2)^2 + B^2 = 0$$

$$\begin{cases} a-2=0 & \Rightarrow \underline{\underline{a=2}} \\ \underline{\underline{B=0}} \end{cases}$$

9.

$$P(x) = 2x - 3$$

$$Q(x) = x^2 - 1$$

$$\textcircled{A} \quad R(x) = 2x P(x) - P(x) Q(x)$$

$$R(x) = 2x(2x-3) - (2x-3)(x^2-1)$$

$$R(x) = 4x^2 - 6x - (2x^3 - 2x - 3x^2 + 3)$$

$$R(x) = 4x^2 - 6x - 2x^3 + 2x + 3x^2 - 3$$

$$R(x) = -2x^3 + 7x^2 - 4x - 3$$

$$\textcircled{B} \quad R(x) = [P(x)]^2 - 3x Q(x)$$

$$R(x) = (2x-3)^2 - 3x(x^2-1)$$

$$R(x) = 4x^2 - 12x + 9 - 3x^3 + 3x$$

$$R(x) = 4x^2 - 3x^3 - 9x + 9$$

$$\textcircled{8} R(x) = P(x) \cdot Q(x) - (Q(x))^2$$

$$R(x) = (2x-3)(x^2-1) - (x^2-1)^2$$

$$R(x) = 2x^3 - 2x - 3x^2 + 3 - (x^4 - 2x^2 + 1)$$

$$R(x) = 2x^3 - 2x - 3x^2 + 3 - x^4 + 2x^2 - 1$$

$$R(x) = 2x^3 - x^2 - 2x + 2$$

$$\textcircled{9} R(x) = (P(x))^3$$

$$R(x) = (2x-3)^3$$

$$R(x) = (2x)^3 - 3 \cdot (2x)^2 \cdot 3 + 3 \cdot 2x \cdot 3^2 - 3^3$$

$$R(x) = 8x^3 - 3 \cdot 4x^2 \cdot 3 + 6x \cdot 9 - 27$$

$$R(x) = 8x^3 - 36x^2 + 54x - 27$$

10. $P(x) = x^2 - x + 1$

$$Q(x) = 2x - 3$$

$$(a) (P(x))^2 = (x^2 - x + 1)^2 = (\cancel{x^2 - x} + 1)^2$$

$$= (x^2 - x)^2 + 2 \cdot (x^2 - x) \cdot 1 + 1^2$$

$$= x^4 - 2x^2 \cdot x + x^2 + 2x^2 - 2x + 1$$

$$= x^4 - 2x^3 + 3x^2 - 2x + 1$$

$$(b) P(Q(x)) = (2x - 3)^2 - (2x - 3) + 1 =$$

$$= 4x^2 - 12x + 9 - 2x + 3 =$$

$$= 4x^2 - 14x + 12.$$

$$(c) Q(Q(x)) = 2(2x - 3) - 3$$

$$= 4x - 6 - 3$$

$$= 4x - 9.$$

12.

$$P(x) = (x^2+1)\lambda^2 + (2x+1)\lambda - 4x^2 + 4x + 2$$

а) $P(x) = (\quad)x^2 + (\quad)x + (\quad)$

$$P(x) = \underline{x^2\lambda^2} + \lambda^2 + \underline{2x\lambda} + \lambda - \underline{4x^2} + \underline{4x} + 2$$

$$P(x) = (\lambda^2 - 4)x^2 + (2\lambda + 4)x + \underbrace{\lambda^2 + \lambda + 2}_{\text{своб. чл}}$$

Σ коэффициентов: $\lambda^2 - 4, 2\lambda + 4, \lambda^2 + \lambda + 2$

б) Σ свободный член: $\lambda^2 + \lambda + 2 \neq 0$

$$\Delta = -7$$

а) $\lambda^2 - 4 = 0$ $\lambda = 2$ $\lambda = -2$ ✓

$2\lambda + 4 = 0$ $\lambda = -2$ ✓

23. (B) $P(x) = (4\lambda^3 - 9\lambda)x^3 + (4\lambda^2 - 9)x^2 - 2\lambda + 3$

$$\rightarrow 4\lambda^3 - 9\lambda = 0$$

$$\lambda(4\lambda^2 - 9) = 0$$

$$\lambda(2\lambda - 3)(2\lambda + 3) = 0$$

$$\boxed{\lambda = 0}$$

or

$$2\lambda - 3 = 0$$

$$\lambda = \frac{3}{2}$$

$$\text{or } 2\lambda + 3 = 0$$

$$\lambda = -\frac{3}{2}$$

1st case: Av $\lambda = 0$

$$P(x) = -9x^2 + 3 \quad \underline{\underline{200}}$$

2nd case: Av $\lambda = \frac{3}{2}$

$$P(x) = 0$$

See ex 4

Badly

3rd case: Av $\lambda = -\frac{3}{2}$

$$P(x) = 6$$

See Badly

4th case: Av $\lambda \neq 0, \frac{3}{2}, -\frac{3}{2}$

See Badly

22. $P(x) = a^2 x^4 + 2ax^3 + (a-1)x + 2$

Но то $P(x)$ не имеет корней.

$$P(-1) = a^2(-1)^4 + 2a(-1)^3 + (a-1)(-1) + 2$$

$$P(-1) = a^2 - 2a - a + 1 + 2$$

$$P(-1) = a^2 - 3a + 3 \neq 0$$

$$\Delta = 9 - 12 = -3 < 0$$

$$P(-1) \neq 0$$

значит $x = -1$ не

является корнем.

21.

$P(x)$ is

- 2ω Bxgpo
- exu p17u 20 0 10u -2
- $P(-1) = 3$

$$P(x) = ax^2 + bx + c$$

$$\left(\begin{array}{l} P(0) = 0 \Rightarrow a \cdot 0^2 + b \cdot 0 + c = 0 \\ \boxed{c = 0} \end{array} \right.$$

$$\underline{\underline{P(x) = ax^2 + bx}}$$

$$P(-2) = 0 \Rightarrow a(-2)^2 + b(-2) = 0$$

$$4a - 2b = 0$$

$$2a - b = 0$$

$$\underline{\underline{b = 2a}}$$

$$P(-1) = 3$$

$$P(-1) = a(-1)^2 + b(-1) = 3$$

$$\underline{\underline{a - b = 3}}$$

$$\begin{cases} B = 201 \\ a - B = 3 \end{cases}$$

$$B = -6$$

$$a - 201 = 3$$

$$-a = 3$$

$$a = -3$$

$$P(x) = -3x^2 - 6x$$

14

$$P(x) = (x^2 + x + 2)A + (2x^2 - x - 1)B - 5x^2 + x$$

$$P(x) = (\quad)x^2 + (\quad)x + (\quad)$$

$$P(x) = \underline{x^2 A} + \underline{x A} + \underline{2A} + \underline{2x^2 B} - \underline{x B} - \underline{B} - \underline{5x^2} + \underline{x}$$

$$P(x) = (A + 2B - 5)x^2 + (A - B + 1)x + 2A - B$$

$$\begin{cases} A + 2B - 5 = 0 \\ A - B + 1 = 0 \\ 2A - B = 0 \end{cases} \rightarrow \begin{aligned} &A + 2 \cdot 2A - 5 = 0 \\ &5A = 5 \\ &A = 1 \end{aligned}$$

$$\Rightarrow \begin{aligned} &B = 2A \\ &B = 2 \end{aligned}$$

15.

$$P(x) = (\lambda^3 - \lambda)x^2 + (\lambda^2 - 1)x + \lambda^2 + \lambda$$

$$\rightarrow \lambda^3 - \lambda = 0$$

$$\lambda(\lambda^2 - 1) = 0$$

$$\lambda(\lambda - 1)(\lambda + 1) = 0$$

$$\lambda = 0$$

$$\lambda = 1$$

$$\lambda = -1$$

$$1_u \ni Av \quad \lambda = 0 \quad P(x) = -x$$

$$2_u \ni Av \quad \lambda = 1 \quad P(x) = 2 \quad \text{σταθιρα} \quad \checkmark$$

$$3_u \ni Av \quad \lambda = -1 \quad P(x) = 0$$

$$\underline{\underline{A_{ps} \quad \lambda = 1}}$$

Εποραιο Μαδουρ

16

(2)

(5)

(7)

(8)

(11)

(13)

(20)

(23)