

Επανάληψη

για το τελευταίο

διαγωνισμό της

σχολικής χρονιάς

στην

Αθήνα.

$$1. \quad \text{Ndo} \quad \log_a \theta_1 \theta_2 = \log_a \theta_1 + \log_a \theta_2$$

$$\alpha^{x_1} = \theta_1$$

(\Rightarrow)

$$x_1 = \log_a \theta_1$$

$$\alpha^{x_2} = \theta_2$$

(\Rightarrow)

$$x_2 = \log_a \theta_2$$

$$\alpha^{x_1} \alpha^{x_2} = \theta_1 \theta_2$$

$$\alpha^{x_1+x_2} = \theta_1 \theta_2$$

$$x_1+x_2 = \log_a \theta_1 \theta_2$$

$$\log_a \theta_1 + \log_a \theta_2 = \log_a \theta_1 \theta_2$$

$$2. \text{ N.B. } \log_a \frac{\theta_1}{\theta_2} = \log_a \theta_1 - \log_a \theta_2$$

$$a^{x_1} = \theta_1$$

(\Rightarrow)

$$x_1 = \log_a \theta_1$$

$$a^{x_2} = \theta_2$$

(\Rightarrow)

$$x_2 = \log_a \theta_2$$

$$\frac{a^{x_1}}{a^{x_2}} = \frac{\theta_1}{\theta_2}$$

$$a^{x_1 - x_2} = \frac{\theta_1}{\theta_2}$$

$$x_1 - x_2 = \log_a \frac{\theta_1}{\theta_2}$$

$$\log_a \theta_1 - \log_a \theta_2 = \log_a \frac{\theta_1}{\theta_2}$$

$$3. \text{ NSo } \log_a \theta^k = k \cdot \log_a \theta$$

$$a^x = \theta$$

(=)

$$x = \log_a \theta$$

$$a^x = \theta \quad (\Rightarrow) \quad (a^x)^k = \theta^k \quad (\Rightarrow) \quad a^{kx} = \theta^k$$

$$kx = \log_a \theta^k$$

$$k \cdot \log_a \theta = \log_a \theta^k$$

1 5 1 0 T H T E S

$$a^x = \theta \Leftrightarrow x = \log_a \theta$$

$\Delta \ll \alpha$ οτι \log \log οτι \log

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

3. $\log_a a = 1$

4. $\log_a 1 = 0$

5. $\log_a (\theta_1 \theta_2) = \log_a \theta_1 + \log_a \theta_2$

6. $\log_a \left(\frac{\theta_1}{\theta_2}\right) = \log_a \theta_1 - \log_a \theta_2$

7. $k \log_a \theta = \log_a \theta^k$

$\log_{10} x = \log x$
 $10^x = \theta \Leftrightarrow x = \log \theta$

$\log_e x = \ln x$
 $e^x = \theta \Leftrightarrow x = \ln \theta$

1. $\log 10^x = x$

2. $10^{\log x} = x$

3. $\log 10 = 1$

4. $\log 1 = 0$

5. $\log \theta_1 \theta_2 = \log \theta_1 + \log \theta_2$

6. $\log \frac{\theta_1}{\theta_2} = \log \theta_1 - \log \theta_2$

7. $k \log \theta = \log \theta^k$

1. $\ln e^x = x$

2. $e^{\ln x} = x$

3. $\ln e = 1$

4. $\ln 1 = 0$

5. $\ln \theta_1 \theta_2 = \ln \theta_1 + \ln \theta_2$

6. $\ln \frac{\theta_1}{\theta_2} = \ln \theta_1 - \ln \theta_2$

7. $k \ln \theta = \ln \theta^k$

1. Να υπολογιστεί η παράσταση

$$A = \frac{1}{3} \log 8 + \frac{1}{2} \log 25 =$$

$$= \log 8^{1/3} + \log 25^{1/2} =$$

$$= \log \sqrt[3]{8} + \log \sqrt{25} = \log 2 + \log 5$$

$$= \log 10 = 1$$

2. (a) Να λυθεί η εξίσωση. $P(x) = 0$

$$x^3 - 2x^2 - 5x + 6 = 0$$

$$\begin{array}{r} 6 \\ \hline \pm 1 \pm 2 \pm 3 \\ \pm 6 \end{array}$$

$$\begin{array}{ccccccc} 1 & -2 & -5 & 6 & \textcircled{1} & & \\ \downarrow & & & & & & \\ 1 & -1 & -6 & 0 & & & \\ 1 & -1 & -6 & 0 & & & \end{array}$$

$$(x-1)(x^2-x-6) = 0$$

$$\textcircled{x=1}$$

$$\textcircled{x=3}$$

$$\textcircled{x=-2}$$

Ⓑ Να λυθεί η εξίσωση

$$\sin^3 x + 2 \cos^2 x - 5 \sin x + 4 = 0$$

$$\text{στο } [0, 2\pi]$$

$$\sin^3 x + 2(1 - \cos^2 x) - 5 \sin x + 4 = 0$$

$$\sin^3 x + 2 - 2 \cos^2 x - 5 \sin x + 4 = 0$$

$$\sin^3 x - 2 \cos^2 x - 5 \sin x + 6 = 0$$

$$P(\sin x) = 0$$

$$\sin x = 1$$

$$\sin x = 3$$

$$\sin x = -2$$

$$\sin x = \sin 0$$

Αδύνατο

αδύνατο.

$$x = 2k\pi \pm 0$$

$$\boxed{x = 2k\pi}$$

$$0 \leq x \leq 2\pi$$

$$0 \leq 2k\pi \leq 2\pi$$

$$0 \leq k \leq 1$$

$$k \in \mathbb{Z}$$

$$k = 0 \Rightarrow \underline{\underline{x = 0}}$$

$$k = 1 \Rightarrow x = 2\pi$$

3. Να λύσει η

i) ελλογισμένη $2^{2x} - 3 \cdot 2^x - 4 = 0$

$2^x = t$

$t^2 - 3t - 4 = 0$

$t = 4 \quad t = -1$

$2^x = 4 \quad 2^x = -1$

$2^x = 2^2$ Αδυνατά

$x = 2$

ii) Ανισωμένη

$2^{2x} - 3 \cdot 2^x - 4 \leq 0$

$t^2 - 3t - 4 \leq 0$

t	-1 4		
$t^2 - 3t - 4$	+	-	+

$-1 \leq t \leq 4$

$-1 \leq 2^x \leq 4$

✓

$2^x \leq 4$

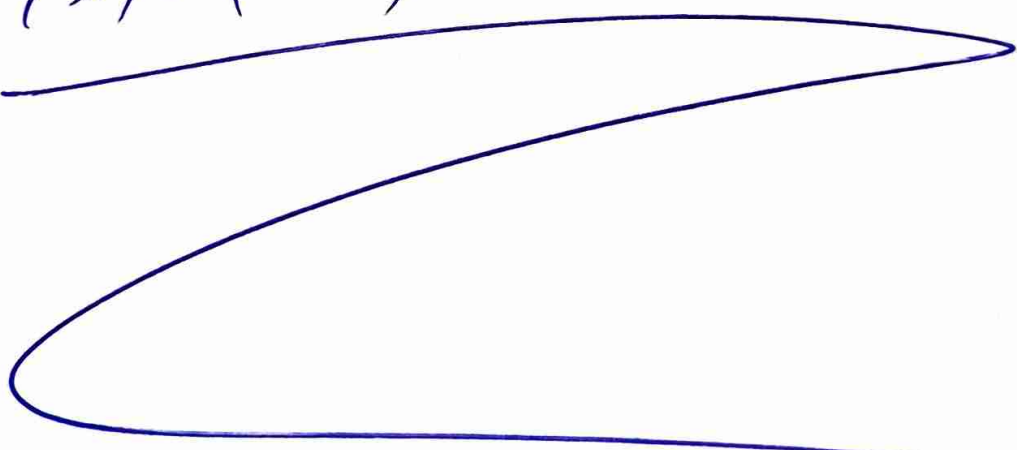
$2^x \leq 2^2$

$x \leq 2$

$x \in (-\infty, 2]$

4. Να γίνει η διαίρεση $P(x) = 2x^3 - 2x^2 + x - 1$
 $P(x) : (x^2 + 4)$ και να γραφεί τωροσητα
εκτελεστω διαίρεση.

$$\begin{array}{r|l} 2x^3 - 2x^2 + x - 1 & x^2 + 4 \\ - (2x^3 + 8x) & \hline \hline -2x^2 - 7x - 1 & \\ - (-2x^2 - 8) & \\ \hline -7x + 7 & \end{array}$$

$$P(x) = (x^2 + 4)(2x - 2) - 7x + 7.$$


5. \hookrightarrow σ_{TW} $P(x) = x^2 - 1$

(a) $\forall x \in [-1, 1] \quad P(x) \leq 0$

$$x^2 - 1 \leq 0$$

x	-1	1
$x^2 - 1$	$+$	$+$

$$x \in [-1, 1] \Leftrightarrow P(x) \leq 0$$

(b) Να λούει η ανίσωση

$$P(9^x - 4 \cdot 3^x + 4) < 0$$

Εβούλα ου το $P(x) \leq 0 \quad \forall x \in [-1, 1]$

αρα προση $-1 < 9^x - 4 \cdot 3^x + 4 < 1$

$$\begin{aligned} & \textcircled{+} \\ -1 < 9^x - 4 \cdot 3^x + 4 \\ \Delta < 0 \end{aligned}$$

$x \in \mathbb{R}$

$$9^x - 4 \cdot 3^x + 4 < 1$$

$$9^x - 4 \cdot 3^x + 3 < 0$$

$$\textcircled{3^x = t}$$

$$t^2 - 4t + 3 < 0$$

t		1	3
$t^2 - 4t + 3$	+	-	+

$$t \in (1, 3)$$

$$1 < t < 3$$

$$3^0 < 3^x < 3^1$$

$$0 < x < 1$$

6. Μια συνάρτηση είναι περιττή
 όταν $f(-x) = -f(x) \quad \forall x \in D_f$.

και (αν $x \in D_f$ και $-x \in D_f$)

α' τρόπο

$$f(-x) = 0000 = -f(x) \quad \checkmark$$

β' τρόπο

$$f(-x) + f(x) = 000 = 0 \quad \checkmark$$

7. Έστω γενική μορφή $f(x)$
 η οποία διασχίζει από τα σημεία

$A(1, 4)$ και $B(-1, -4)$

Το μορφή έχει;

Αφού είναι γενική μορφή

είναι $f \nearrow$ ή $f \searrow$

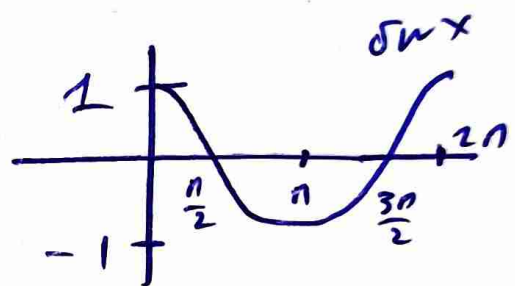
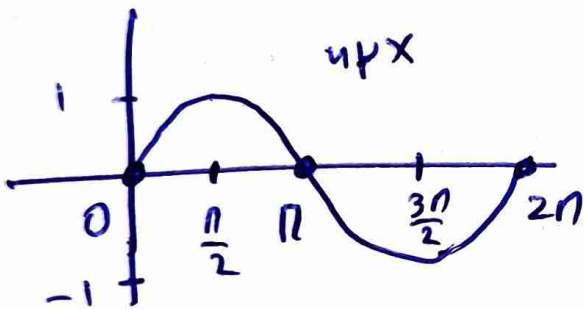
$$\text{οπότε } 1 > -1 \quad (\Leftrightarrow) \quad \begin{matrix} f(1) > f(-1) \\ \parallel & \parallel \\ 4 & -4 \end{matrix}$$

$\Rightarrow f \nearrow$

Τριγωνομετρική Επίσωση

1. $\eta\mu x = \eta\mu \theta \Leftrightarrow x = 2k\pi + \theta$ ή $x = 2k\pi + \pi - \theta$
2. $\sigma\omega x = \sigma\omega \theta \Leftrightarrow x = 2k\pi + \theta$ ή $x = 2k\pi - \theta$
3. $\epsilon\varphi x = \epsilon\varphi \theta \Leftrightarrow x = k\pi + \theta$
4. $\delta\varphi x = \delta\varphi \theta \Leftrightarrow x = k\pi + \theta$

Ημίτονο - Σημειότονο



x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\eta\mu x$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
$\sigma\omega x$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
$\epsilon\varphi x$	$\sqrt{3}/3$	1	$\sqrt{3}$
$\delta\varphi x$	$\sqrt{3}$	1	$\sqrt{3}/3$

$$\sigma\omega(-x) = -\sigma\omega x$$

$$\epsilon\varphi(\pi + x) = -\epsilon\varphi x$$

$$\eta\mu(\pi - x) = \eta\mu x$$

$$\eta\mu(\frac{\pi}{2} - x) = \sigma\omega x$$

SOS

$$(2^x)^2 = (2^2)^x = 4^x = 2^{2x}$$

8. \triangle $\triangle ABC$ \triangle
-отн $\triangle ABC$.

$$NB \quad n\mu(B-\Gamma) = n\mu(A+2\Gamma)$$

$$A+B+\Gamma=180$$

$$B=180-A-\Gamma$$

$$B-\Gamma=180-A-2\Gamma$$

$$B-\Gamma=180-(A+2\Gamma)$$

$$n\mu(B-\Gamma) = n\mu(180-(A+2\Gamma)) =$$

$$= n\mu(n - (A+2\Gamma)) =$$

$$= n\mu(A+2\Gamma) \quad \checkmark$$

9. Έστω ότι $f(x) = \ln\left(\frac{x-1}{x+1}\right)$

Πεδίο ορισμού

$$\frac{x-1}{x+1} > 0$$

Νόσος f αφ'εαυτής

$$x \in D_f$$

$$-x \in D_f$$

x	-1	1
x-1	-	+
x+1	-	+
$\frac{x-1}{x+1}$	+	+

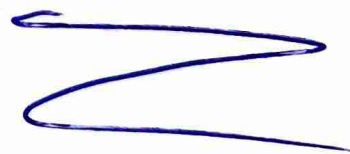
$$x \in (-\infty, -1) \cup (1, +\infty)$$

$$f(-x) = \ln\left(\frac{-x-1}{-x+1}\right) = \ln\left(\frac{-(x+1)}{1-x}\right) =$$

$$= \ln\left(\frac{x+1}{x-1}\right) = \ln\left(\frac{x-1}{x+1}\right)^{-1} =$$

$$= - \ln \frac{x-1}{x+1} = -f(x)$$

αφ'εαυτής



$$\text{Nδο } f\left(\varepsilon\psi \frac{A+r}{2}\right) + f\left(-\sigma\psi \frac{B}{2}\right) = 0$$

$$\bullet A+B+r=180$$

$$A+r+B=180$$

$$\frac{A+r}{2} + \frac{B}{2} = 90$$

$$\Rightarrow \frac{A+r}{2} = 90 - \frac{B}{2}$$

$$\varepsilon\psi \frac{A+r}{2} = \varepsilon\psi \left(90 - \frac{B}{2}\right)$$

$$\boxed{\varepsilon\psi \frac{A+r}{2} = \sigma\psi \frac{B}{2}}$$

f συναρτησια

$$f\left(\varepsilon\psi \frac{A+r}{2}\right) = f\left(\sigma\psi \frac{B}{2}\right)$$

$$f\left(\varepsilon\psi \frac{A+r}{2}\right) - f\left(\sigma\psi \frac{B}{2}\right) = 0$$

Αφού f η φλττυ

$$- f(x) = f(-x)$$

$$f\left(\varepsilon\psi \frac{A+r}{2}\right) + f\left(-\sigma\psi \frac{B}{2}\right) = 0$$

✓

$$\sigma_W \left(\frac{3}{2} (A+B) \right) + nP \left(\frac{3}{2} r \right) = 0$$

$$A + B + r = 180$$

$$A + B = 180 - r$$

$$\frac{3}{2} (A+B) = (180 - r) \frac{3}{2}$$

$$\frac{3}{2} (A+B) = 180 \cdot \frac{3}{2} - \frac{3}{2} r$$

$$\frac{3}{2} (A+B) = 270 - \frac{3}{2} r$$

$$\sigma_W \frac{3}{2} (A+B) = \sigma_W \left(\frac{30}{2} - \frac{3}{2} r \right)$$

$$\sigma_W \frac{3}{2} (A+B) = \sigma_W \left(\frac{20}{2} + \frac{1}{2} - \frac{3}{2} r \right)$$

$$\sigma_W \frac{3}{2} (A+B) = \sigma_W \left(10 + \frac{1}{2} - \frac{3}{2} r \right)$$

$$\sigma_W \frac{3}{2} (A+B) = -\sigma_W \left(\frac{1}{2} - \frac{3}{2} r \right)$$

$$\sigma_W \frac{3}{2} (A+B) = -nP \left(\frac{3}{2} r \right)$$

10. Να λύσει η εξίσωση

$$\varepsilon\psi x - 3\sigma\psi x = -2 \quad \sigma\psi$$

$$\left[0, \frac{\pi}{2}\right]$$

$$\varepsilon\psi x \cdot \sigma\psi x = 1$$

$$\sigma\psi x = \frac{1}{\varepsilon\psi x}$$

$$\varepsilon\psi x - 3 \cdot \frac{1}{\varepsilon\psi x} = -2$$

$$\varepsilon\psi x = t$$

$$t - \frac{3}{t} = -2 \Rightarrow t^2 - 3 = -2t$$

$$t^2 + 2t - 3 = 0$$

$$t = -3$$

$$t = 1$$

$$\varepsilon\psi x \neq -3$$

$$\varepsilon\psi x = 1$$

$$\varepsilon\psi x = \varepsilon\psi \frac{\pi}{4}$$

$$x = k\pi + \frac{\pi}{4}$$

$$0 \leq x \leq \frac{n}{2}$$

$$0 \leq kn + \frac{n}{4} \leq \frac{n}{2}$$

$$0 \leq k + \frac{1}{4} \leq \frac{1}{2}$$

$$-\frac{1}{4} \leq k \leq \frac{1}{2} - \frac{1}{4}$$

$$-\frac{1}{4} \leq k \leq \frac{1}{4}$$

$$\underline{\underline{k \in \mathbb{Z}}}$$

$$k=0 \Rightarrow x = \frac{n}{4}$$