

Ενότητα 10

11. α) $(\lambda - 2)x = \lambda - 3$

1. Αν $\lambda = 2$ τότε $0x = -1$ Αδύνατο,

2. Αν $\lambda \neq 2$ τότε $x = \frac{\lambda - 3}{\lambda - 2}$

β) $(\lambda + 1)x = \lambda(\lambda + 1)$

1. Αν $\lambda = -1$ τότε $0x = 0$ Αόριστο,

2. Αν $\lambda \neq -1$ τότε $x = \lambda$

12. α) $\lambda^2(x - 1) = 5(5x - \lambda)$

$$\lambda^2 x - \lambda^2 = 25x - 5\lambda$$

$$\lambda^2 x - 25x = \lambda^2 - 5\lambda$$

$$(\lambda^2 - 25)x = \lambda^2 - 5\lambda$$

$$\boxed{(\lambda - 5)(\lambda + 5)x = \lambda(\lambda - 5)}$$

1. Av $\lambda = 5$ tot $0x = 0$ Answer!

2. Av $\lambda = -5$ tot $0x = 50$ Answer!

3. Av $\lambda \neq 5, \lambda \neq -5$ tot $x = \frac{\lambda(\cancel{\lambda-5})}{(\cancel{\lambda-5})(\lambda+5)}$

$$x = \frac{\lambda}{\lambda+5}$$

$$\textcircled{B} \lambda^2(x+1) = 2(2-\lambda x).$$

$$\lambda^2 x + \lambda^2 = 4 - 2\lambda x$$

$$\lambda^2 x + 2\lambda x = 4 - \lambda^2$$

$$(\lambda^2 + 2\lambda) x = 4 - \lambda^2$$

$$\lambda(\lambda+2) x = 4 - \lambda^2$$

$$\boxed{\lambda(\lambda+2)x = (2-\lambda)(2+\lambda)}$$

3. Av $\lambda \neq 0, \lambda \neq -2$

$$x = \frac{2-\lambda}{\lambda}.$$

1. Av $\lambda = 0$ $0x = 4$ Answer

2. Av $\lambda = -2$ $0x = 0$ Answer

$$\textcircled{8} \quad \lambda^3(x-1) = 4\lambda x - 8$$

$$\lambda^3 x - \lambda^3 = 4\lambda x - 8$$

$$\lambda^3 x - 4\lambda x = \lambda^3 - 8$$

$$(\lambda^3 - 4\lambda)x = \lambda^3 - 8$$

$$\lambda(\lambda^2 - 4)x = (\lambda - 2)(\lambda^2 + 2\lambda + 4)$$

$$\lambda(\lambda - 2)(\lambda + 2)x = (\lambda - 2)(\lambda^2 + 2\lambda + 4)$$

1. Av $\lambda = 0$ wcc $0x = -8$ Advers

2. Av $\lambda = 2$ wcc $0x = 0$ Advers

3. Av $\lambda = -2$ wcc $0x = -4 \cdot 4$

$0x = -16$ Advers

4. Av $\lambda \neq 0, 2, -2$

$$x = \frac{\lambda^2 + 2\lambda + 4}{\lambda(\lambda + 2)}$$

$$\textcircled{8} \quad \lambda^2(x-1) + \lambda = 4x - 2$$

$$\lambda^2 x - \lambda^2 + \lambda = 4x - 2$$

$$\lambda^2 x - 4x = \lambda^2 - \lambda - 2$$

$$\begin{aligned} \lambda^2 - \lambda - 2 &= \lambda^2 + \lambda - 2\lambda - 2 \\ &= \lambda(\lambda+1) - 2(\lambda+1) \\ &= (\lambda-2)(\lambda+1) \end{aligned}$$

$$(\lambda^2 - 4)x = \lambda^2 - \lambda - 2$$

$$(\lambda-2)(\lambda+2)x = (\lambda-2)(\lambda+1)$$

1. Av $\lambda=2$ wrt $0x=0$ Answer

2. Av $\lambda=-2$ wrt $0x=4$ Answer

3. Av $\lambda \neq 2, -2$ wrt $x = \frac{\lambda+1}{\lambda+2}$

$$24. \textcircled{01} \quad x^2(x-2) + 2x(x-2) + x-2 = 0$$

$$(x-2)(x^2 + 2x + 1) = 0$$

$$x-2=0$$

u'

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$$x+1=0$$

$$x=-1$$

$$\textcircled{x=2}$$

$$\underline{\underline{x=-1}}$$

$$\textcircled{Y} \quad (2x-1)^2 = 2x-1$$

$$(2x-1)^2 - (2x-1) = 0$$

$$(2x-1)(2x-1-1) = 0$$

$$2x-1=0 \quad \text{u'} \quad 2x-2=0$$

$$x = \frac{1}{2}$$

$$x=1$$

$$\underline{\underline{x=1}}$$

$$\textcircled{E} \quad (x^2 - 9)(x - 1) = (x^2 - 1)(x - 3),$$

$$(x - 3)(x + 3)(x - 1) = (x - 1)(x + 1)(x - 3)$$

$$(x - 3)(x + 3)(x - 1) - (x - 1)(x + 1)(x - 3) = 0$$

$$(x - 3)(x - 1)(x + 3 - (x + 1)) = 0$$

$$x - 3 = 0$$

$$\boxed{x = 3}$$

$$\sim x - 1 = 0$$

$$\boxed{x = 1}$$

$$\textcircled{J} \quad x(x - 1)^2 = x^2 - 2x + 1$$

$$x(x - 1)^2 = (x - 1)^2$$

$$x(x - 1)^2 - (x - 1)^2 = 0$$

$$(x - 1)^2(x - 1) = 0$$

$$(x - 1)^3 = 0$$

$$x - 1 = 0$$

$$\boxed{x = 1}$$

$$\textcircled{0} \quad x^3 - 3x^2 = (2x-1)(x-3)$$

$$x^2(x-3) - (2x-1)(x-3) = 0$$

$$(x-3)(x^2 - 2x + 1) = 0$$

$$x-3=0$$

$$\textcircled{x=3}$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$\textcircled{x=1}$$

Exercice 12

1. (a) $x^3 - 1 = 0 \Rightarrow x^3 = 1 \Rightarrow x^3 = 1^3$
 $x = 1$

(b) $x^3 + 8 = 0 \Rightarrow x^3 = -8$
 $x^3 = (-2)^3$
 $x = -2$

(c) $x^5 - 32 = 0$
 $x^5 = 32$
 $x^5 = 2^5$
 $x = 2$

(d) $x^3 + 27 = 0$
 $x^3 = -27$ $x^3 = (-3)^3$
 $x = -3$

(e) $x^3 - 2 = 0$
 $x^3 = 2$
 $x = \sqrt[3]{2}$

(62)

$$x^2 - 25 = 0$$

$$x^2 = 25$$

$$x = 5$$

$$x = -5$$

(7) $x^2 - 3 = 0$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

(4) $x^4 - 81 = 0$

$$x^4 = 81$$

$$x^4 = 3^4$$

$$x = 3$$

$$x = -3$$

(9)

$$x^2 + 1 = 0$$

$$x^2 = -1$$

Admission.

Ενότητα 11

1. (a) $|x| - 2 = 0$

$$|x| = 2$$

$$x = 2$$

$$x = -2$$

(b) $2|x| - 3 = 0$

$$2|x| = 3$$

$$|x| = \frac{3}{2}$$

$$x = \frac{3}{2}$$

$$x = -\frac{3}{2}$$

(c) $2 - 5|x| = 0$

$$\frac{2}{5} = |x|$$

$$x = \frac{2}{5}$$

$$x = -\frac{2}{5}$$

(d) $2(|x| - 1) = |x| - 2$

$$2|x| - 2 = |x| - 2$$

$$|x| = 0$$

$$x = 0$$

$$\textcircled{51} \frac{|x|}{2} - \frac{|x|}{3} = 1$$

$$3|x| - 2|x| = 6$$

$$|x| = 6$$

$$\textcircled{x=6} \quad \textcircled{x=-6}$$

$$\textcircled{52} \frac{|x|-2}{3} + |x| = -1$$

$$|x|-2 + 3|x| = -3$$

$$4|x| = -1$$

Абсурд

Επαλήθευση στις εξισώσεις

1.

$$8 - x(5x+6) = (x+1)^3 - x(x+4)^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$8 - 5x^2 - 6x = x^3 + 3x^2 + 3x + 1 - x(x^2 + 8x + 16)$$

$$8 - 5x^2 - 6x = \cancel{x^3} + 3x^2 + 3x + 1 - \cancel{x^3} - 8x^2 - 16x$$

$$\cancel{8} - \cancel{5x^2} - 6x = -\cancel{5x^2} - 13x + 1$$

$$13x - 6x = 1 - 8$$

$$7x = -7$$

$$\underline{\underline{x = -1}}$$

$$2. \quad (x^2 + 3x)(x-1) = (2x+6)(x^2-1)$$

$$x(x+3)(x-1) - (2x+6)(x^2-1) = 0$$

$$x(x+3)(x-1) - 2(x+3)(x-1)(x+1) = 0$$

$$(x-1)(x+3) (x - 2(x+1)) = 0$$

$$(x-1)(x+3)(x-2x-2) = 0$$

$$(x-1)(x+3)(-x-2) = 0$$

$$x-1=0$$

$$x=1$$

or

$$x+3=0$$

$$x=-3$$

$$-x-2=0$$

$$x=-2$$

$$3. \quad \frac{4}{x+2} - \frac{3x}{2-x} = \frac{3x^2-8}{x^2-4}$$

$$\frac{4}{x+2} + \frac{3x}{x-2} = \frac{3x^2-8}{(x-2)(x+2)},$$

$$\bullet \quad x+2=0 \Rightarrow \boxed{x=-2}$$

$$\bullet \quad x-2=0 \Rightarrow \boxed{x=2}$$

$$\text{Prima } \begin{matrix} x \neq 2 \\ x \neq -2 \end{matrix}$$

$$\bullet \quad (x-2)(x+2)=0 \Rightarrow x-2=0 \vee x+2=0$$

$$\boxed{x=2}$$

$$\boxed{x=-2}$$

$$\in \mathbb{N} \text{ } (x-2)(x+2)$$

$$\cancel{(x-2)(x+2)} \frac{4}{\cancel{x+2}} + (x-2)(x+2) \frac{3x}{x-2} = (x-2)(x+2) \frac{3x^2-8}{(x-2)(x+2)}$$

$$4(x-2) + 3x(x+2) = 3x^2-8$$

$$4x-8 + \cancel{3x^2} + 6x = \cancel{3x^2} - 8$$

$$10x = 0$$

$$\boxed{x=0}$$

$$4. \quad 2\lambda^2 x - \lambda^2 (\lambda^2 x - 1) = -2\lambda (\lambda x - 1)$$

$$2\lambda^2 x - \lambda^4 x + \lambda^2 = -2\lambda^2 x + 2\lambda$$

$$2\lambda^2 x - \lambda^4 x + 2\lambda^2 x = -\lambda^2 + 2\lambda$$

$$(-\lambda^4 + 4\lambda^2) x = -\lambda^2 + 2\lambda$$

$$\lambda^2 (4 - \lambda^2) x = -\lambda (\lambda - 2)$$

$$\lambda^2 (2 - \lambda)(2 + \lambda) x = -\lambda (\lambda - 2)$$

$$1. \text{ Av } \lambda = 0 \quad \text{wz} \quad 0x = 0 \quad \text{Aopisc}$$

$$2. \text{ Av } \lambda = 2 \quad \text{wz} \quad 0x = 0 \quad \text{Aopisc}$$

$$3. \text{ Av } \lambda = -2 \quad \text{wz} \quad 0x = -8 \quad \text{Aduwa}$$

$$4. \text{ Av } \lambda \neq 0, 2, -2 \quad \text{wz} \quad x = + \frac{1}{\lambda(\lambda+2)}$$

$$5. (2x-1)^3 = -27$$

$$(2x-1)^3 = (-3)^3$$

$$2x-1 = -3$$

$$2x = -3 + 1$$

$$2x = -2$$

$$x = -1$$

$$6. (3-2x)^4 - 81 = 0$$

$$(3-2x)^4 = 81$$

$$(3-2x)^4 = 3^4$$

$$\cancel{3-2x} = \cancel{3}$$

or

$$3-2x = -3$$

$$-2x = 0$$

$$x = 0$$

$$-2x = -6$$

$$x = 3$$

$$7. \frac{|4-x|-2}{2} = 5 - \frac{|x-4|+10}{9}$$

$$18. \frac{|4-x|-2}{2} = 18 \cdot 5 - 18 \cdot \frac{|x-4|+10}{9}$$

$$9(|4-x|-2) = 90 - 2(|x-4|+10)$$

$$9|4-x|-18 = 90 - 2|x-4| - 20.$$

$$9|x-4| + 2|x-4| = 18 + 90 - 20.$$

$$11|x-4| = 88$$

$$|x-4| = 8$$

$$x-4 = 8$$

$$x = 12$$

$$\text{or } x-4 = -8$$

$$x = -4$$

$$8. \quad |5x-20| - 14 = |12-3x|$$

$$5|x-4| - 14 = 3|4-x|$$

$$5|x-4| - 3|x-4| = 14$$

$$2|x-4| = 14$$

$$|x-4| = 7$$

$$x-4 = 7 \quad \text{и} \quad x-4 = -7$$

$$x = 11$$

$$x = -3$$

$$9. \quad |x+2| = x+6.$$

$$\text{при } x+6 \geq 0$$

$$\underline{\underline{x \geq -6}}$$

~~$$x+2 = x+6$$~~

~~$$\text{и } x+2 = -x-6$$~~

$$2 = 6$$

Аналог

$$2x = -6-2$$

$$2x = -8$$

$$x = -4$$

$$10. \quad d(2x, 5) = d(x, -1)$$

$$|2x - 5| = |x + 1|$$

$$2x - 5 = x + 1$$

$$\underline{\underline{x = 6}}$$

$$\text{or } 2x - 5 = -x - 1$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}}$$

$$11. \quad |x^2 - 3x| + |x^2 - 9| = 0$$

$$x^2 - 3x = 0$$

or

$$x^2 - 9 = 0$$

$$x(x - 3) = 0$$

$$(x - 3)(x + 3) = 0$$

$$\cancel{\boxed{x = 0}}$$

$$\boxed{x = 3}$$

$$\boxed{x = 3}$$

$$\cancel{\boxed{x = -3}}$$

$$12. \left| |x| - 3 \right| - 5 = 1.$$

$$|x| - 3 - 5 = 1 \quad \vee \quad |x| - 3 - 5 = -1$$

$$|x| - 3 = 6$$

$$|x| - 3 = 6 \quad \vee \quad |x| - 3 = -6$$

$$|x| = 9$$

$$|x| = -3$$

Answer,

$$x = 9 \quad x = -9$$

$$|x| - 3 = 4$$

$$|x| - 3 = 4 \quad \vee \quad |x| - 3 = -4$$

$$|x| = 7$$

$$|x| = -1$$

$$x = 7$$

Answer

$$x = -7$$