Ολοκληρωματα

1. $\int_0^1 2x\,dx = (x^2)_0^1 = 1^2 - 0^2 = 1 - 0 = 1$

2. $\int_0^{\pi} ημx\,dx = (-συνx)_0^{\pi} = -(συνx)_0^{\pi} =$

$= -(συνπ - συν0) = -(-1-1) =$

$= 2$

3. $\int_0^1 e^x\,dx = (e^x)_0^1 = e^1 - e^0 = e - 1$

4. $\int_1^2 x^2\,dx = \left(\frac{x^3}{3}\right)_1^2 = \frac{1}{3}(x^3)_1^2 =$

$= \frac{1}{3}(2^3 - 1^3) =$

$= \frac{1}{3} 7$

Γενικος κανονας

$$\int x^{\rho}\,dx = \frac{x^{\rho+1}}{\rho+1}$$

5. $\int_{-1}^{1} \frac{1}{x^2}\,dx = \left(-\frac{1}{x}\right)_{-1}^{1} = -\left(\frac{1}{x}\right)_{-1}^{1} =$

$= -\left(\frac{1}{1} - \frac{1}{-1}\right) = -(1+1) = -2$

6. $\int_{-1}^{1} \frac{1}{x^4}\,dx = \int_{-1}^{1} x^{-4}\,dx = \left(\frac{x^{-4+1}}{-4+1}\right)_{-1}^{1}$

$= \left(\frac{x^{-3}}{-3}\right)_{-1}^{1} = -\frac{1}{3}\left(\frac{1}{x^3}\right)_{-1}^{1} = -\frac{1}{3}\left(1-(-1)\right)$

$= -\frac{2}{3}.$

7. $\int_{-1}^{1} \frac{1}{x}\,dx = \left(\ln|x|\right)_{-1}^{1} = \ln 1 - \ln|-1| =$

$= 0 - \ln 1 = 0-0=0$

8. $\int_{1}^{2} \frac{x\sqrt{x}}{\sqrt[3]{x^2}}\,dx = \int_{1}^{2} \frac{x \cdot x^{\frac{1}{2}}}{x^{\frac{2}{3}}}\,dx = \int_{1}^{2} \frac{x^{\frac{3}{2}}}{x^{\frac{2}{3}}}\,dx$

$= \int_{1}^{2} x^{\frac{3}{2}-\frac{2}{3}}\,dx = \int_{1}^{2} x^{\frac{5}{6}}\,dx = \left(\frac{x^{\frac{5}{6}+1}}{\frac{5}{6}+1}\right)_{1}^{2}$

$$= \left(\frac{x^{\frac{11}{6}}}{\frac{11}{6}}\right)_1^2 = \frac{1}{\frac{11}{6}} \cdot \left(x^{11/6}\right)_1^2 = \frac{6}{11}\left(\sqrt[6]{x^{11}}\right)_1^2$$

$$= \frac{6}{11}\left(\sqrt[6]{x^6 \cdot x^5}\right)_1^2 = \frac{6}{11}\left(\sqrt[6]{x^6} \cdot \sqrt[6]{x^5}\right)_1^2$$

$$= \frac{6}{11}\left(x\sqrt[6]{x^5}\right)_1^2 = \frac{6}{11}\left(2\sqrt[6]{2^5} - 1\right).$$

9. $\int_0^{\pi} 2ημx - 3συνx \, dx = \int_0^{\pi} 2ημx \, dx - \int_0^{\pi} 3συνx \, dx$

$$= 2\int_0^{\pi} ημx \, dx - 3\int_0^{\pi} συνx \, dx$$

$$= 2(-συνx)_0^{\pi} - 3(ημx)_0^{\pi} =$$

$$= -2(συνx)_0^{\pi} - 3(ημπ - ημ0) =$$

$$= -2(συνπ - συν0) - 3(0 - 0) =$$

$$= -2(-1-1) = 4.$$

10. $\int_0^1 e^{-x}\,dx = \left(-e^{-x}\right)_0^1 = -\left(e^{-x}\right)_0^1$

$= -\left(e^{-1}-1\right)$

$= -\frac{1}{e}+1$

<u>Γενικώς κανόνας</u>

$\int e^{ax+\beta}\,dx = \frac{1}{a} e^{ax+\beta}$

$\int \eta\mu(ax+\beta) = -\frac{1}{a}\,\sigma\upsilon\nu(ax+\beta)$

$\int \sigma\upsilon\nu(ax+\beta) = -\frac{1}{a}\,\eta\mu(ax+\beta)$

$\int \frac{1}{ax+\beta}\,dx = \frac{1}{a}\,\ln|ax+\beta|.$

11. $\int_0^{\pi} 2\eta\mu(2x-1) - \frac{1}{2}e^{3-x}\,dx =$

$= 2\int_0^{\pi} \eta\mu(2x-1)\,dx - \frac{1}{2}\int_0^{\pi} e^{3-x}\,dx$

$= 2\cdot\left[-\frac{1}{2}\sigma\upsilon\nu(2x-1)\right]_0^{\pi} - \frac{1}{2}\left(\frac{1}{-1}e^{3-x}\right)_0^{\pi}$

$= -\left(\sigma\upsilon\nu(2\pi-1) - \sigma\upsilon\nu(1)\right) + \frac{1}{2}\left(e^{3-\pi} - e^{3}\right)$

Ρητα ολοκληρωματα

1. $\int_1^2 \frac{(x-1)^2}{x}\,dx = \int_1^2 \frac{x^2-2x+1}{x}\,dx =$

$= \int_1^2 \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x}\,dx = \int_1^2 x - 2 + \frac{1}{x}\,dx$

$= \int_1^2 x\,dx - \int_1^2 2\,dx + \int_1^2 \frac{1}{x}\,dx$

$= \left(\frac{x^2}{2}\right)_1^2 - (2x)_1^2 + (\ln|x|)_1^2$

$= \frac{1}{2}(x^2)_1^2 - 2(x)_1^2 + \ln 2 - \ln 1$

$= \frac{1}{2}(4-1) - 2\cdot 1 + \ln 2 =$

$= \frac{3}{2} - 2 + \ln 2.$

2. $\int_0^1 \frac{2x+3}{x^2+3x+4}\,dx = \left(\ln|x^2+3x+4|\right)_0^1$

Παρατηρω οτι ο αριθμητης ειναι η παραγωγος του παρονομαστη.

$= \ln 8 - \ln 4 = \ln \frac{8}{4} = \ln 2$

3. $\int_1^2 \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int_1^2 \frac{2x+2}{x^2+2x}\,dx$

$= \frac{1}{2}\left(\ln|x^2+2x|\right)_1^2 =$

$= \frac{1}{2}\left(\ln 8 - \ln 3\right) = \frac{1}{2}\ln\frac{8}{3}.$

4. $\int_2^3 \frac{x+3}{x^2-1}\,dx = \int_2^3 \frac{2}{x-1} - \frac{1}{x+1}\,dx \overset{(*)}{=}$

Παρατηρω οτι ο παρονομαστης παραγοντοποιειται και ο αριθμητης εχει μικροτερο βαθμο απο τον παρονομαστη.

Μεθοδος των A, B.

$\frac{x+3}{x^2-1} = \boxed{\frac{x+3}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}.}$

$x+3 = A(x+1) + B(x-1)$

$x+3 = Ax + A + Bx - B$

$x+3 = (A+B)x + A - B.$

$\begin{cases} A+B=1 \\ A-B=3 \end{cases} \oplus \quad 2A=4 \Rightarrow A=2$

$\hookrightarrow 2-B=3 \quad B=-1$

$(*)\ \int_2^3 \frac{2}{x-1}\,dx - \int_2^3 \frac{1}{x+1}\,dx = 2\left(\ln|x-1|\right)_2^3 - \left(\ln|x+1|\right)_2^3$

$= 2(\ln 2 - \ln 1) - (\ln 4 - \ln 3) = 2\ln 2 - \ln\frac{4}{3}.$

5. $\int_0^1 \frac{x^3}{x^2+1}\,dx = \int_0^1 \frac{(x^2+1)x - x}{x^2+1}\,dx \overset{*}{=}$

Ο βαθμος του αριθμητη ειναι μεγαλυτερος ή ισος του βαθμου του παρονομαστη.

$$\begin{array}{r|l} x^3 & x^2+1 \\ \hline -(x^3+x) & x \\ \hline -x & \end{array}$$

$$\boxed{x^3 = (x^2+1)\cdot x - x}$$

$\Delta = \delta \cdot \pi + \upsilon$

$$\overset{*}{=} \int_0^1 \frac{\cancel{(x^2+1)}x}{\cancel{x^2+1}} - \frac{x}{x^2+1}\,dx$$

$$= \int_0^1 x\,dx - \int_0^1 \frac{x}{x^2+1}\,dx$$

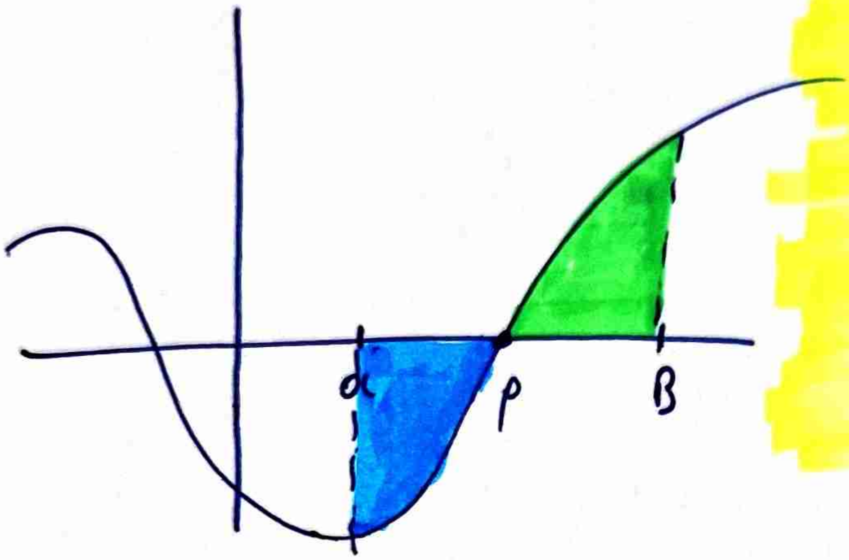
$$= \left(\frac{x^2}{2}\right)_0^1 - \frac{1}{2}\int_0^1 \frac{2x}{x^2+1}\,dx$$

$$= \frac{1}{2}\left(x^2\right)_0^1 - \frac{1}{2}\left(\ln|x^2+1|\right)_0^1$$

$$= \frac{1}{2} - \frac{1}{2}\ln 2.$$

Εμβαδον Επιπεδου χωριου

1.



Γενικος Τυπος

$E: C_f, x'x, x=\alpha, x=\beta$

$$E=\int_\alpha^\beta |f(x)|\,dx$$

$E: C_f,\ x'x,\ x=\rho,\ x=\beta$

$$E=\int_\rho^\beta |\overset{+}{f(x)}|\,dx=\int_\rho^\beta f(x)\,dx$$

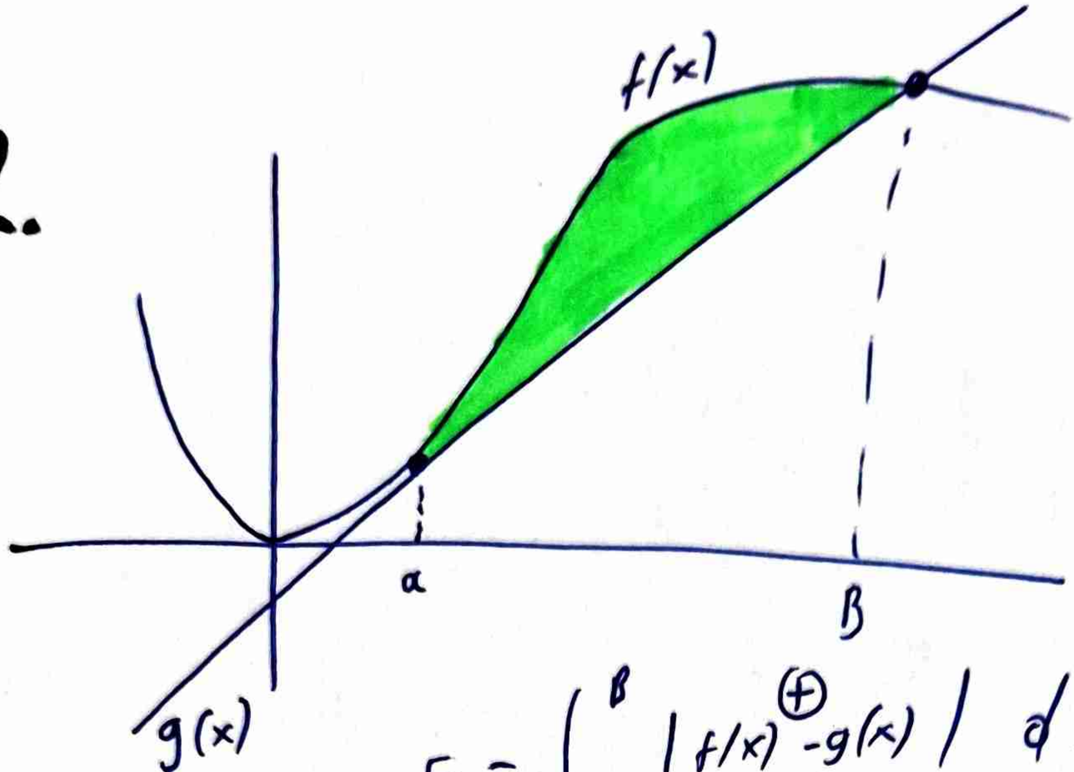
$E: C_f,\ x'x,\ x=\alpha,\ x=\rho$

$$E=\int_\alpha^\rho |\overset{-}{f(x)}|\,dx=\int_\alpha^\rho -f(x)\,dx$$

$E: C_f,\ x'x,\ x=\alpha,\ x=\beta$

$$E=\int_\alpha^\beta |f(x)|\,dx=\int_\alpha^\rho -f(x)\,dx+\int_\rho^\beta f(x)\,dx$$

2.



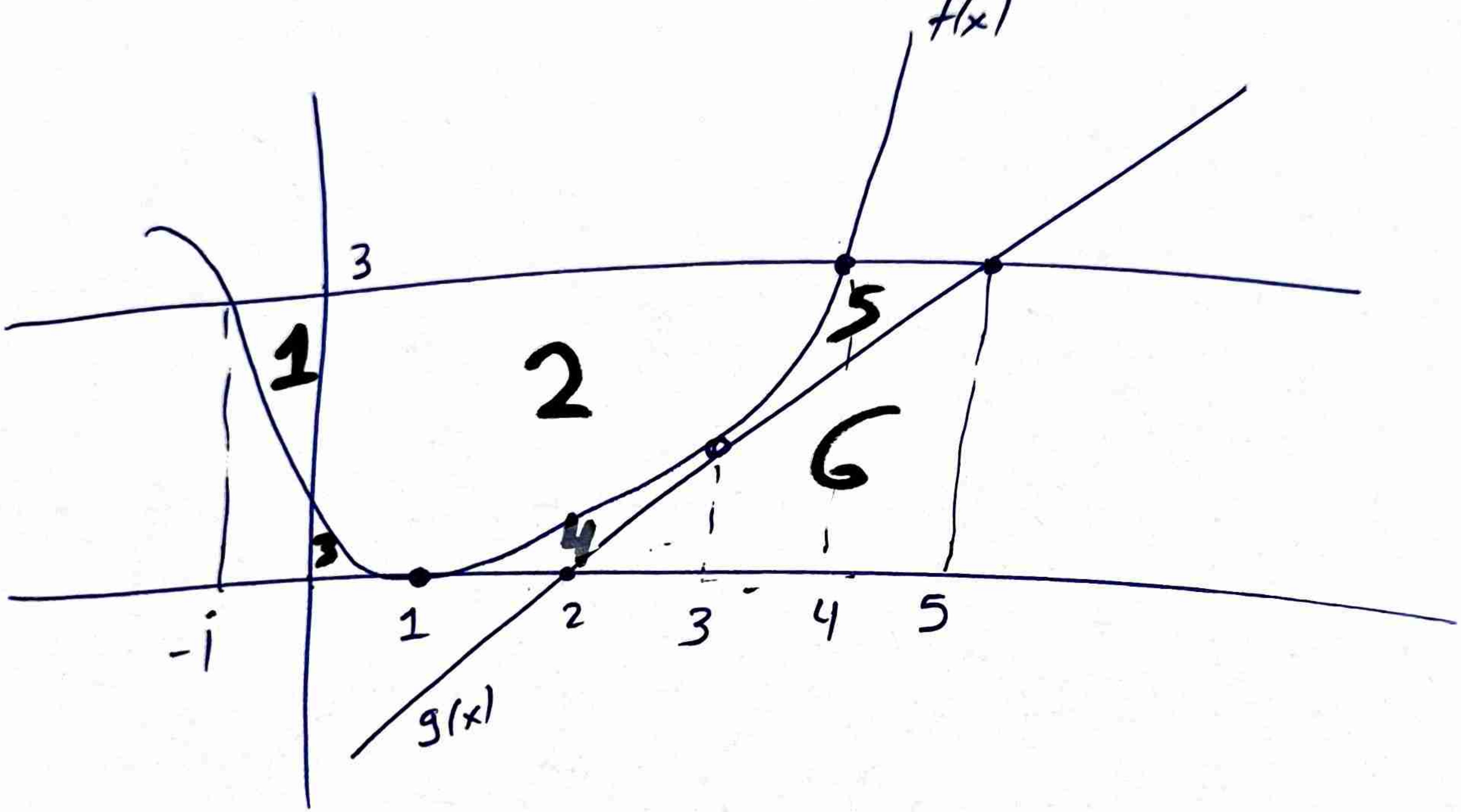
$$E = \int_{\alpha}^{\beta} \left| f(x) - g(x) \right| dx$$

$$E = \int_{\alpha}^{\beta} f(x) - g(x) \, dx$$

Γενικός κανόνας

E : C_f, C_g, $x = \alpha$, $x = \beta$

$$E = \int_{\alpha}^{\beta} \left| f(x) - g(x) \right| dx$$



f(x)
3
1
2
5
6
3
4
-1
1
2
3
4
5
g(x)

1: $\in = \int_{-1}^{0} 3 - f(x)\, dx$

2: $\in = \int_{0}^{4} 3 - f(x)\, dx$

3: $\in = \int_{0}^{1} f(x)\, dx$

4: $\in = \int_{1}^{2} f(x)\, dx + \int_{2}^{3} f(x) - g(x)\, dx$

5: $\in = \int_{3}^{4} f(x) - g(x)\, dx + \int_{4}^{5} 3 - g(x)\, dx$

6: $\in = \int_{2}^{5} g(x)\, dx$

kanonas

$$\int_{\alpha}^{\beta} f(x)\,dx = \int_{\alpha}^{\gamma} f(x)\,dx + \int_{\gamma}^{\beta} f(x)\,dx$$

Θεμελιωδη Θεωρημα ολοκληρωτικου λογισμου

$$\int_\alpha^\beta f(x)\,dx = \left[G(x)\right]_\alpha^\beta = G(\beta) - G(\alpha)$$

Αποδειξη

Συναρτηση ολοκληρωμα $F(x) = \int_\alpha^x f(t)\,dt$

ειναι παραγουσα της $f(x)$

Εστω $G(x)$ μια αλλη παραγουσα της f

$$\left.\begin{aligned} F'(x) &= f(x) \\ G'(x) &= f(x) \end{aligned}\right\} \quad F'(x) = G'(x) \quad \Rightarrow F(x) = G(x) + c$$

Αρα $F(x) = G(x) + c \longrightarrow F(x) = G(x) - G(\alpha)$

$$\underline{x = \alpha}$$

$$F(\alpha) = G(\alpha) + c$$

$$\int_\alpha^\alpha f(t)\,dt = G(\alpha) + c$$

$$0 = G(\alpha) + c$$

$$c = -G(\alpha)$$

$$\underline{x = \beta}$$

$$F(\beta) = G(\beta) - G(\alpha)$$

$$\int_\alpha^\beta f(t)\,dt = G(\beta) - G(\alpha)$$

Σχολιο

$$\int_\alpha^\beta f(x)\,dx = \int_\alpha^\beta f(t)\,dt = \int_\alpha^\beta f(\omega)\,d\omega$$

$$\int_\alpha^\alpha f(x)\,dx = 0$$

Επόμενο Μάθημα

34	35
6	20
7	21
8	22
9	23
10	
11	