Σελ 89

1. (B). Αν $\alpha > 2$ και $1 < \beta$

- $\alpha > 2 \Rightarrow \alpha - 2 > 0$
- $1 < \beta \Rightarrow 0 < \beta - 1$
- $\frac{\alpha - 2}{\beta - 1} > 0$

(+) $\alpha - 2$, (+) $\beta - 1$, (+)

(+) $\alpha - 2$, (−) $\beta - 1$, (−)

2. (B) $2 < x < 3$

- $2 < x < 3 \Leftrightarrow 2 - 2 < x - 2 < 3 - 2$

$$\boxed{0 < x - 2 < 1.}$$

(+) $x - 2$, (−) $x - 3$

- $2 < x < 3 \Leftrightarrow 2 - 3 < x - 3 < 3 - 3$

$$\boxed{-1 < x - 3 < 0}$$

- $(x-2)(x-3) < 0$

(+) $x - 2$, (−) $x - 3$, (−)

(−) $x - 2$, (−) $x - \frac{1}{3}$, (+)

3. (β) Νδο $a^2+25 \geq 10a$

$\longrightarrow a^2-10a+25 \geq 0$

$(a-5)^2 \geq 0$ που ισχύει!

(σε) νδο $2(a^2+9) \geq (a+3)^2$

$2a^2+18 \geq a^2+6a+9$

$2a^2+18-a^2-6a-9 \geq 0$

$a^2-6a+9 \geq 0$

$(a-3)^2 \geq 0$ που ισχύει,

4. Νδο $\frac{x^2}{x+1} < 0$ $\forall x < -1$

- $x^2 \geq 0$
- $x < -1 \Rightarrow x+1 < 0$

$\frac{x^2}{x+1} < 0$

$\oplus$

$\ominus$

Επομενο Μαθημα

Πεμπτη 5:30 - 7.

1. Γενικη επαναληψη σε
   - → Δυναμεις
   - → Ταυτοτητες
   - → Παραγοντοποιηση.

2. Καλο διαβασμα στα 8 Β' θεματα των Τ.Θ που αφορουν το 2.1.

Σελ 89

5. (B) $x^2+y^2-4x+6y \geq -13$.

$x^2-4x + y^2+6y +13 \geq 0$

$\underbrace{x^2-4x+4} + \underbrace{y^2+6y+9}$

$(x-2)^2 + (y+3)^2 \geq 0$

που ισχύει!

6. (B) $x^2+y^2-4x+2y+5=0$

$x^2-4x + y^2+2y+5=0$

$\underbrace{x^2-4x+4} + \underbrace{y^2+2y+1} = 0$

$(x-2)^2 + (y+1)^2 = 0$

Ισότητα

$\alpha^2+\beta^2=0$

$\Leftrightarrow \alpha=0$ και $\beta=0$

$x-2=0$ και $y+1=0$

$x=2$ $y=-1$

(8) $x^2+4y^2-2x-4y+2 \leq 0$

$x^2-2x+4y^2-4y+2 \leq 0$

$\underbrace{x^2-2x+1}+\underbrace{4y^2-4y+1} \leq 0$

$(x-1)^2+(2y-1)^2 \leq 0.$

$\Rightarrow (x-1)^2+(2y-1)^2=0$

$x-1=0$ and $2y-1=0$

$x=1$

$2y=1$

$y=\frac{1}{2}$

23. (B) Av $\alpha < \beta$ vδo $\alpha^3 - \beta^3 \leq \alpha\beta^2 - \alpha^2\beta$

$$\alpha^3 - \beta^3 - \alpha\beta^2 + \alpha^2\beta \leq 0 .$$

$$\alpha(\alpha^2 - \beta^2) + \beta(\alpha^2 - \beta^2) \leq 0$$

$$(\alpha^2 - \beta^2)(\alpha + \beta) \leq 0$$

$$(\alpha - \beta)(\alpha + \beta)(\alpha + \beta) \leq 0$$

$$(\alpha - \beta)(\alpha + \beta)^2 \leq 0 \quad \checkmark$$

$(-)$ $(+)$

$\rightarrow \alpha < \beta \Rightarrow \alpha - \beta < 0$

7. $x^2+y^2+z^2-6x+2y-4z=-14$

$x^2-6x+y^2+2y+z^2-4z+14=0.$

$x^2-6x+9+y^2+2y+1+z^2-4z+4=0$

$(x-3)^2+(y+1)^2+(z-2)^2=0.$

Άρα τω $x-3=0$ και $y+1=0$ και $z-2=0$

$x=3$ $y=-1$ $z=2.$

25. Αν $x>1$ νδο $x^2-3x+3>\frac{1}{x}$

Αφού $x>1$ το x θετικός.

$$x^3-3x^2+3x>1$$

$$x^3-3x^2+3x-1>0.$$

$$(x-1)^3>0$$

(+) ✓

Αφού $x>1 \Rightarrow x-1>0 \Rightarrow (x-1)^3>0$

26. Β). Νδο $x^2y^2+3x^2+y^2+3\geq 2xy^2+6x$ $\forall x,y\in\mathbb{R}$

$$x^2y^2+3x^2+y^2+3-2xy^2-6x\geq 0.$$

$$x^2(y^2+3)+y^2(1-2x)+3(1-2x)\geq 0$$

$$x^2(y^2+3)+(1-2x)(y^2+3)\geq 0$$ ✓

$$(y^2+3)(x^2+1-2x)\geq 0 \Rightarrow (y^2+3)(x-1)^2\geq 0$$

(+) (+)

28. (β) $9x^2-6x+2>0$

$9x^2-6x+1+1>0$

$(3x-1)^2+1>0$ ✓

(δ) $\alpha^2+\alpha\beta+\beta^2\geq 0$

$2\alpha^2+2\alpha\beta+2\beta^2\geq 0$

$\alpha^2+\alpha^2+2\alpha\beta+\beta^2+\beta^2\geq 0$

$\alpha^2+(\alpha+\beta)^2+\beta^2\geq 0$ ✓

32. (β) Αν $\alpha<\beta<0$ τότε $\alpha^2+\frac{2}{\alpha}>\beta^2+\frac{2}{\beta}$.

$\Leftrightarrow \begin{matrix}\alpha<0\\ \beta<0\end{matrix}\Big\} \alpha\beta>0$

$\alpha^3\beta+2\beta>\alpha\beta^3+2\alpha$

$\alpha^3\beta-2\beta-\alpha\beta^3-2\alpha>0$

$\alpha\beta(\alpha^2-\beta^2)-2(\alpha-\beta)>0$

$\alpha\beta(\alpha-\beta)(\alpha+\beta)-2(\alpha-\beta)>0$

$(\alpha-\beta)(\alpha\beta(\alpha+\beta)-2)>0$

$$(\overset{(-)}{\alpha - \beta})(\underbrace{\overset{(+)}{\alpha\beta}(\alpha+\beta)}_{(-)} - 2) > 0.$$

- $\alpha < \beta \Rightarrow \alpha - \beta < 0$

- $\left.\begin{matrix}\alpha < 0 \\ \beta < 0\end{matrix}\right\} \alpha\beta > 0$

$\left.\begin{matrix}\alpha < 0 \\ \beta < 0\end{matrix}\right\} \alpha + \beta < 0$

Apa $\alpha\beta(\alpha+\beta) < 0$

$\alpha\beta(\alpha+\beta) - 2 < 0$

Επαναληψη Μαθημα

Σελ 89

(1) α

(2) α

(3) α γ δ ε

(5) α

(6) α γ

(23) α

(26) α

(28) α γ

(32) α

Σελ 59

1 i) Νδο $a^2+9 \geq 6a$

$$a^2-6a+9 \geq 0$$

$$(a-3)^2 \geq 0 \quad \text{που ισχύει!}$$

ii) Νδο $2(a^2+\beta^2) \geq (a+\beta)^2$

$$2a^2+2\beta^2 \geq a^2+2a\beta+\beta^2$$

$$2a^2+2\beta^2-a^2-2a\beta-\beta^2 \geq 0$$

$$a^2+\beta^2-2a\beta \geq 0 \Rightarrow a^2-2a\beta+\beta^2 \geq 0$$

$$(a-\beta)^2 \geq 0 \quad \text{που ισχύει.}$$

2 Νδο $a^2+\beta^2-2a+1 \geq 0$.

$$a^2-2a+1+\beta^2 \geq 0$$

$$\boxed{(a-1)^2+\beta^2 \geq 0} \quad \text{που ισχύει!}$$

Η ισότητα ισχύει αν $a=1$ και $\beta=0$

γιατί $(a-1)^2 \geq 0$

$\beta^2 \geq 0$

3. i) $(x-2)^2 + (y+1)^2 = 0$

Προσπ $x-2=0$ και $y+1=0$

$x=2$ $y=-1$

ii). $x^2+y^2-2x+4y+5=0$

$x^2-2x+y^2+4y+5=0$

$x^2-2x+1+y^2+4y+4=0$

$(x-1)^2 + (y+2)^2 = 0$

$x-1=0$ και $y+2=0$

$x=1$ $y=-2$.

(4)

$4,5 < x < 4,6$

$5,3 < y < 5,4$.

(01) $x+y$.

- $4,5 < x < 4,6$
- $5,3 < y < 5,4$

} (+) $\boxed{9,8 < x+y < 10.}$

(B) . $x-y$

- $4,5 < x < 4,6 \Rightarrow 4,6 > x > 4,5$
- $5,3 < y < 5,4 \Rightarrow -5,3 > -y > -5,4$

} (+)

$\boxed{-0,7 > x-y > -0,9}$

(1) $\frac{x}{y}$

- $4,5 < x < 4,6 \Rightarrow 4,6 > x > 4,5$
- $5,3 < y < 5,4 \Rightarrow \frac{1}{5,3} > \frac{1}{y} > \frac{1}{5,4}$

} (.) $\boxed{\frac{4,6}{5,3} > \frac{x}{y} > \frac{4,5}{5,4}}$

(δ) x^2+y^2

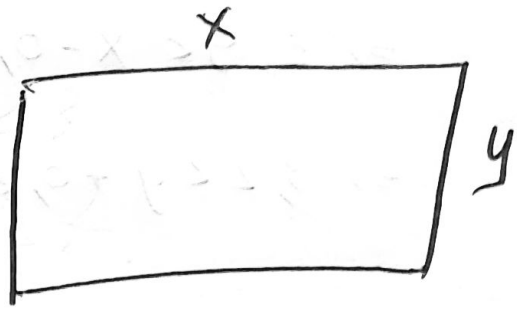
- $4,5 < x < 4,6 \Rightarrow 4,5^2 < x^2 < 4,6^2$
- $5,3 < y < 5,4 \Rightarrow 5,3^2 < y^2 < 5,4^2$

} (+)

$$4,5^2+5,3^2 < x^2+y^2 < 4,6^2+5,4^2$$

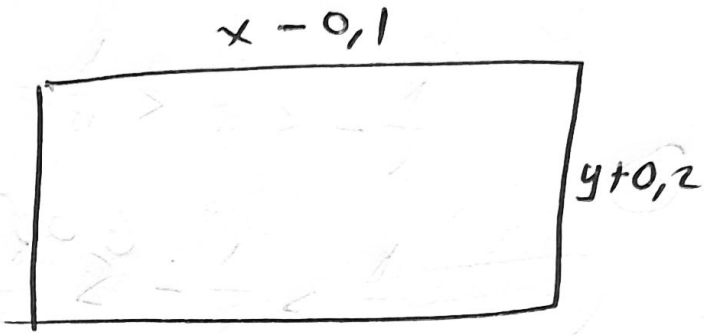
$$\boxed{48,34 < x^2+y^2 < 50,32}$$

5.



$2 < x < 3$

$3 < y < 5$



$$\Pi = 2(x-0,1) + 2(y+0,2)$$

$$\Pi = 2x - 0,2 + 2y + 0,4$$

$$\boxed{\Pi = 2x+2y+0,2}$$

$$\boxed{\epsilon = (x-0,1)(y+0,2)}$$

- $2 < x < 3 \Rightarrow 4 < 2x < 6$
- $3 < y < 5 \Rightarrow 6 < 2y < 10$

$\Big\}\oplus$ $\quad 10 < 2x+2y < 16$

$10,2 < 2x+2y+0,2 < 16,2$

$$10,2 < \Pi < 16,2$$

- $2 < x < 3 \Rightarrow 1,9 < x - 0,1 < 2,9$
- $3 < y < 5 \Rightarrow 3,2 < y + 0,2 < 5,2$

$\Big\} \odot$

$$6,08 < (x-0,1)(y+0,2) < 15,08$$

$$6,08 < \in < 15,08$$

6. $0 \le \alpha < \beta$ να δω $\frac{\alpha}{1+\alpha} < \frac{\beta}{1+\beta}$.

$\alpha(1+\beta) < \beta(1+\alpha)$

$\alpha + \alpha\beta < \beta + \alpha\beta$

$\alpha < \beta$ που ισχύει !.

7. Έστω $x > 5$

$5x > 25$

$5x - x^2 > 25 - x^2$

$x(5-x) > (5-x)(5+x)$

$x > 5 + x$

$0 > 5$.

• Αφού $x > 5$ το $0 > 5 - x$

Άρα το $5 - x$ που διαιρούμε είναι αρνητικό οπότε επρεπε να αλλάξει φορά

Β' ομάδα

① α, β, γ θετικοί.

i) Αν $\frac{\alpha}{\beta} < 1$ τότε $\frac{\alpha+\gamma}{\beta+\gamma} > \frac{\alpha}{\beta}$

↓

$\alpha < \beta$

$(\alpha+\gamma)\beta > \alpha(\beta+\gamma)$

$\alpha\beta + \beta\gamma > \alpha\beta + \alpha\gamma$

$\beta\gamma > \alpha\gamma$

$\beta > \alpha$ ✓

ii). Αν $\frac{\alpha}{\beta} > 1$

$\Rightarrow \alpha > \beta$ τότε $\frac{\alpha+\gamma}{\beta+\gamma} < \frac{\alpha}{\beta}$.

$\beta(\alpha+\gamma) < \alpha(\beta+\gamma)$

$\beta\alpha + \beta\gamma < \alpha\beta + \alpha\gamma$

$\beta\gamma < \alpha\gamma$

$\beta < \alpha$ ✓

2. Αν $\alpha > 1 > \beta$

ν.δ.ο. $\alpha + \beta > 1 + \alpha\beta$.

$$\alpha + \beta - 1 - \alpha\beta > 0$$

$$\alpha - 1 - \beta(\alpha - 1) > 0$$

$$(\alpha - 1)(1 - \beta) > 0.$$

Άρα ν.δ.ο. $(\alpha - 1)(1 - \beta)$ θετικό ✓
($\alpha - 1$: ⊕, $1 - \beta$: ⊕)

- $\alpha > 1 \Rightarrow \alpha - 1 > 0$

- $1 > \beta \Rightarrow 1 - \beta > 0$

3. $a, b > 0$ için $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right) \geq 4$

$a \cdot \frac{1}{a} + a \cdot \frac{1}{b} + b \cdot \frac{1}{a} + b \cdot \frac{1}{b} \geq 4$

$1 + \frac{a}{b} + \frac{b}{a} + 1 \geq 4$

$\frac{a}{b} + \frac{b}{a} \geq 2$

$ab > 0$ (+)

$ab \cdot \frac{a}{b} + ab \cdot \frac{b}{a} \geq 2ab$

$a^2 + b^2 \geq 2ab$

$a^2 - 2ab + b^2 \geq 0$

$(a-b)^2 \geq 0$

4. i) vdo $a^2+aB+B^2 \geq 0$

$$2a^2+2aB+2B^2 \geq 0$$

$$a^2+a^2+2aB+B^2+B^2 \geq 0.$$

$$a^2+(a+B)^2+B^2 \geq 0$$

ii). $a^2-aB+B^2 \geq 0$

$$2a^2-2aB+2B^2 \geq 0$$

$$a^2+a^2-2aB+B^2+B^2 \geq 0$$

$$a^2+(a-B)^2+B^2 \geq 0$$

✓

26

(a) Να δ.ο $x^3y^2 + x^3 \geq 2x^3y$ $\forall x \geq 0$.

$$x^3y^2 + x^3 - 2x^3y \geq 0$$

$$x^3\left(y^2 + 1 - 2y\right) \geq 0$$

$$x^3 (y-1)^2 \geq 0$$

$\oplus$ $\oplus$ ✓

- $x \geq 0 \Rightarrow x^3 \geq 0$
- $(y-1)^2 \geq 0$ ✓

23

$a < 1$

vso $a(a-1)^2 < 1 - a^2$

$a(a^2 - 2a + 1) < 1 - a^2$

$a^3 - 2a^2 + a < 1 - a^2$

$a^3 - 2a^2 + a - 1 + a^2 < 0$

$a^3 - a^2 + a - 1 < 0$

$a^2(a-1) + a - 1 < 0$

$(a-1)(a^2+1) < 0$

$\ominus \quad \oplus$ ✓

- $a < 1 \Rightarrow a - 1 < 0$

28 (01) $x^2 - 4x + 5 > 0$.

$x^2 - 4x + 4 + 1 > 0$

$(x-2)^2 + 1 > 0$ ✓

(02) $2x^2 + 2x + 1 > 0$.

$x^2 + x^2 + 2x + 1 > 0$

$x^2 + (x+1)^2 > 0$ ✓

32

(α) $0 < \alpha < \beta$ να δ.ο $\alpha^2 - \frac{1}{\alpha} < \beta^2 - \frac{1}{\beta}$.

• $\alpha > 0$
• $\beta > 0$
$\Big\} \alpha\beta > 0$

α' τρόπος

$$\alpha\beta \cdot \alpha^2 - \alpha\beta \cdot \frac{1}{\alpha} < \alpha\beta \cdot \beta^2 - \alpha\beta \frac{1}{\beta}$$

$$\beta\alpha^3 - \beta < \alpha\beta^3 - \alpha$$

$$\beta\alpha^3 - \beta - \alpha\beta^3 + \alpha < 0$$

$$\alpha\beta(\alpha^2 - \beta^2) + \alpha - \beta < 0$$

$$\alpha\beta(\alpha-\beta)(\alpha+\beta) + \alpha - \beta < 0$$

$$(\underset{(-)}{\alpha-\beta}) \cdot (\underset{(+)}{\alpha\beta}\underset{(+)}{(\alpha+\beta)} + \underset{(+)}{1}) < 0 \quad \checkmark$$

• $\alpha < \beta \Rightarrow \alpha - \beta < 0$

β' τρόπος

Αν $0<\alpha<\beta$ τότε $\alpha^2-\frac{1}{\alpha}<\beta^2-\frac{1}{\beta}$.

• $\alpha<\beta \Rightarrow \alpha^2<\beta^2$ (+)

Αν $x<y$ τότε $x^2<y^2$ εφόσον $x,y>0$
Αν $x<y$ τότε $x^2>y^2$ αν $x,y<0$

• $\alpha<\beta \Rightarrow \frac{1}{\alpha}>\frac{1}{\beta} \Rightarrow -\frac{1}{\alpha}<-\frac{1}{\beta}$.

$$\alpha^2-\frac{1}{\alpha}<\beta^2-\frac{1}{\beta}$$ ✓

31 Αν $\alpha < \beta$.

(α) νδο $\alpha^3 + \alpha < \beta^3 + \beta$.

$$\alpha^3 + \alpha - \beta^3 - \beta < 0$$

$$(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) + \alpha - \beta < 0$$

$$(\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2 + 1) < 0$$

(−) (+) (+) (+)

• $\alpha < \beta \Rightarrow \alpha - \beta < 0$

Χασαρε!

Πως βγαινει;

$\boxed{\alpha^2 + \alpha\beta + \beta^2}$ (+) τριωνυμο ως προς α.

$\Delta = \beta^2 - 4 \cdot 1 \cdot \beta^2$

$\Delta = \beta^2 - 4\beta^2 = -3\beta^2 < 0$

Β' τρόπος

Αν $\alpha < \beta$ τότε $\alpha^3 + \alpha < \beta^3 + \beta$

- $\alpha < \beta$ ⊕
- $\alpha < \beta \Rightarrow \alpha^3 < \beta^3$

$$\alpha + \alpha^3 < \beta + \beta^3 \quad \checkmark$$

1. Νδο $(\alpha-\beta)^2 + 8\beta^2 \geq 4\alpha\beta$

$\longrightarrow$ $\alpha^2 - 2\alpha\beta + \beta^2 + 8\beta^2 \geq 4\alpha\beta$

$\alpha^2 - 2\alpha\beta + 9\beta^2 - 4\alpha\beta \geq 0$

$\alpha^2 - 6\alpha\beta + 9\beta^2 \geq 0$

$(\alpha - 3\beta)^2 \geq 0$ που ισχύει!

2. Νδο $(\alpha+2\beta)^2 \geq 8\alpha\beta$

$\longrightarrow$ $(\alpha+2\beta)^2 \geq 8\alpha\beta$

$\alpha^2 + 4\alpha\beta + 4\beta^2 \geq 8\alpha\beta$

$\alpha^2 - 4\alpha\beta + 4\beta^2 \geq 0$

$(\alpha - 2\beta)^2 \geq 0$ που ισχύει!

3. Ndo $(a+3\beta)^2 \geq 2\beta(5a+2\beta)$

$\longrightarrow$ $a^2+6a\beta+9\beta^2 \geq 10a\beta+4\beta^2$

$a^2+6a\beta+9\beta^2-10a\beta-4\beta^2 \geq 0$

$a^2-4a\beta+5\beta^2 \geq 0$

$a^2-4a\beta+4\beta^2+\beta^2 \geq 0$

$(a-2\beta)^2+\beta^2 \geq 0$ που ισχυει!

4. $x^2-4x+5>0$

$x^2-4x+4+1>0$

$(x-2)^2+1>0$ που ισχυει.

5. $2a^2+2a+3>0$

$a^2+a^2+2a+1+2>0$

$a^2+(a+1)^2+2>0$ που ισχυει!

6. Νδο $x^2 + \frac{1}{x^2+1} \geq 1$. Επειδή x^2+1 (+)

$(x^2+1)x^2 + (x^2+1)\frac{1}{x^2+1} \geq (x^2+1)\cdot 1$

$x^4 + x^2 + 1 \geq x^2 + 1$

$x^4 \geq 0$ που ισχύει!.

---

7 Αν $x>2$ νδο $x^3 > 2x^2 - x + 2$.

$\longrightarrow \quad x^3 - 2x^2 + x - 2 > 0$

$x^2(x-2) + x - 2 > 0$

$(x-2)(x^2+1) > 0$ ✓ (με $x-2$ (+), x^2+1 (+))

• $x^2 \geq 0 \Rightarrow x^2+1 \geq 1 \Rightarrow x^2+1 > 0$

• $x > 2 \Rightarrow x - 2 > 0$

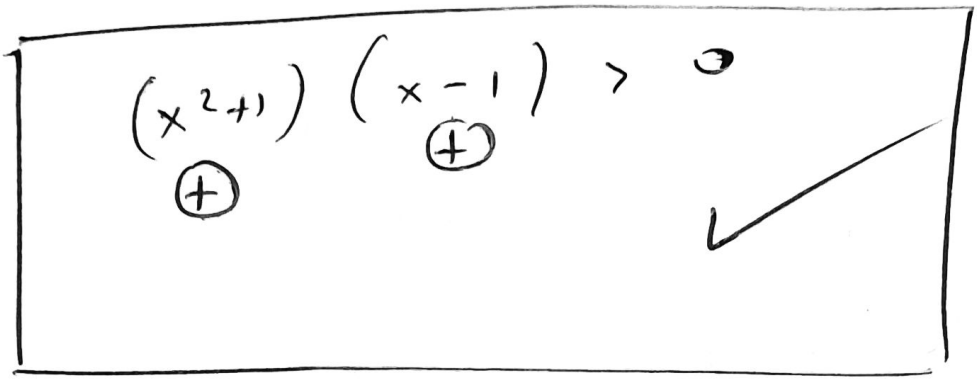
8. Αν $x>1$ νδο $x^3+x > x^2+1$

$\longrightarrow$ $x^3+x-x^2-1 > 0$

$x(x^2+1)-(x^2+1) > 0$

$(x^2+1)(x-1) > 0$

(x^2+1) ⊕ $(x-1)$ ⊕



- $x^2 \geq 0 \Rightarrow x^2+1 \geq 1 \Rightarrow x^2+1 > 0$
- $x>1 \Rightarrow x-1>0$

9. Αν $x>1$ νδο $\frac{x+1}{1-x} < 1$.

$1-x$ ⊖

- Αφού $x>1$

$0>1-x$

$x+1 > 1-x$

$x+1-1+x > 0$

$2x > 0$

$x>0$ που ισχύει,

Αφού $x>1$ προφανώς $x>0$

10. Αν $\alpha < 3 < \beta$ τότε $\alpha\beta - 3\alpha - 3\beta + 9 < 0$

$\longrightarrow$ $\alpha\beta - 3\alpha - 3\beta + 9 < 0$

$\beta(\alpha-3) - 3(\alpha-3) < 0$

$(\alpha-3)(\beta-3) < 0$

$\ominus \quad \oplus$

- $\alpha < 3 \Rightarrow \alpha - 3 < 0$
- $3 < \beta \Rightarrow 0 < \beta - 3$

11. Αν $x>0$, $y>0$ τότε $\frac{1}{2} \le \frac{x^2+y^2}{(x+y)^2} < 1$.

Αρκεί τότε $\frac{1}{2} \le \frac{x^2+y^2}{(x+y)^2}$ και $\frac{x^2+y^2}{(x+y)^2} < 1$.

$(x+y)^2 \le 2(x^2+y^2)$.

$x^2+2xy+y^2 \le 2x^2+2y^2$

$-x^2+2xy-y^2 \le 0$.

$x^2-2xy+y^2 \ge 0$

$(x-y)^2 \ge 0$.

$x^2+y^2 < (x+y)^2$

$x^2+y^2 < x^2+2xy+y^2$

$0 < 2xy$

$x>0 \quad y>0$

12. Αν $0<\alpha<\beta$ να δ. $\alpha^7-\frac{1}{\alpha}<\beta^7-\frac{1}{\beta}$

- $\alpha<\beta \Rightarrow \alpha^7<\beta^7$
- $\alpha<\beta \Rightarrow \frac{1}{\alpha}>\frac{1}{\beta} \Rightarrow -\frac{1}{\alpha}<-\frac{1}{\beta}$.

$\oplus$

$$\alpha^7-\frac{1}{\alpha}<\beta^7-\frac{1}{\beta}$$

Όταν αντιστρέφω ορισμένη ανισότητα

αλλαγή φορά.

Προσοχή

- $\alpha<\beta \Leftrightarrow \alpha^\nu<\beta^\nu$ α,β,ν θετικοί
- $\alpha<\beta \Leftrightarrow \alpha^\nu<\beta^\nu$ $\alpha,\beta\in\mathbb{R}$ και ν περιττός.

13. Αν $\alpha < \beta < 0$ να δ.ο $\alpha^8 + \frac{1}{\alpha} > \beta^8 + \frac{1}{\beta}$

- $\alpha < \beta \rightarrow \alpha^8 > \beta^8$
- $\alpha < \beta \Rightarrow \frac{1}{\alpha} > \frac{1}{\beta}$.

$\Big\}$ ⊕ $\boxed{\alpha^8 + \frac{1}{\alpha} > \beta^8 + \frac{1}{\beta}}$

Προσοχή

Αν $\alpha < \beta \Rightarrow \alpha^2 < \beta^2$ μόνο αν α, β θετικοί

Αν $\alpha < \beta \Rightarrow \alpha^2 > \beta^2$ μόνο αν α, β αρνητικοί

14. Να βρεθούν $\alpha, \beta \in \mathbb{R}$ ώστε

$$\alpha^2+\beta^2-4\alpha+4=0$$

$$\underbrace{\alpha^2-4\alpha+4}_{(\alpha-2)^2}+\beta^2=0$$

$$\alpha-2=0 \quad \text{και} \quad \beta=0$$

$$\alpha=2$$

Βασική ιδιότητα

$$\alpha^2+\beta^2=0$$

$$\Leftrightarrow$$

$$\alpha=0 \quad \text{και} \quad \beta=0$$

15. Να βρεθούν $x, y \in \mathbb{R}$ ώστε

$$x^2+y^2-6x+4y+13=0.$$

$$\underbrace{x^2-6x+9}_{(x-3)^2}+y^2+4y+4=0$$

$$(x-3)^2+(y+2)^2=0$$

$$x-3=0 \quad \text{και} \quad y+2=0$$

$$x=3 \qquad y=-2$$

16. Να βρεθούν τα $x, y \in \mathbb{R}$ ώστε

$$x^2+y^2-2(x-y)+2 \leq 0 .$$

$$x^2+y^2-2x+2y+2 \leq 0$$

$$\underbrace{x^2-2x+1} + y^2+2y+1 \leq 0$$

$$(x-1)^2 + (y+1)^2 \leq 0 .$$

$$(x-1)^2 + (y+1)^2 = 0$$

$x-1=0$ και $y+1=0$

$x=1$ $y=-1$

17. Βρεί $x, y \in \mathbb{R}$ ωστε

$$3x^2+y^2+z^2+4=2x(y-z+2).$$

$$3x^2+y^2+z^2+4=2xy-2xz+4x$$

$$3x^2+y^2+z^2+4-2xy+2xz-4x=0$$

$$\underbrace{x^2-2xy+y^2}+\underbrace{x^2+2xz+z^2}+\underbrace{x^2-4x+4}=0$$

$$(x-y)^2+(x+z)^2+(x-2)^2=0$$

$x-y=0$ και $x+z=0$ και $x-2=0$

$2-y=0$ $\quad$ $2+z=0$ $\quad$ $x=2$

$y=2$ $\quad$ $z=-2$

Λυρσα Ασκηση 1

Εστω $2 < x < 4$

(α) $x+1$

- $2 < x < 4 \Rightarrow 2+1 < x+1 < 4+1$

$$\boxed{3 < x+1 < 5.}$$

(β) $2x$

- $2 < x < 4 \Rightarrow 2 \cdot 2 < 2 \cdot x < 4 \cdot 2 \Rightarrow \boxed{4 < 2x < 8}$

(γ) $-3x$

- $2 < x < 4 \Rightarrow -3 \cdot 2 > -3 \cdot x > -3 \cdot 4$

$$\boxed{-6 > -3x > -12}$$

(δ) $2x^2$

- $2 < x < 4 \Rightarrow 2^2 < x^2 < 4^2 \Rightarrow 4 < x^2 < 16$

$$\boxed{8 < 2x^2 < 32}$$

(ε) $-\frac{3}{x}$

• $2 < x < 4 \Rightarrow \frac{1}{2} > \frac{1}{x} > \frac{1}{4} \Rightarrow -\frac{3}{2} < -\frac{3}{x} < -\frac{3}{4}$

Βασικοί κανόνες

1. $\alpha < \beta \Rightarrow \alpha^2 < \beta^2$ $\quad \alpha, \beta > 0$
2. $\alpha < \beta \Rightarrow \alpha^2 > \beta^2$ $\quad \alpha, \beta < 0$
3. $\alpha < \beta \Rightarrow \alpha^7 < \beta^7$
4. $\alpha < \beta \rightarrow \frac{1}{\alpha} > \frac{1}{\beta}$ $\quad \alpha, \beta$ ομόσημοι.

Λυμένα Άσκηση 2

Έστω $1 < \alpha < 3$ και $2 < \beta < 4$

1. $2\alpha - 3\beta$

- $1 < \alpha < 3 \Rightarrow 2 < 2\alpha < 6 \Rightarrow 6 > 2\alpha > 2$
- $2 < \beta < 4 \Rightarrow -6 > -3\beta > -12$ $\Big| \oplus$

$6 - 6 > 2\alpha - 3\beta > 2 - 12$

$$0 > 2\alpha - 3\beta > -10$$

2. $\alpha^2 - 2\alpha\beta$

- $1 < \alpha < 3 \Rightarrow 1^2 < \alpha^2 < 3^2 \Rightarrow 1 < \alpha^2 < 9$

$9 > \alpha^2 > 1$

- $1 < \alpha < 3$
- $2 < \beta < 4$

$\Big\} \odot \; 2 < \alpha\beta < 12 \quad \Big/ \oplus$

$-4 > -2\alpha\beta > -24$

$$5 > \alpha^2 - 2\alpha\beta > -23$$

34. $1<a<3$ $\quad -4<\beta<-2$

(α) $a-\beta$,

- $1<a<3 \Rightarrow 3>a>1$
- $-4<\beta<-2 \Rightarrow 4>-\beta>2$

$\Big\}\oplus$ $\boxed{7>a-\beta>3}$

(β) $a\beta$

- $1<a<3 \Rightarrow 3>a>1$
- $-4<\beta<-2 \Rightarrow 4>-\beta>2$

$\Big\}\odot$ $12>-a\beta>2$

$\boxed{-12<a\beta<-2}$

(γ) $\frac{a}{\beta}$

- $1<a<3 \Rightarrow -1>-a>-3$
- $-4<\beta<-2 \Rightarrow -\frac{1}{4}>\frac{1}{\beta}>-\frac{1}{2}$

$\Big\}\odot$ $\frac{1}{4}<-\frac{a}{\beta}<\frac{3}{2}$

$-\frac{1}{4}>\frac{a}{\beta}>-\frac{3}{2}$

⑥ a^2+B^2

- $1<a<3 \Rightarrow 1<a^2<9 \Rightarrow 9>a^2>1$
- $-4<B<-2 \Rightarrow 16>B^2>4$ ⊕

$$25>a^2+B^2>5$$

⑦ $\frac{a}{a+1}$

- $1<a<3 \Rightarrow 2<a+1<4 \Rightarrow \frac{1}{2}>\frac{1}{a+1}>\frac{1}{4}$
- $1<a<3 \Rightarrow 3>a>1$ ⊗

$$\frac{3}{2}>\frac{a}{a+1}>\frac{1}{4}$$

33. Σελ 93

$$-3 < x < -2$$

$$-2 < y < -1$$

(α) xy

- $-3 < x < -2$
- $-2 < y < -1$

$\Big\}\odot$ $6 > xy > 2$

(β) $\frac{x}{y}$

- $-3 < x < -2 \Rightarrow -2 > x > -3$
- $-2 < y < -1 \Rightarrow -\frac{1}{2} > \frac{1}{y} > -\frac{1}{1}$

$\Big\}\odot$

$$2 \cdot \frac{1}{2} < x \cdot \frac{1}{y} < 3 \cdot 1$$

$$1 < \frac{x}{y} < 3$$

(f) x^2+y^3

- $-3 < x < -2 \Rightarrow 9 > x^2 > 2 \Rightarrow 2 < x^2 < 9$
- $-2 < y < -1 \Rightarrow -8 < y^3 < -1$ ⊥ ⊕

$$-6 < x^2+y^3 < 8$$

Επομενο Μαθημα

Σελ 90

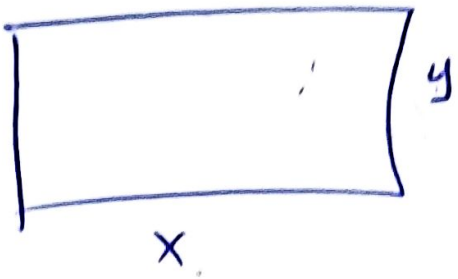
(11).

(12)

(13).

(14)

18.



- $4 \le x \le 7$
- $2 \le y \le 3$

(α) Περίμετρος και εμβαδόν.

Περίμετρος: $\Pi = 2x+2y$

$4 \le x \le 7 \Rightarrow 8 \le 2x \le 14$

$2 \le y \le 3 \Rightarrow 4 \le 2y \le 6$

$\Rightarrow 12 \le 2x+2y \le 20 \Rightarrow$

$\Rightarrow \boxed{12 \le \Pi \le 20}$

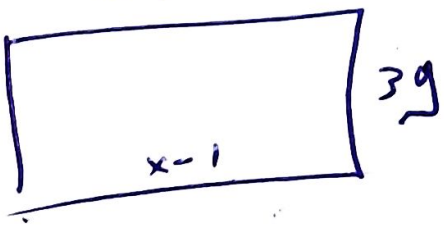
Εμβαδόν: $E = xy$

$4 \le x \le 7$

$2 \le y \le 3$

$\Rightarrow 8 \le xy \le 21 \Rightarrow \boxed{8 \le E \le 21}$

(β) Εάν το μήκος x μειωθεί κατά 1 και το y τριπλασιαστεί βρείτε Π και E:



Περίμετρος: $\Pi = 2(x-1)+6y$

$\Pi = 2x+6y-2$

$4 \le x \le 7 \Rightarrow 3 \le x-1 \le 6$

$2 \le y \le 3 \Rightarrow 6 \le 3y \le 9$

$\Rightarrow 6 \le 2(x-1) \le 12$

$\Rightarrow 12 \le 6y \le 18$

$\Rightarrow 18 \le 6y+2(x-1) \le 30$

Εμβαδόν: $6 \le 3y \le 9$

$3 \le x-1 \le 6$

$\Rightarrow 18 \le 3y(x-1) \le 54$

14

(a)

- $2 < x \leq 3 \Rightarrow -2 \geq -x \geq -3$
- $4 \leq y < 5 \Rightarrow 5 > y \geq 4$

} (+)

$$3 > y - x \geq 1$$

(b)

- $2 < x \leq 3$
- $4 \leq y < 5 \Rightarrow 16 \leq y^2 \leq 25$

- $2 < x \leq 3$
- $4 \leq y < 5$

} (·) $8 < xy < 15$.

$-8 > -xy > -15$

(+)

$$-15 < -xy < -8$$

$$1 < y^2 - xy < 17$$

(7) $2 < x \leq 3$
$4 \leq y < 5$ $\Rightarrow \frac{1}{4} \geq \frac{1}{y} > \frac{1}{5}$

$-\frac{1}{4} \leq \frac{-1}{y} < \frac{-1}{5}$

$2 - \frac{1}{4} < x - \frac{1}{y} < 3 - \frac{1}{5}$.

$\frac{7}{4} < x - \frac{1}{y} < \frac{14}{5}$.

(8) . $2 < x \leq 3$ $\Rightarrow 4 < x^2 \leq 9$

. $4 \leq y < 5$ $\Rightarrow 64 \leq y^3 < 125$.

$-64 \geq -y^3 > -125$

$-125 < -y^3 \leq -64$

$-121 < x^2 - y^3 \leq -55$

36. $x \in [3,5]$ $\quad$ $y \in [-2,-1]$

$3 \le x \le 5$ $\quad$ $-2 \le y \le -1$

(A) $y - x$

~~$3 \le x$~~ $\quad -3 \ge -x \ge -5$

$-1 \ge y \ge -2$

$-4 \ge y - x \ge -7$

(B) $x^2 + y^2$ ✓

$3^2 \le x^2 \le 5^2$ $\quad$ ~~$(-2)^2 \ge y^2 \ge (-1)^2$~~

$1 \le y^2 \le 4$ $\quad$ $4 \ge y^2 \ge 1$

$10 \le y^2 + x^2 \le 29$

(C) $\frac{x}{y}$ $\quad$ $-\frac{1}{2} \ge \frac{1}{y} \ge -1$ $\quad$ $\frac{1}{2} \le -\frac{1}{y} \le 1$ $\}\odot$

$5 \ge x \ge 3$ $\quad$ $3 \le x \le 5$

$-\frac{5}{2} \ge \frac{x}{y} \ge -3$ $\quad$ $\frac{3}{2} \le -\frac{x}{y} \le 5$.

$\frac{3}{2} \ge \frac{x}{y} \ge 5$.

35 Αν $x \in [2,3]$ και $y \in [1,2]$

$2 \le x \le 3$ $\quad$ $1 \le y \le 2$

(α) $x+y$

$$\left.\begin{array}{l} 2 \le x \le 3 \\ 1 \le y \le 2 \end{array}\right\} 3 \le x+y \le 5$$

(β) $2x-3y$

$$\left.\begin{array}{l} 2 \le x \le 3 \\ 1 \le y \le 2 \end{array}\right\} \begin{array}{l} 4 \le 2x \le 6 \\ -3 \ge -3y \ge -6 \end{array} \left\} \begin{array}{l} 4 \le 2x \le 6 \\ -6 \le -3y \le -3 \end{array}\right\} -2 \le 2x-3y \le 3$$

(γ) $\dfrac{x}{y}$

$$\left.\begin{array}{l} 2 \le x \le 3 \\ 1 \le y \le 2 \end{array}\right\} \begin{array}{l} 2 \le x \le 3 \\ \frac{1}{1} \ge \frac{1}{y} \ge \frac{1}{2} \end{array} \left\} \begin{array}{l} 2 \le x \le 3 \\ \frac{1}{2} \le \frac{1}{y} \le \frac{1}{1} \end{array}\right\} \frac{2}{2} \le \frac{x}{y} \le \frac{3}{1}$$

$\Rightarrow 1 \le \frac{x}{y} \le 3$

(δ) x^2-y^2

$$\left.\begin{array}{l} 2 \le x \le 3 \\ 1 \le y \le 2 \end{array}\right\} \begin{array}{l} 4 \le x^2 \le 9 \\ 1 \le y^2 \le 4 \end{array} \left\} \begin{array}{l} 4 \le x^2 \le 9 \\ -1 \le -y^2 \ge -4 \end{array} \right\} \begin{array}{l} 4 \le x^2 \le 9 \\ -4 \le -y^2 \le -1 \end{array}$$

$\Rightarrow 0 \le x^2-y^2 \le 8$

8. $2 < x < 3$

(A) $x+1$

- $2 < x < 3 \Rightarrow 3 < x+1 < 4$

(B) $2x-3$

- $2 < x < 3 \Rightarrow 4 < 2x < 6 \Rightarrow 4-3 < 2x-3 < 6-3 \Rightarrow 1 < 2x-3 < 3$

(Γ) $-3x+1$

- $2 < x < 3 \Rightarrow -6 > -3x > -9 \Rightarrow -6+1 > -3x+1 > -9+1 \Rightarrow$

$\Rightarrow -5 > -3x+1 > -8$

(Δ) $2-x$

- $2 < x < 3 \Rightarrow -2 > -x > -3 \Rightarrow 0 > 2-x > -1$

(ε) $\frac{x}{3} - 1$

• $2 < x < 3 \Rightarrow \frac{6}{3} < \frac{x}{3} < \frac{9}{3} \Rightarrow \frac{6}{3} - 1 < \frac{x}{3} - 1 < \frac{9}{3} - 1 \Rightarrow$

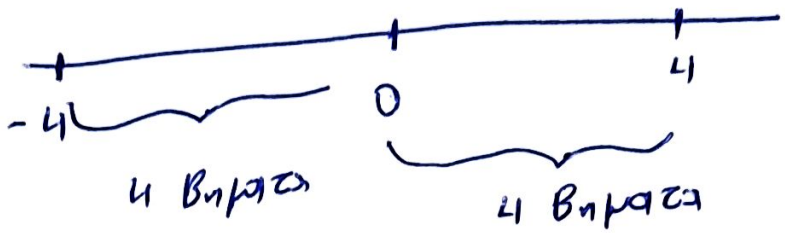
$\Rightarrow 1 < \frac{x}{3} - 1 < 2$

(29) $\frac{2}{x}$

• $2 < x < 3 \Rightarrow \frac{1}{2} > \frac{1}{x} > \frac{1}{3} \Rightarrow \frac{2}{2} > \frac{2}{x} > \frac{2}{3} \Rightarrow 1 > \frac{2}{x} > \frac{2}{3}$

Απόλυτη Τιμή

1.



$|-4| = 4$. $|4| = 4$

2. $|0| = 0$ 0 θετικός

3. $|\alpha| = -\alpha$ α αρνητικός

4. $|x| = |-x| \geq 0$

5. $|x| \geq x$ και $|-x| \geq x$.

6. $|\alpha|^2 = \alpha^2$

7. $|\alpha \cdot \beta| = |\alpha| \cdot |\beta|$

8. $\left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|}$

9. $|\alpha + \beta| \leq |\alpha| + |\beta|$.

1. (Σ) $\big|\,|1+|-2|\,|-|3-|-5|\,\big| =$

$= |1+2|-|3-5|$

$= |3|-|-2| = 3-2 = 1$.

2. (Σ) $|\sqrt{2}-1|$. $\overset{(+)}{=} \sqrt{2}-1$.

Έστω $\sqrt{2} < 1 \Rightarrow 2 < 1$ ΑΤΟΠΟ!

Άρα $\sqrt{2} > 1 \Rightarrow \sqrt{2}-1 > 0$

(Σ2) $|\sqrt{5}-3| \overset{(-)}{=} 3-\sqrt{5}$.

Έστω $\sqrt{5} < 3 \Rightarrow 5 < 9$ που ισχύει!.

$\sqrt{5}-3 < 0$

3. (в) $|\overset{(-)}{\pi-4}| + |\overset{(-)}{\sqrt{2}-2}| - |\overset{(-)}{\sqrt{2}-\sqrt{3}}|$ $\overset{(+)}{=}$

- $\pi < 4 \Rightarrow \pi - 4 < 0$
- $2 > \sqrt{2} \Rightarrow 0 > \sqrt{2} - 2$
- $\sqrt{2} < \sqrt{3} \Rightarrow \sqrt{2} - \sqrt{3} < 0$

$\overset{(*)}{=} -\pi + 4 - \sqrt{2} + 2 - (-\sqrt{2} + \sqrt{3})$

$= -\pi + 4 - \cancel{\sqrt{2}} + 2 + \cancel{\sqrt{2}} - \sqrt{3} = 6 - \sqrt{3} - \pi.$

4. (б) $|-x^2 + 10x - 25| = |-(x^2 - 10x + 25)|$

$= |\overset{(-)}{-(x-5)^2}| = (x-5)^2$

(г) $|\overset{(+)}{|x| + 1}| - |x| =$

$= \cancel{|x|} + 1 - \cancel{|x|} = 1.$

7. (B) $\left|2-\overset{(+)}{|x-3|}\right| = \left|2-(x-3)\right|$ (*)

- $3<x<5 \Rightarrow \boxed{0<x-3<2}$

(*) $\left|2-x+3\right| = \left|\overset{(+)}{5-x}\right| = 5-x$.

- $3<x<5 \Rightarrow -3>-x>-5$

$5-3>5-x>0$

$\boxed{2>5-x>0}$

(1) $\left|\overset{(+)}{x-2}\right| + \left|\overset{(-)}{x-6}\right| + \left|\overset{(+)}{x}\right|$ (*)

- $3<x<5 \Rightarrow \boxed{1<x-2<3}$
- $3<x<5 \Rightarrow \boxed{-3<x-6<-1}$
- $\boxed{3<x<5}$

(*) $x-2-x+6+x = x+4$

$-1 < x < 3$

9. (B) $\left|6 - |x-5|\right| = \left|6-(-x+5)\right| = |6+x-5| =$

$= |x+1| =$

$= x+1$

$-1 < x < 3$

$-1-5 < x-5 < 3-5$

$-6 < x-5 < -2$

$-1 < x < 3$

$-1+1 < x+1 < 3+1$

$0 < x+1 < 4$

8. (B) $A = |x-2| - |x-3|$

- $x < 2 \Rightarrow x - 2 < 0$

- $x < 2 \Rightarrow x - 3 < -1$

$A = -x + 2 - (-x + 3) = -x + 2 + x - 3 = -1$ ✓

(B) $A = |x-2| - |x-3|$

- $x > 3 \Rightarrow x - 2 > 1$

- $x > 3 \Rightarrow x - 3 > 0$

$A = x + 2 - (x + 3) = x + 2 - x - 3 = -1$ ✓

10. $\alpha < \beta < \gamma$

(B) $\beta = \left| |\alpha - \beta| + |\beta - \gamma| \right| \Rightarrow$

- $\alpha < \beta \Rightarrow \alpha - \beta < 0$
- $\beta < \gamma \Rightarrow \beta - \gamma < 0$

$\Longrightarrow \beta = |-\alpha + \beta - \beta + \gamma|$

$\beta = |\gamma - \alpha| = \gamma - \alpha$ ✓

- $\gamma > \alpha \Rightarrow \gamma - \alpha > 0$

11. (B) $\alpha < -1 < \beta$

$|\alpha + 1| - |\beta + 1| + |\beta - \alpha|$ (⊖, ⊕, ⊕)

$\alpha + 1 < 0$

$\beta + 1 > 0$

$\beta - \alpha > 0$

$\Rightarrow -\alpha - 1 - \beta - 1 + \beta - \alpha = -2\alpha - 2 = -2(\alpha + 1)$

Εποραω Μαθημα

(1) α β γ δ

(2) α β γ δ

(3) αι.

(4) α β γ

(5)

(6)

(7) αι

(8) αι.

(9) α

(10) αι

(11) α.

18. (B) Ndo $|2\alpha-6|-6\left|1-\frac{\alpha}{3}\right|=0$

$\Rightarrow |2\alpha-6|-\left|6-6\frac{\alpha}{3}\right|=0$

$|2\alpha-6|-|6-2\alpha|=0$

$|2\alpha-6|-|2\alpha-6|=0$

$0=0$ ✓

Σημαντικά ιδιώματα

$|\alpha\cdot\beta|=|\alpha|\cdot|\beta|$ $\quad \left|\frac{\alpha}{\beta}\right|=\frac{|\alpha|}{|\beta|}$ $\quad |\alpha+\beta|\le|\alpha|+|\beta|$.

17. (B) Να δ.ο $|a+1|^2 + |a-1|^2 = 2|a|^2 + 2$.

Απόδειξη $(a+1)^2 + (a-1)^2 = 2a^2 + 2$

$a^2 + 2a + 1 + a^2 - 2a + 1 = 2a^2 + 2$

$2a^2 + 2 = 2a^2 + 2$

$0 = 0$.

Π.Χ

$2^2 = 4$

$|2|^2 = 2^2 = 4$

$|2^2| = |4| = 4$

$|x|^2 = x^2$

19. (B) $B = 4\left|\frac{x}{2} - 1\right| + 8\left|\frac{x}{4} - \frac{1}{2}\right|$ (−) (−) (+)

- $x < 2 \Rightarrow \frac{x}{2} < \frac{2}{2} \Rightarrow \frac{x}{2} < 1 \Rightarrow \frac{x}{2} - 1 < 0$
- $x < 2 \Rightarrow \frac{x}{4} < \frac{2}{4} \Rightarrow \frac{x}{4} < \frac{1}{2} \Rightarrow \frac{x}{4} - \frac{1}{2} < 0$

(*) $4\left(-\frac{x}{2} + 1\right) + 8\left(-\frac{x}{4} + \frac{1}{2}\right) =$

$= -4\frac{x}{2} + 4 - 8\frac{x}{4} + 8 \cdot \frac{1}{2} =$

$= -2x + 4 - 2x + 4$

$= -4x + 8$.

13. (α) $A = |3x-y| \overset{(+)}{=} 3x-y$.

. $1 < x < 3 \Rightarrow 3 < 3x < 9$

. $2 < y < 3 \Rightarrow -2 > -y > -3$ (+)

$\Rightarrow 9 > 3x > 3$

$7 > 3x-y > 0$

14. $A = |x-2| + 3x - 1$.

1η. Αν $x-2 \geq 0$ δηλαδή $x \geq 2$ τοτε

$A = |\overset{(+)}{x-2}| + 3x - 1$

$A = x-2+3x-1$

$A = 4x-3$

2η. Αν $x-2 < 0 \Rightarrow x < 2$ τοτε

$A = |\overset{(-)}{x-2}| + 3x - 1$

$A = -x+2+3x-1$

$A = 2x+1$

Αρα $A = \begin{cases} 4x-3, & x \geq 2 \\ 2x+1, & x < 2 . \end{cases}$

15. (B). $B = \frac{|2x-1|}{|1-2x|} - 3\frac{|3y+2|}{|-2-3y|} + 2\frac{|x-y+z|}{|y-x-z|}$.

$|x| = |-x|$

$B = \frac{|2x-1|}{|2x-1|} - 3\frac{|3y+2|}{|3y+2|} + 2\frac{|x-y+z|}{|x-y+z|}$.

$B = 1 - 3 + 2 = 0$.

16. $A = \big||x-1|x|| - |x+1|x|\big|$.

α' τρόπος

(1η) Αν $x \geq 0$ τότε $A = \big|x-1\overset{(+)}{|x|}\big| - \big|x+1\overset{(+)}{|x|}\big|$

$A = |x-x| - |x+x| = -|\overset{(+)}{2x}| = -2x$.

(2η) Αν $x < 0$ τότε $A = \big|x-1\overset{(-)}{|x|}\big| - \big|x+1\overset{(-)}{|x|}\big|$

$A = |x-(-x)| - |x+(-x)| = |\overset{(-)}{2x}| - 0 = -2x$

$\boxed{A = -2x}$

B' τρόπος

$$A = \left| \overset{\ominus}{x - |x|} \right| - \left| \overset{\oplus}{x + |x|} \right| \overset{(*)}{=}$$

$|x| \geq x$

$|x| \geq -x$

- $|x| \geq x \Rightarrow 0 \geq x - |x|$
- $|x| \geq -x \Rightarrow |x| + x \geq 0$

$$\overset{(*)}{=} -x + |x| - (x + |x|) = -x + \cancel{|x|} - x - \cancel{|x|}$$

$$= -2x$$

Επομενο Μαθημα

Σελ 112

(12) .

(13) Βδ.

(15) α

(17) α

(8) α

(19) α .

Μεγάλη Προσοχή

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1. $|x| < \theta \iff -\theta < x < \theta$
   ή $x \in (-\theta, \theta)$.

2. $|x| > \theta \iff x > \theta$ ή $x < -\theta$
   $x \in (-\infty, -\theta) \cup (\theta, +\infty)$.

3. $|x| = \theta \iff x = \theta$ ή $x = -\theta$.

4. $|x| = |y| \iff x = y$ ή $x = -y$.

5. $|x - y| = d(x, y)$.

21. (α) $|x| = 1 \Rightarrow x = 1$ ή $x = -1$.

(β) $|x| - 3 = 0 \Rightarrow |x| = 3 \Rightarrow x = 3$ ή $x = -3$

(γ) $3|x| - 5 = 0 \Leftrightarrow 3|x| = 5 \Rightarrow |x| = \frac{5}{3}$

$x = \frac{5}{3}$ ή $x = -\frac{5}{3}$

(δ) $|x| + 2 = 0 \Rightarrow |x| = -2$. Αδύνατη!

(ε) $2|x| - 5|-x| + 6 = 0 \Leftrightarrow 2|x| - 5|x| + 6 = 0$

$|-x| = |x|$

$-3|x| + 6 = 0$

$-3|x| = -6$

$|x| = 2$

$x = 2$ ή $x = -2$

(23) $|2x| - \left|\frac{x}{2}\right| - 1 = 0$.

$|a \cdot \beta| = |\alpha||\beta|$

$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$.

$|2||x| - \frac{|x|}{|2|} = 1$

$2|x| - \frac{1}{2}|x| = 1$

$4|x| - |x| = 2$

$3|x| = 2$

$|x| = \frac{2}{3}$

$x = \frac{2}{3}$ ή $x = -\frac{2}{3}$.

(J). $x^2 - 2|x| = 0 \Rightarrow |x|^2 - 2|x| = 0$

$x^2 = |x|^2$

$|x|(|x|-2) = 0$

$|x| = 0$ ni $|x|-2 = 0$

$x = 0$ $|x| = 2$

$x = 2$ ni $x = -2$

22. (a) $|x-1| = 5$

$x-1 = 5$ ni $x-1 = -5$

$x = 6$ $x = -4$

(b) $|x-2| - 1 = 0$

$|x-2| = 1$

$x-2 = 1$ ni $x-2 = -1$

$x = 3$ $x = 1$

(i) $3|x-2| - 2|2-x| = 5$

$3|x-2| - 2|x-2| = 5$

$|x-2| = 5$ ni $x-2 = -5$

$x-2 = 5$ $x = -3$

$x = 7$

(δ) $|3x-6| - |4-2x| = 1$

$3|x-2| - 2|2-x| = 1$

$3|x-2| - 2|x-2| = 1$

$|x-2| = 1$

$x-2=1$ ή $x-2=-1$

$x=3$ $x=1$

23. (01) $|2x-1| = |x-2|$.

$2x-1 = x-2$ ή $2x-1 = -x+2$

$x=-1$ $3x=3$

$x=1$

(β) $|3x-2| - |2x-3| = 0$

$|3x-2| = |2x-3|$

$3x-2 = 2x-3$ ή $3x-2 = -2x+3$

$x=-1$ $5x=5$

$x=1$

(f) $3|x-2| - 2|x| = 0$

$3|x-2| = 2|x|$

$|3x-6| = |2x|$

$3x-6 = 2x$ ó $3x-6 = -2x$.

$\underline{x = 6}$ $\qquad 5x = 6$

$x = \frac{6}{5}$

24. (a) $|x| < 5 \Rightarrow -5 < x < 5$

$x \in (-5, 5)$.

(b) $|x| \geq 2 \Rightarrow x \geq 2$ ó $x \leq -2$

$x \in (-\infty, -2] \cup [2, +\infty)$.

(c) $3|x| - 2 < 0 \Rightarrow 3|x| < 2 \Rightarrow |x| < \frac{2}{3}$.

$-\frac{2}{3} < x < \frac{2}{3}$ $\qquad x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$.

(d) $2 - 3|x| \leq 0 \Rightarrow 2 \leq 3|x| \Rightarrow \frac{2}{3} \leq |x|$

$x \geq \frac{2}{3}$ ó $x \leq -\frac{2}{3}$ $\qquad x \in \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{2}{3}, +\infty\right)$

25. (α) $|x-1|<3$

$-3<x-1<3$

$-2<x<4$

$x\in(-2,4)$.

(β) $|x-2|\geq 5$

$x-2\geq 5$ ή $x-2\leq -5$

$x\geq 7$ ή $x\leq -3$

$x\in(-\infty,-3]\cup[7,+\infty)$.

(δ) $|3x-2|-1<0$

$|3x-2|<1$

$-1<3x-2<1$

$1<3x<3$

$\frac{1}{3}<x<1$

$x\in(\frac{1}{3},1)$.

26. (a) $d(x,-5) \geq 2$.

$$|x-(-5)| \geq 2$$

$$|x+5| \geq 2$$

$x+5 \geq 2$ ni $x+5 \leq -2$

$x \geq -3$ ni $x \leq -7$

$x \in (-\infty, -7] \cup [-3, +\infty)$.

(B) $|x-1| < |x-3|$.

$$|x-1|^2 < |x-3|^2$$

$$(x-1)^2 < (x-3)^2$$

$$x^2-2x+1 < x^2-6x+9$$

$$4x < 8$$

$$x < 2$$

$x \in (-\infty, 2)$.

⑦ $d(2x,1) > d(2x,3)$

$|2x-1| > |2x-3|$

$|2x-1|^2 > |2x-3|^2$

$(2x-1)^2 > (2x-3)^2$

$4x^2-4x+1 > 4x^2-12x+9$

$8x > 8$

$x > 1$

$x \in (1, +\infty)$.

⑧ $x^2 < 9$.

$x^2 < 3^2$

$|x| < |3|$

$|x| < 3$.

$-3 < x < 3$

$x \in (-3, 3)$.

$x^2 < y^2$

$\Leftrightarrow$

$|x| < |y|$.

(ε) $x^2 > 1$

$x^2 > 1^2$

$|x| > |1|$

$|x| > 1$

$x > 1$ ú $x < -1$

$x \in (-\infty, -1) \cup (1, +\infty)$.

(02) $4x^2 < 25$.

$(2x)^2 < 5^2$

$|2x| < |5|$

$|2x| < 5$

$-5 < 2x < 5$

$-\frac{5}{2} < x < \frac{5}{2}$

$x \in (-\frac{5}{2}, \frac{5}{2})$.

Να λυθούν οι παρακάτω εξισώσεις - ανισώσεις

(α) $|x-4|=6$

(β) $|2x-3|=5$

(γ) $|18-3x|=0$

(δ) $|2x-2024|=-2$

(ε) $|x+3|-5=1$

(ζ) $3|2-2x|-10=2$

(η) $|4x-8|<-2$

(θ) $|4-2x|=|3x-6|$

(ι) $\frac{4|x-1|-1}{5}=7$

(κ) $|2x-4|<2$

(λ) $|4-|x||=|x|+3|$

(μ) $|3-|2x+1||\leq 2$

(ν) $|x-1|-|x-4|<0$

(ξ) $||x|-3|=3$

(ο) $|x-1|-\frac{|1-x|-1}{4}=\frac{|2x-2|-1}{2}$

(π) $3|3-2x|-13<2$

(ρ) $|2x-3|-|x|=0$

(σ) $|x+1|=|x-5|$

(τ) $|x-1|>-2024$

(υ) $|5-7x|\geq -2$

(φ) $1\leq |3x-6|<2$

(χ) $|4-x|=|5x+6|$

(ψ) $2|6-2x|-6\leq 2$

(ω) $|5x-4|>6$

1. ⓐ $|x-4|=6$

$x-4=6$ ή $x-4=-6$

$\boxed{x=10}$ $\boxed{x=-2}$.

Ⓑ $|2x-3|=5$

$2x-3=5$ ή $2x-3=-5$

$2x=8$ $2x=-2$

$\boxed{x=4}$ $\boxed{x=-1}$.

Ⓖ $|18-3x|=0$

$18-3x=0$

$18=3x$

$\boxed{x=6}$

Ⓓ $|2x-2024|=-2$

Αδύνατη!

Ⓔ $|x+3|-5=1$

$|x+3|=6$

$x+3=6$ ή $x+3=-6$

$x=3$. $x=-9$

(ζ) $3|2-2x|-10=2$

$3|2-2x|=12$

$|2-2x|=4$

$2-2x=4$ ή $2-2x=-4$

$-2x=2$ $-2x=-6$

$x=-1$ $x=3$

(η) $|4x-8|<-2$ Αδύνατη.

(θ) $|4-2x|=|3x-6|$.

$4-2x=3x-6$ ή $4-2x=-3x+6$

$-5x=-10$ $x=2$

$x=2$

(ι) $\frac{4|x-1|-1}{5}=7$ $\Rightarrow 4|x-1|-1=35$

$4|x-1|=36$ $\Rightarrow |x-1|=9$

$x-1=9$ ή $x-1=-9$

$x=10$ $x=-8$.

(κ) $|2x-4|<2$

$-2<2x-4<2$

$2<2x<6$

$1<x<3$ $\quad x\in(1,3)$.

(λ) $|4-|x||=|x|+3$

$4-|x|=|x|+3$ ή $4-|x|=-|x|-3$

Αδύνατη

$-2|x|=-1$

$|x|=\frac{1}{2}$

$x=\frac{1}{2}$ ή $x=-\frac{1}{2}$

(μ) $|3-|2x+1||\leq 2$.

$-2\leq 3-|2x+1|\leq 2$

$-5\leq -|2x+1|\leq -1$

$5\geqslant |2x+1|\geqslant 1$.

$5 \geq |2x+1|$ και $|2x+1| \geq 1$.

$-5 \leq 2x+1 \leq 5$

$-6 \leq 2x \leq 4$

$-3 \leq x \leq 2$

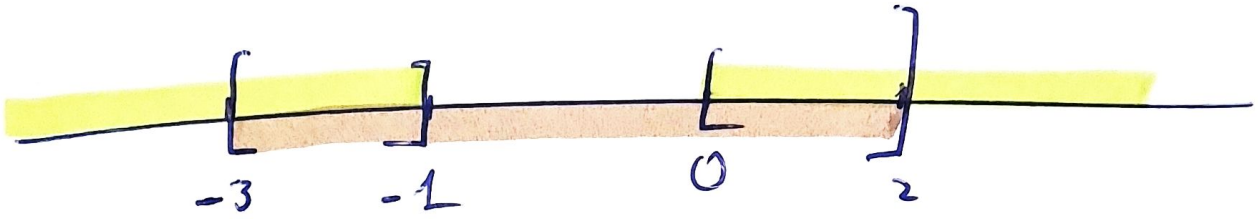
$x \in [-3, 2]$

$2x+1 \geq 1$ ή $2x+1 \leq -1$

$2x \geq 0$ $\quad 2x \leq -2$

$x \geq 0$ ή $x \leq -1$

$x \in (-\infty, -1] \cup [0, +\infty)$.



$x \in [-3, -1] \cup [0, 2]$.

(v) $|x-1| - |x-4| < 0$

$|x-1| < |x-4|$

$|x-1|^2 < |x-4|^2$

$(x-1)^2 < (x-4)^2$

$x^2 - 2x + 1 < x^2 - 8x + 16$

$6x < 15$

$x < \frac{16}{5}$

(f) $||x|-3| = 3$.

$|x|-3 = 3$ or $|x|-3 = -3$

$|x| = 6$ or $|x| = 0$

$x = 6$ or $x = -6$ or $x = 0$

(g) $|x-1| - \frac{|1-x|-1}{4} = \frac{|2x-2|-1}{2}$

$4|x-1| - (|x-1|-1) = 2(|2x-2|-1)$

$4|x-1| - |x-1| + 1 = 2|2x-2| - 2$

$3|x-1| + 1 = 4|x-1| - 2$

$3|x-1| - 4|x-1| = -2-1$

$-|x-1| = -3$

$|x-1| = 3$

$x-1 = 3$ or $x-1 = -3$

$x = 4$ or $x = -2$.

(п) $3|3-2x|-13<2$

$3|3-2x|<15$

$|3-2x|<5$

$-5<3-2x<5$

$-8<-2x<2$

$4>x>-1$

$x\in(-1,4)$

(е) $|2x-3|-|x|=0$

$|2x-3|=|x|$

$2x-3=x$ и $2x-3=-x$

$x=3$ $3x=3$

$x=1$

(б) $|x+1|=|x-5|$

$x+1=x-5$ и $x+1=-x+5$

Абсцисы $2x=4$

$x=2$

(τ) $|x-1| > -2024$ Αόριστη $x \in \mathbb{R}$.

(υ) $|5-7x| \le -2$ Αδύνατη!.

(φ). $1 \le |3x-6| < 2$.

$1 \le |3x-6|$ <u>και</u> $|3x-6| < 2$

$-2 < 3x-6 < 2$

$3x-6 \ge 1$ ή $3x-6 \le -1$

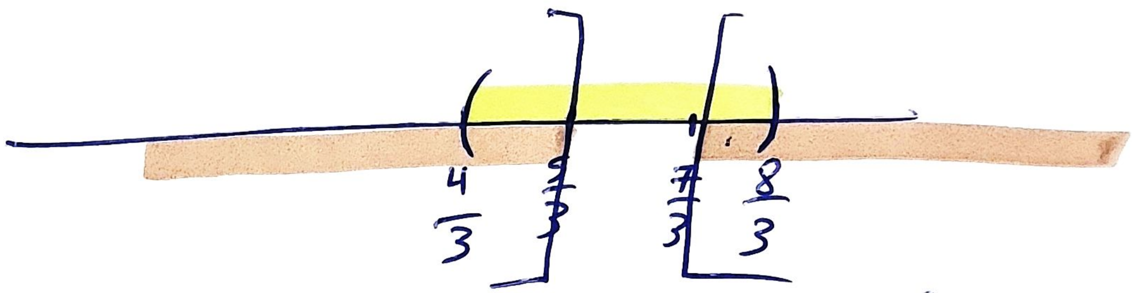
$4 < 3x < 8$

$3x \ge 7$ $\quad$ $3x \le 5$

$\frac{4}{3} < x < \frac{8}{3}$

$x \ge \frac{7}{3}$ $\quad$ $x \le \frac{5}{7}$

$x \in (-\infty, \frac{5}{3}] \cup [\frac{7}{3}, +\infty)$ $\quad$ $x \in (\frac{4}{3}, \frac{8}{3})$



$x \in (\frac{4}{3}, \frac{5}{3}] \cup [\frac{7}{3}, \frac{8}{3})$.

(χ) $|4-x| = |5x+6|$

$4-x = 5x+6$ ή $4-x = -5x-6$

$-6x = 2$ $\quad$ $4x = -10$

$x = -\frac{1}{3}$ $\quad$ $x = -\frac{5}{2}$,

(4). $2|6-2x|-6 \leq 2$

$|6-2x|-3 \leq 1$

$|6-2x| \leq 4$

$-4 \leq 6-2x \leq 4$

$-10 \leq -2x \leq -2$

$5 \geq x \geq 1$ $\quad x \in [1,5]$.

(w). $|5x-4| > 6$

$5x-4 > 6$ 或 $5x-4 < -6$.

$5x > 10$ $\quad$ $5x < -2$

$x > 2$ 或 $x < -\frac{2}{5}$

$x \in (-\infty, -\frac{2}{5}) \cup (2, +\infty)$.

28. $B = |x-3| + x-1$

1. Αν $x-3 \geq 0$ δηλαδή $x \geq 3$ τότε

$$B = |\overset{\oplus}{x-3}| + x-1$$

$$B = x-3+x-1$$

$$B = 2x-4.$$

2. Αν $x-3 < 0$ δηλαδή $x < 3$

$$B = |\overset{\ominus}{x-3}| + x-1$$

$$B = -x+3+x-1$$

$$B = 2$$

$$B = \begin{cases} 2x-4, & x \geq 3 \\ 2, & x < 3 \end{cases}$$

(8) . $\Delta = |x| - |x-1| + |x-2|$

x		0		1		2	
x	−	0	+	+	+	+	
x-1	−	−	0	+	+	+	
x-2	−	−	−	0	+	+	

1. Av $x < 0$

$\Delta = \overset{\ominus}{|x|} - \overset{\ominus}{|x-1|} + \overset{\ominus}{|x-2|} = -x-(-x+1)+(-x+2)$

$\Delta = -x+x-1-x+2 = 1-x$ $\quad \Delta = 1-x$

2. Av $0 \le x \le 1$

$\Delta = \overset{\oplus}{|x|} - \overset{\ominus}{|x-1|} + \overset{\ominus}{|x-2|} = x-(-x+1)-x+2$

$\Delta = x+x-1-x+2$

$\Delta = x+1$

3. Av $1 < x < 2$ 2020

$\Delta = \overset{\oplus}{|x|} - \overset{\oplus}{|x-1|} + \overset{\ominus}{|x-2|} = x-(x-1)-x+2$

$\Delta = 3-x$

4. Αν $x \geq 2$ τότε

$$\Delta = \overset{\oplus}{|x|} - \overset{\oplus}{|x-1|} + \overset{\oplus}{|x-2|}$$

$$\Delta = x - (x-1) + (x-2)$$

$$\Delta = \cancel{x} - \cancel{x} + 1 + x - 2$$

$$\Delta = x - 1$$

$$\Delta = \begin{cases} 1 - x, & x < 0 \\ x + 1, & 0 \leq x \leq 1 \\ 3 - x, & 1 < x < 2 \\ x - 1, & x \geq 2. \end{cases}$$

50. $A = \dfrac{|x|^3 - 2x^2}{|x| - 2}$

(α) Για να ορίζεται η παράσταση A

πρέπει $|x| - 2 \neq 0$

$\longrightarrow |x| - 2 = 0 \Rightarrow |x| = 2 \Rightarrow \underline{\underline{x = 2}}$ ή $\underline{\underline{x = -2}}$

Η A ορίζεται αν $x \neq 2$ και $x \neq -2$

(β) $A = \dfrac{|x|^3 - 2|x|^2}{|x| - 2} = \dfrac{|x|^2 (|x| - 2)}{|x| - 2} = |x|^2 = x^2$

(γ) $A < 9$

$x^2 < 9,$

$x^2 < 3^2$

$|x| < |3|$

$|x| < 3$

$\underline{\underline{-3 < x < 3}}$

$x \in (-3, 3).$

51. (α) $A = \dfrac{x^2-6|x|+9}{x^2-3|x|}$ Η Α ορίζεται

αν $x \neq 0, 3, -3$.

$\longrightarrow x^2-3|x|=0$

$|x|^2-3|x|=0$

$|x|(|x|-3)=0$

$|x|=0$ ή $|x|-3=0$

$|x|=3$

$x=0$ $x=3$ ή $x=-3$

(β) $A = \dfrac{|x|^2-6|x|+9}{|x|^2-3|x|} = \dfrac{(|x|-3)^2}{|x|(|x|-3)} = \dfrac{|x|-3}{|x|}$

(γ) i) $|A| = \dfrac{1}{2} \Leftrightarrow \left|\dfrac{|x|-3}{|x|}\right| = \dfrac{1}{2}$

$\dfrac{|x|-3}{|x|} = \dfrac{1}{2}$ ή $\dfrac{|x|-3}{|x|} = -\dfrac{1}{2}$

$2|x|-6=|x|$ $-2|x|+6=|x|$

$|x|=6$ $6=3|x|$

$|x|=2$

$x=6$ $x=-6$ $x=2$ ή $x=-2$.

38. (a) $|x+2| + |3x-2y-1| = 0$.

$x+2=0$ hoặc $3x-2y-1=0$.

$x=-2$

$-6-2y-1=0$

$-7 = 2y$

$y = -\frac{7}{2}$

(b) $|x-y+1| + x^2 = 2x-1$.

$|x-y+1| + x^2-2x+1 = 0$

$|x-y+1| + (x-1)^2 = 0$

$x-y+1=0$ hoặc $x-1=0$

$x=1$

$1-y+1=0$

$y=2$

(8). $|2x+y-3| + |3x-y-2| = 0$

$2x+y-3=0$ kan $3x-y-2=0$.

$$\begin{cases} 2x+y-3=0 \\ 3x-y-2=0 \end{cases}$$

$\Rightarrow \boxed{y = 3-2x}$ ✓

$3x-(3-2x)-2=0$

$3x-3+2x-2=0$

$5x=5$

$\boxed{x=1}$

$y = 3-2\cdot 1$

$\boxed{y=1}$

31. $A = \frac{|x|}{x} - \frac{|y|}{y}$

1. Av $x>0$ kai $y>0$

$$A = \frac{\overset{\oplus}{|x|}}{x} - \frac{\overset{\oplus}{|y|}}{y} = \frac{x}{x} - \frac{y}{y} = 1-1 = 0$$

2. Av $x<0$ kai $y<0$

$$A = \frac{\overset{\ominus}{|x|}}{x} - \frac{\overset{\ominus}{|y|}}{y} = -\frac{x}{x} + \frac{y}{y} = -1+1 = 0.$$

3. Av $x>0$ kai $y<0$

$$A = \frac{\overset{\oplus}{|x|}}{x} - \frac{\overset{\ominus}{|y|}}{y} = \frac{x}{x} + \frac{y}{y} = 1+1 = 2$$

4. Av $x<0$ kai $y>0$

$$A = \frac{\overset{\ominus}{|x|}}{x} - \frac{\overset{\oplus}{|y|}}{y} = -\frac{x}{x} - \frac{y}{y} = -2.$$

40. (a) Nđo $a^2+\beta^2 \geq 2|a\beta|$.

$$|a|^2+|\beta|^2 \geq 2|a||\beta|$$

$$|a|^2-2|a||\beta|+|\beta|^2 \geq 0$$

$$(|a|-|\beta|)^2 \geq 0 \quad .$$

→ Av

$|a|=|\beta|$

$\underline{\alpha=\beta}$ i $\underline{a=-\beta}$

(b) $|a|+\left|\frac{1}{a}\right| \geq 2$.

$$|a|+\frac{|1|}{|a|} \geq 2$$

$$|a|+\frac{1}{|a|} \geq 2$$

$$|a|^2+1 \geq 2|a|$$

$$|a|^2-2|a|+1 \geq 0$$

$$(|a|-1)^2 \geq 0$$

41. (a) Nso $\left|\frac{2a}{a^2+1}\right| \leq 1$.

$$\frac{|2a|}{|a^2+1|} \leq 1$$

$$\frac{2|a|}{a^2+1} \leq 1$$

$$2|a| \leq a^2+1$$

$$0 \leq a^2 - 2|a| + 1$$

$$0 \leq |a|^2 - 2|a| + 1$$

$$0 \leq (|a|-1)^2$$

(B) $\left|\frac{a}{a^2+9}\right| \leq \frac{1}{6}$

$$-\frac{1}{6} \leq \frac{a}{a^2+9} \leq \frac{1}{6}$$

$-\frac{1}{6} \leq \frac{a}{a^2+9}$ και $\frac{a}{a^2+9} \leq \frac{1}{6}$

$-(a^2+9) \leq 6a$ $\quad$ $6a \leq a^2+9$

$0 \leq a^2+6a+9$ $\quad$ $0 \leq a^2-6a+9$

$(a+3)^2 \geq 0$ ✓ $\quad$ $0 \leq (a-3)^2$ ✓

29. Αν $|\alpha - 2\beta| = 1$

τότε $\alpha = 2\beta + 1$ ή $\alpha = 2\beta - 1$

$\alpha - 2\beta = 1$ ή $\alpha - 2\beta = -1$

$\alpha = 2\beta + 1$ ✓ $\alpha = 2\beta - 1$ ✓

30. Αν $|2\alpha - \beta| = |\alpha - 2\beta|$

(α) - τότε α, β είναι ή αντίθετοι

$2\alpha - \beta = \alpha - 2\beta$ ή $2\alpha - \beta = -\alpha + 2\beta$

$\alpha = -\beta$ $3\alpha = 3\beta$

$\alpha = \beta$

(B) Av $|3\alpha-\beta|-|\alpha+2\beta|=0$

vdo $\frac{\alpha}{\beta}=\frac{3}{2}$ ή $\frac{\alpha}{\beta}=-\frac{1}{4}$

$|3\alpha-\beta|=|\alpha+2\beta|$

$3\alpha-\beta=\alpha+2\beta$ ή $3\alpha-\beta=-\alpha-2\beta$

$2\alpha=3\beta$ $4\alpha=-\beta$

$\frac{\alpha}{\beta}=\frac{3}{2}$ $\frac{\alpha}{\beta}=-\frac{1}{4}$

11). $|A| < \frac{1}{3}$

$$\left| \frac{|x|-3}{|x|} \right| < \frac{1}{3}$$

$$-\frac{1}{3} < \frac{|x|-3}{|x|} < \frac{1}{3}$$

$-\frac{1}{3} < \frac{|x|-3}{|x|}$ $\underline{\text{sau}}$ $\frac{|x|-3}{|x|} < \frac{1}{3}$.

$-|x| < 3|x| - 9$

$9 < 4|x|$

$\frac{9}{4} < |x|$

$x > \frac{9}{4}$; $x < -\frac{9}{4}$

$3|x| - 9 < |x|$

$2|x| < 9$

$|x| < \frac{9}{2}$

$-\frac{9}{2} < x < \frac{9}{2}$

$x \in (-\frac{9}{2}, \frac{9}{2})$.

$x \in (-\infty, -\frac{9}{4}) \cup (\frac{9}{4}, +\infty)$.

$x \in (-\frac{9}{2}, -\frac{9}{4}) \cup (\frac{9}{4}, \frac{9}{2})$