Διαστήματα

$x \in (\alpha, \beta) \iff \alpha < x < \beta$

$x \in [\alpha, \beta] \iff \alpha \leq x \leq \beta$

$x \in [\alpha, \beta) \iff \alpha \leq x < \beta$

$x \in [\alpha, +\infty) \iff x \geq \alpha$

$x \in (-\infty, +\infty)$

Άσκηση 1

Έστω $2 \leq x \leq 4$

(α) $4x$

• $2 \leq x \leq 4 \Rightarrow 8 \leq 4x \leq 16$

(β) $-2x$

• $2 \leq x \leq 4 \Rightarrow -4 \geq -2x \geq -8$

(γ) $x+5$

• $2 \leq x \leq 4 \Rightarrow 7 \leq x+5 \leq 9$

(δ) $5-3x$

• $2 \leq x \leq 4 \Rightarrow -6 \geq -3x \geq -12$

$-1 \geq 5-3x \geq -7$

(ε) x^3

- $2 \le x \le 4 \Rightarrow 2^3 \le x^3 \le 4^3$

$$8 \le x^3 \le 64$$

(ζ) $2 - x^2$

- $2 \le x \le 4 \rightarrow 2^2 \le x^2 \le 4^2$

$$4 \le x^2 \le 16$$

$$-4 \ge -x^2 \ge -16$$

$$2 - 4 \ge 2 - x^2 \ge 2 - 16$$

$$-2 \ge 2 - x^2 \ge -14$$

(η) $-\frac{3}{x}$

- $2 \le x \le 4 \Rightarrow \frac{1}{2} \ge \frac{1}{x} \ge \frac{1}{4} \Rightarrow \frac{3}{2} \ge \frac{3}{x} \ge \frac{3}{4}$

$$-\frac{3}{2} \le -\frac{3}{x} \le -\frac{3}{4}$$

Άσκηση 2

Έστω ότι $2 < x < 3$

(α) $x^2 + 3x$

- $2 < x < 3 \Rightarrow 4 < x^2 < 9$
- $2 < x < 3 \Rightarrow 6 < 3x < 9$

$\Big\}\ \oplus$

$$10 < x^2 + 3x < 18$$

(β) $\frac{2}{x} - x^2$

- $2 < x < 3 \Rightarrow \frac{1}{2} > \frac{1}{x} > \frac{1}{3} \Rightarrow \boxed{\frac{2}{2} > \frac{2}{x} > \frac{2}{3}}$
- $2 < x < 3 \Rightarrow 4 < x^2 < 9 \Rightarrow \boxed{-4 > -x^2 > -9}$

$(+)$

$$-3 > \frac{2}{x} - x^2 > \frac{2}{3} - 9$$

$$-3 > \frac{2}{x} - x^2 > -\frac{25}{3}$$

Άσκηση 3

Έστω $1 \le \alpha \le 3$ και $2 \le \beta \le 3$

(α) $2\alpha + 3\beta$

- $1 \le \alpha \le 3 \Rightarrow 2 \le 2\alpha \le 6$
- $2 \le \beta \le 3 \Rightarrow 6 \le 3\beta \le 9$

$\Big\}$ (+)

$$8 \le 2\alpha + 3\beta \le 15$$

(β) $3\alpha - 2\beta$

- $1 \le \alpha \le 3 \Rightarrow \boxed{3 \le 3\alpha \le 9}$
- $2 \le \beta \le 3 \Rightarrow -4 \ge -2\beta \ge -6$

$\Big)$ (+)

$$\boxed{-6 \le -2\beta \le -4}$$

$$\boxed{-3 \le 3\alpha - 2\beta \le 5}.$$

(γ) $\alpha\beta - 1$

- $1 \leq \alpha \leq 3$
- $2 \leq \beta \leq 3$

$\Big\}\odot$ $2 \leq \alpha\beta \leq 9$

$1 \leq \alpha\beta - 1 \leq 8$

(δ) $\frac{\alpha}{\beta}$

- $1 \leq \alpha \leq 3 \Rightarrow 3 \geq \alpha \geq 1$
- $2 \leq \beta \leq 3 \Rightarrow \frac{1}{2} \geq \frac{1}{\beta} \geq \frac{1}{3}$

$\Big\}\odot$

$\frac{3}{2} \geq \frac{\alpha}{\beta} \geq \frac{1}{3}$

(E) $\dfrac{\alpha^2+\beta}{\beta^2+\alpha}$

- $1 \le \alpha \le 3$ $\Rightarrow 1 \le a^2 \le 9$,
- $2 \le \beta \le 3$ $\quad \lrcorner \oplus$

$$3 \le a^2+\beta \le 12$$

- $1 \le \alpha \le 3$ $\quad \lrcorner \oplus$
- $2 \le \beta \le 3$ $\Rightarrow 4 \le \beta^2 \le 9$

$$5 \le \beta^2+a \le 12$$

$$\frac{1}{5} \ge \frac{1}{\beta^2+a} \ge \frac{1}{12}$$

$$\frac{1}{12} \le \frac{1}{\beta^2+a} \le \frac{1}{5}$$

$$\frac{3}{12} \le \frac{a^2+\beta}{\beta^2+a} \le \frac{12}{5}$$

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$x \in (-\infty, +\infty)$

Επομενο Μαθημα

7

(8)

(9)

(11)

(12)

(13)

(14)

(33)

(34)