48. (α) Αν $\alpha>1$ νδο $a^2<a^3$

$\rightarrow a^2<a^3 \Rightarrow a^2-a^3<0 \Rightarrow a^2(1-a)<0$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (+) \quad (-)$

(β) Αν $A=x^2-2x+\frac{21}{10}$ Σύγκριση A^2, A^3.

έστω $A^2<A^3$

$$A^2-A^3<0$$

$$A^2(1-A)<0$$
$$(+) \quad\quad (-)$$

$$A=x^2-2x+1+\frac{21}{10}-1$$

$$A=(x-1)^2+\frac{11}{10}=(x-1)^2+\frac{10}{10}+\frac{1}{10}$$

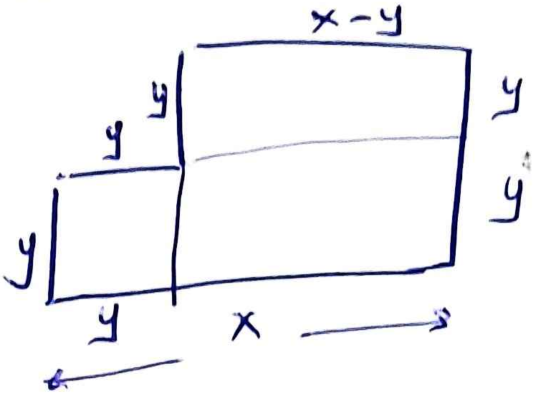
$$A=1+(x-1)^2+\frac{1}{10} \Rightarrow A>1$$

$1-A<0$

39.

$1,9 < x < 2,1$

$0,7 < y < 0,8$



$\Pi = x + 2y + x \cancel{-y} \cancel{+y} + y + y$

$\Pi = 2x + 4y$

- $1,9 < x < 2,1 \Rightarrow 3,8 < 2x < 4,2$
- $0,7 < y < 0,8 \Rightarrow 2,8 < 4y < 3,2$

$\left.\begin{matrix} 3,8 < 2x < 4,2 \\ 2,8 < 4y < 3,2 \end{matrix}\right\} \oplus$

$6,6 < \Pi < 7,4$

42. Αν $\alpha, \beta > 0$ και $\boxed{\alpha + 2\beta = 2}$

$\alpha = 2 - 2\beta$

$2\beta = 2 - \alpha$

$\beta = \frac{2-\alpha}{2}$

(α) νδο $\alpha < 2$ και $\beta < 1$.

$\alpha < 2$

$\cancel{2} - 2\beta < \cancel{2}$

$-2\beta < 0$

$\beta > 0$ ✓

$\beta < 1$

$\frac{2-\alpha}{2} < 1$

$\cancel{2} - \alpha < \cancel{2}$

$0 < \alpha$ ✓

(β) νδο $\alpha\beta \leq \frac{1}{2}$

$(2-2\beta)\beta \leq \frac{1}{2}$

$2\beta - 2\beta^2 \leq \frac{1}{2}$

$4\beta - 4\beta^2 \leq 1$

$0 \leq 4\beta^2 - 4\beta + 1$

$0 \leq (2\beta - 1)^2$

✓

45. (α) Αν $\alpha, \beta > 0$ νδο $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \geq 2$..

$$\alpha^2 + \beta^2 \geq 2\alpha\beta$$

$$\alpha^2 - 2\alpha\beta + \beta^2 \geq 0$$

$$(\alpha - \beta)^2 \geq 0$$

(β) Αν $x, y, z > 0$ νδο $\frac{x+y}{z} + \frac{y+z}{x} + \frac{z+x}{y} \geq 6$.

$$\left.\begin{aligned} \frac{x}{y} + \frac{y}{x} &\geq 2 \\ \frac{y}{z} + \frac{z}{y} &\geq 2 \\ \frac{z}{x} + \frac{x}{z} &\geq 2 \end{aligned}\right\} (+)$$

$$\frac{x}{y} + \frac{y}{x} + \frac{y}{z} + \frac{z}{y} + \frac{z}{x} + \frac{x}{z} \geq 6$$

$$\frac{x+z}{y} + \frac{y+z}{x} + \frac{y+x}{z} \geq 6$$

50. $\alpha > 0$ $\beta > 0$

(α) ΝΔΟ $a + \frac{4}{a} \geq 4$

$a^2 + 4 \geq 4a$

$a^2 - 4a + 4 \geq 0$

$(a-2)^2 \geq 0$

(β) ΝΔΟ $\left(a + \frac{4}{a}\right)\left(\beta + \frac{4}{\beta}\right) \geq 16.$

$\left.\begin{array}{l} a + \frac{4}{a} \geq 4 \\ \beta + \frac{4}{\beta} \geq 4 \end{array}\right\} \odot$ $\forall a > 0$

$\left(a + \frac{4}{a}\right)\left(\beta + \frac{4}{\beta}\right) \geq 16$ ✓

51. ⓐ $2x$ και $5x$, $x>0$

$\to 2<5 \Rightarrow \boxed{2x<5x}$

ⓑ x^3 και x όταν $0<x<1$

Έστω $x^3<x \Rightarrow x^3-x<0$

$x(x^2-1)<0$

$$\boxed{\begin{array}{l} x(x-1)(x+1)<0 \\ \oplus \quad \ominus \quad \oplus \end{array}}$$

- $x>0$
- $0<x<1 \Rightarrow -1<x-1<0$
- $0<x<1 \Rightarrow 1<x+1<2$

ⓒ 8^{10} και 9^{15}.

$8^{10} \to (2^3)^{10} \to 2^{30}$

$9^{15} \to (3^2)^{15} \to 3^{30}$

Αφού $2<3 \Rightarrow 2^{30}<3^{30}$

(δ) 7 και $5\sqrt{2}$

Έστω $7 > 5\sqrt{2}$.

$7^2 > (5\sqrt{2})^2$

$49 > 25 \cdot 2$

$49 > 50$ Άτοπο!

Άρα $7 < 5\sqrt{2}$

19. (a) $A = |3x-6| - |4-2x| + x$

- $x < 2 \Rightarrow 3x < 6 \Rightarrow 3x - 6 < 0$
- $x < 2 \Rightarrow -2x > -4 \Rightarrow 4 - 2x > 0$

$A = -3x + 6 - (4 - 2x) + x$

$A = -3x + 6 - 4 + 2x + x$

$A = 2$

(B) $B = 4\left|\frac{x}{2} - 1\right| + 8\left|\frac{x}{4} - \frac{1}{2}\right|$

$B = \left|4\frac{x}{2} - 4\right| + \left|8\frac{x}{4} - 8\frac{1}{2}\right|$

$B = \cancel{|2x-4| - |2x-4|}$

$B = 0$

$$|x| \geq \theta$$
$$x \geq \theta \text{ ή } x \leq -\theta$$

26. (a) $d(x,-5) \geq 2$

$$|x+5| \geq 2$$

$$x+5 \geq 2 \quad \text{ή} \quad x+5 \leq -2$$

$$x \geq -3 \qquad\qquad x \leq -7$$

$$x \in (-\infty, -7] \cup [-3, +\infty)$$

(B) $|x-1| < |x-3|$

$$|x-1|^2 < |x-3|^2$$

$$(x-1)^2 < (x-3)^2$$

$$\cancel{x^2} - 2x + 1 < \cancel{x^2} - 6x + 9$$

$$4x < 8$$

$$x < 2$$

γ) $d(2x,1) > d(2x,3)$

$$|2x-1| > |2x-3|$$

$$|2x-1|^2 > |2x-3|^2$$

$$(2x-1)^2 > (2x-3)^2$$

$$\cancel{4x^2}-4x+1 > \cancel{4x^2}-12x+9$$

$$8x > 8 \qquad x>1$$

δ) $x^2 < 9 \Rightarrow x^2 < 3^2 \Rightarrow |x| < |3|$

$$|x| < 3$$

$$-3 < x < 3$$

ε) $x^2 > 1$

$$x^2 > 1^2$$

$$|x| > |1| \Rightarrow |x| > 1$$

$x > 1$ ή $x < -1$.

20. $|\alpha| = 2$ $\quad |\beta| = 3$ $\quad |\gamma| = 4$

(α) Ν.δ.ο. $|\alpha+\beta+\gamma| \leq 9$

Τριγωνική ανισότητα

$$|\alpha+\beta| \leq |\alpha| + |\beta|$$

Γενικά $|\alpha+\beta+\gamma| \leq |\alpha| + |\beta| + |\gamma|$.

$$|\alpha+\beta+\gamma| \leq 2+3+4$$

$$|\alpha+\beta+\gamma| \leq 9$$

(β) ν.δ.ο. $|\alpha-\beta+3\gamma| \leq 17$.

$$|\alpha+(-\beta)+(3\gamma)| \leq 17$$

$\longrightarrow |\alpha+(-\beta)+(3\gamma)| \leq |\alpha| + |-\beta| + |3\gamma|$

$|\alpha-\beta+3\gamma| \leq 2 + |\beta| + 3|\gamma| \Leftrightarrow |\alpha-\beta+3\gamma| \leq 2+3+3\cdot 4$

$|\alpha-\beta+3\gamma| \leq 17$.

Τετραγωνική Ρίζα

1. Έστω ότι ψάχνω τη λύση της εξίσωσης $x^2 = \theta \iff x = \sqrt{\theta}$
$x = -\sqrt{\theta}$

<u>π.χ</u>

$\sqrt{16} = 4$ γιατί $4^2 = 16$

$\sqrt{9} = 3$ γιατί $3^2 = 9$

2. <u>Βασικές ρίζες</u>

$\sqrt{0} = 0$
$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$

$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$
$\sqrt{169} = 13$
$\sqrt{196} = 14$
$\sqrt{225} = 15$

$\sqrt{256} = 16$
$\sqrt{289} = 17$
$\sqrt{324} = 18$
$\sqrt{361} = 19$
$\sqrt{400} = 20$
$\sqrt{441} = 21$
$\sqrt{484} = 22$
$\sqrt{529} = 23$
$\sqrt{576} = 24$
$\sqrt{625} = 25.$

3. $\sqrt{x} \geq 0$ και $x \geq 0$

4. $\sqrt{a}\sqrt{\beta} = \sqrt{a \cdot \beta}$

5. $\frac{\sqrt{a}}{\sqrt{\beta}} = \sqrt{\frac{a}{\beta}}$

6. $\sqrt{x}^{2} = x$, $x \geq 0$

7. $\sqrt{x^{2}} = |x|$, $x \in \mathbb{R}$.

8. $\sqrt{a} + \sqrt{\beta} \neq \sqrt{a+\beta}$

$\boxed{\text{Προσοχη}}$

$\sqrt{x}^{3} = \sqrt{x}^{2}\sqrt{x} = x\sqrt{x}$

7. Νδο $(\overset{a}{\sqrt{x^2+3}}-\overset{b}{\sqrt{x^2+1}})(\overset{a}{\sqrt{x^2+3}}+\overset{b}{\sqrt{x^2+1}})=2$.

$$(\sqrt{x^2+3})^2-(\sqrt{x^2+1})^2=2$$

$$\cancel{x^2}+3-\cancel{x^2}-1=2$$

$$3-1=2 \quad \checkmark$$

42. (α) Νδο $(2-\sqrt{3})^2(7+4\sqrt{3})=1$.

$4-2\cdot2\cdot\sqrt{3}+\sqrt{3}^2(7+4\sqrt{3})=1$

$4-4\sqrt{3}+3(7+4\sqrt{3})=1$

$\cancel{21+16\sqrt{3}}-\cancel{28}\sqrt{3}\ \cancel{-16\sqrt{3}+21}+12\sqrt{3}$

$1=1$

$$(2^2-2\cdot2\cdot\sqrt{3}+\sqrt{3}^2)\cdot(7+4\sqrt{3})=1$$

$$(4-4\sqrt{3}+3)(7+4\sqrt{3})=1$$

$$(7-4\sqrt{3})(7+4\sqrt{3})=1$$

$$7^2-(4\sqrt{3})^2=1$$

$$49-16\cdot3=1$$

$$49-48=1 \qquad 1=1.$$

41. (α) $(1+2\sqrt{5})^2 = 1^2 + 2\cdot 1\cdot 2\sqrt{5} + (2\sqrt{5})^2 =$

$1 + 4\sqrt{5} + 4\cdot 5 = 1 + 4\sqrt{5} + 20 = 21 + 4\sqrt{5}.$

$(1-2\sqrt{5})^2 = 1^2 - 2\cdot 1\cdot 2\sqrt{5} + (2\sqrt{5})^2 =$

$1 - 4\sqrt{5} + 20 = 21 - 4\sqrt{5}$

(β) $\sqrt{21+4\sqrt{5}} - \sqrt{21-4\sqrt{5}} =$

$\sqrt{(1+2\sqrt{5})^2} - \sqrt{(1-2\sqrt{5})^2}$

$= |\overset{(+)}{1+2\sqrt{5}}| - |\overset{(-)}{1-2\sqrt{5}}| =$

$= 1+2\sqrt{5} - (-1+2\sqrt{5}) =$

$= 1 + \cancel{2\sqrt{5}} + 1 - \cancel{2\sqrt{5}}$

$= 2.$

12. (γ) Νδο $\sqrt{3}\sqrt{2\sqrt{2}-\sqrt{5}}\sqrt{2\sqrt{2}+\sqrt{5}}=3$.

$\cancel{\sqrt{3\cdot 2\sqrt{2}-\sqrt{5}\cdot 2\sqrt{2}+\sqrt{5}}=3}$

$\cancel{\sqrt{6\sqrt{2}-\sqrt{5}\cdot 2\sqrt{2}+\sqrt{5}}}$

$$\sqrt{3\cdot(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})}=3$$

$$\sqrt{3\cdot\left((2\sqrt{2})^2-\sqrt{5}^2\right)}=3$$

$$\sqrt{3\cdot(4\cdot 2-5)}=3$$

$$\sqrt{3\cdot 3}=3$$

$$\sqrt{9}=3$$

$$3=3.$$

Σελ 136

2. $$\sqrt{8\cdot\sqrt{1+\sqrt{2+\sqrt{49}}}} =$$

$$= \sqrt{8\cdot\sqrt{1+\sqrt{2+7}}} =$$

$$= \sqrt{8\cdot\sqrt{1+\sqrt{9}}}$$

$$= \sqrt{8\cdot\sqrt{1+3}} = \sqrt{8\cdot\sqrt{4}}$$

$$= \sqrt{8\cdot 2} = \sqrt{16} = 4 .$$

3. $$B = \sqrt{(\sqrt{2}-1)^2} + \sqrt{(\sqrt{2}-2)^2} =$$

$$= |\overset{+}{\sqrt{2}-1}| + |\overset{-}{\sqrt{2}-2}| = \sqrt{2}-1-\sqrt{2}+2$$

$$= 1$$

- $\sqrt{2}>1 \Rightarrow \sqrt{2}-1>0$
- $\sqrt{2}<2 \Rightarrow \sqrt{2}-2<0$

12. 8) $\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2-\sqrt{2+\sqrt{3}}} = 1$

$$\sqrt{(2+\sqrt{3})(2+\sqrt{2+\sqrt{3}})(2-\sqrt{2+\sqrt{3}})} = 1$$

$$\sqrt{(2+\sqrt{3})\left[2^2 - (\sqrt{2+\sqrt{3}})^2\right]} = 1$$

$$\sqrt{(2+\sqrt{3})(4-2-\sqrt{3})} = 1$$

$$\sqrt{(2+\sqrt{3})(2-\sqrt{3})} = 1$$

$$\sqrt{2^2-\sqrt{3}^2} = 1$$

$$\sqrt{4-3} = 1$$

$$\sqrt{1} = 1$$

$$1 = 1.$$

37. Αν $d(3x,-1)<2$

ⓐ $A=\sqrt{x^2+2x+1}-\sqrt{1-4x+4x^2}$

$d(\alpha,\beta)=|\alpha-\beta|$.

$|3x-(-1)|<2$

$|3x+1|<2$

$-2<3x+1<2$

$-3<3x<1$

$-1<x<\frac{1}{3}$

$A=\sqrt{(x+1)^2}-\sqrt{(1-2x)^2}$

$A=|\overset{+}{x+1}|-|\overset{+}{1-2x}|=x+1-(1-2x)=x+1-1+2x$

$=3x$

• $-1<x<\frac{1}{3}\Rightarrow 0<x+1<\frac{4}{3}$

• $-1<x<\frac{1}{3}\Rightarrow 2>-2x>-\frac{2}{3}$

$3>1-2x>\frac{1}{3}$

(γ) $\Gamma = \sqrt{\frac{x^2}{25}} + \sqrt{x^2 - 4x + 4} =$

$= \sqrt{\left(\frac{x}{5}\right)^2} - \sqrt{(x-2)^2} =$

$= \left|\frac{x}{5}\right| - |x-2|$

6. Αν $x = 2\sqrt{3} - 1$ και $y = 2\sqrt{3} + 1$.

$x^2 - xy = (2\sqrt{3} - 1)^2 - (2\sqrt{3} - 1)(2\sqrt{3} + 1) =$

$= 4\sqrt{3}^2 - 2 \cdot 4\sqrt{3} + 1^2 - (4\sqrt{3}^2 + 2\sqrt{3} - 2\sqrt{3} - 1)$

$= \cancel{0}\,\cancel{4\sqrt{3}^2} - 8\sqrt{3} + 1 \cancel{- 4\sqrt{3}^2} \cancel{- 2\sqrt{3}} \cancel{+ 2\sqrt{3}} + 1$

$= -8\sqrt{3} + 2$

42. B $(\sqrt{2}+1)^3(5\sqrt{2}-7)=1$.

$$\left((\sqrt{2})^3+3(\sqrt{2})^2+3(\sqrt{2})+1\right)(5\sqrt{2}-7)=1$$

$$(2\sqrt{2}+6+3\sqrt{2}+1)(5\sqrt{2}-7)=1$$

$$(5\sqrt{2}+7)(5\sqrt{2}-7)=1$$

$$(5\sqrt{2})^2-49=1$$

$$5^2\cdot 2-49=1$$

$$50-49=1$$

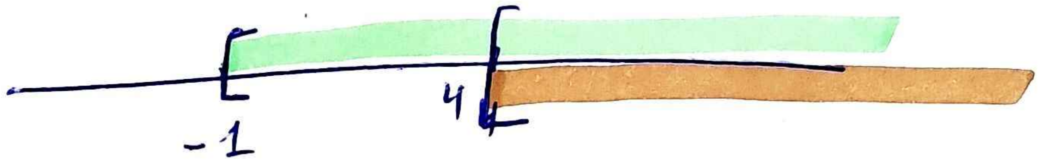
39.

$$A = \left(\sqrt{x-4} + \sqrt{x+1}\right)\left(\sqrt{x-4} - \sqrt{x+1}\right)$$

Για να ορίζεται η A πρέπει

$x-4 \geq 0$ και $x+1 \geq 0$

$x \geq 4$ και $x \geq -1$.



Πρέπει $x \geq 4$ δηλαδή $x \in [4,+\infty)$

$$A = \sqrt{x-4}^{\,2} - \sqrt{x+1}^{\,2}$$

$$A = x-4-x-1$$

$$A = -5$$

Επόμενο Μάθημα

Σελ 136

1

2 α β γ.

3 α

4

5

12 α β.

36.

26.

ⓐ $d(x, -5) \geq 2$.

$|x-(-5)| \geq 2$

$|x+5| \geq 2$

$x+5 \geq 2$ ή $x+5 \leq -2$

$\underline{x \geq -3}$ $\quad$ $\underline{x \leq -7}$

ⓑ $|x-1| < |x-3|$.

$|x-1|^2 < |x-3|^2$

$(x-1)^2 < (x-3)^2$

$\cancel{x^2}-2x+1 < \cancel{x^2}-6x+9$

$6x-2x < 8$

$4x < 8$

$\underline{x < 2}$

(γ) $d(2x,1) > d(2x,3)$.

$|2x-1| > |2x-3|$

$(2x-1)^2 > (2x-3)^2$

$(2x)^2 - 2\cdot 2x\cdot 1 + 1 > (2x)^2 - 2\cdot 2\cdot 3 + 3^2$

$\cancel{4x^2} - 4x + 1 > \cancel{4x^2} - 12x + 9$

$-4x + 1 > -12x + 9$

$12x - 4x > 9 - 1$

$8x > 8$

$x > 1$.

(δ) $x^2 < 9 \Rightarrow x^2 < 3^2 \Rightarrow |x| < |3|$

$|x| < 3$

$-3 < x < 3$

(ε) $x^2 > 1 \Rightarrow x^2 > 1^2 \Rightarrow |x| > |1|$

$|x| > 1$

$x > 1$ ή $x < -1$

(στ) $4x^2 < 25$.

$(2x)^2 < 5^2$

$|2x| < |5| \Rightarrow |2x| < 5$

$-5 < 2x < 5$ $\quad -\frac{5}{2} < x < \frac{5}{2}$

8. α) $\sqrt{2}\sqrt{8} = \sqrt{2 \cdot 8} = \sqrt{16} = 4$.

β) $\sqrt{49 \cdot 81} = \sqrt{49}\sqrt{81} = 7 \cdot 9 = 63$

γ) $\dfrac{\sqrt{72}}{\sqrt{2}} = \sqrt{\dfrac{72}{2}} = \sqrt{36} = 6$.

δ) $\dfrac{\sqrt{6}\sqrt{18}}{\sqrt{12}} = \dfrac{\cancel{\sqrt{6}}\ \sqrt{18}}{\sqrt{2}\cancel{\sqrt{6}}} = \dfrac{\cancel{\sqrt{2}}\sqrt{9}}{\cancel{\sqrt{2}}} = 3$.

ε) $\dfrac{\sqrt{18} \cdot \sqrt{48}}{\sqrt{24}} = \dfrac{\sqrt{18} \cdot \sqrt{48}}{\sqrt{4} \cdot \sqrt{6}} = \dfrac{\sqrt{18} \cdot \cancel{\sqrt{6}} \cdot \sqrt{8}}{2 \cdot \cancel{\sqrt{6}}} = \dfrac{\sqrt{18} \cdot \sqrt{8}}{2} =$

$= \dfrac{\sqrt{18} \cdot \sqrt{4} \cdot \cancel{\sqrt{2}}}{\cancel{2}} =$

$= \sqrt{18 \cdot 2} =$

$= \sqrt{36} = 6$.

στ) $\dfrac{\sqrt{8}\ \sqrt{75}}{\sqrt{150}} = \dfrac{\sqrt{4} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{25}}{\sqrt{6} \cdot \sqrt{25}} =$

$= \dfrac{2 \cdot \sqrt{2} \cdot \cancel{5}\sqrt{3}}{\sqrt{6} \cdot \cancel{5}} = \dfrac{2\cancel{\sqrt{2}}\cancel{\sqrt{3}}}{\cancel{\sqrt{2}}\cancel{\sqrt{3}}} = 2$.

14. (a) $\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{2(\sqrt{5}-\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} =$

$= \frac{\cancel{2}(\sqrt{5}-\sqrt{3})}{\cancel{2}}$

(β) $\frac{8}{\sqrt{3}-\sqrt{7}} = \frac{8\cdot(\sqrt{3}-\sqrt{7})}{(\sqrt{3}-\sqrt{7})\cdot(\sqrt{3}+\sqrt{7})} = \frac{8(\sqrt{3}-\sqrt{7})}{\sqrt{3}^2-\sqrt{7}^2} = \frac{8(\sqrt{3}-\sqrt{7})}{-4} =$

$= -2(\sqrt{3}-\sqrt{7})$

(γ) $\frac{3}{\sqrt{2}+1} = \frac{3\cdot(\sqrt{2}+1)}{(\sqrt{2}+1)\cdot(\sqrt{2}-1)} = \frac{3\cdot(\sqrt{2}+1)}{\sqrt{2}^2-1^2} = \frac{3(\sqrt{2}+1)}{1} = 3(\sqrt{2}+1)$

(δ) $\frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{(\sqrt{5}-2)(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)} =$

$\frac{(\sqrt{5}-2)^2}{\sqrt{5}^2-4} = \frac{(\sqrt{5}-2)^2}{5-4} = \frac{(5-2)^2}{1}$

10. $$A = \frac{\sqrt{20} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{45} - 2\sqrt{18} + 3\sqrt{27}}.$$

$$A = \frac{\sqrt{4}\cdot\sqrt{5} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{5}\cdot\sqrt{9} - 2\sqrt{18} + 3\sqrt{27}}$$

$$A = \frac{2\sqrt{5} - 2\sqrt{8} + 3\sqrt{12}}{3\sqrt{5} - 2\sqrt{18} + 3\sqrt{27}}$$

$$A = \frac{2\sqrt{5} - 2\sqrt{2}\sqrt{4} + 3\sqrt{3}\sqrt{4}}{3\sqrt{5} - 2\sqrt{2}\sqrt{9} + 3\sqrt{3}\sqrt{9}}$$

$$A = \frac{2\sqrt{5} - 4\sqrt{2} + 6\sqrt{3}}{3\sqrt{5} - 6\sqrt{2} + 9\sqrt{3}} = \frac{2\cancel{(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}}{3\cancel{(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}}$$

$$= \frac{2}{3}.$$

16. γ $\frac{5}{3-2\sqrt{2}}+\frac{2}{7-\sqrt{50}}=1$. (=)

$$\frac{5(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}+\frac{2(7+\sqrt{50})}{(7-\sqrt{50})(7+\sqrt{50}}=1$$

$$\frac{5(3+2\sqrt{2})}{9-4\cdot 2}+\frac{2(7+\sqrt{50})}{49-\cancel{\sqrt{50}}}=1$$

$$\frac{5(3+2\sqrt{2})}{9-8=1}+\frac{2(7+\sqrt{50})}{-1}=1$$

$$5(3+2\sqrt{2})-2(7+\sqrt{50})=1$$

$$(15+10\sqrt{2})-2(7+\sqrt{2}+\sqrt{25})=1$$

$$(15+10\sqrt{2})-2(7+5\sqrt{2})=1$$

$$(15+10\sqrt{2})-14-10\sqrt{2}=1$$

$$15+\cancel{10\sqrt{2}}-14\cancel{-10\sqrt{2}}=1$$

$$1=1$$

13. (α) $\frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{1\sqrt{3}}{3}$

(β) $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$.

(γ) $\frac{3}{\sqrt{8}} = \frac{3 \cdot \sqrt{8}}{\sqrt{8} \cdot \sqrt{8}} = \frac{3\sqrt{8}}{8}$

15. (β) $\frac{1+2\sqrt{5}}{1-2\sqrt{5}} = \frac{(1+2\sqrt{5}) \cdot (1+2\sqrt{5})}{(1-2\sqrt{5}) \cdot (1+2\sqrt{5})}$

$$= \frac{(1+2\sqrt{5})^2}{1^2-(2\sqrt{5})^2} = \frac{1+2 \cdot 2\sqrt{5}+(2\sqrt{5})^2}{1^2 - 2^2\sqrt{5}^2}$$

$$= \frac{5\sqrt{5}+20}{1-20} =$$

9. $A = \sqrt{8} - \sqrt{12} - \sqrt{50} + \sqrt{75}$

$A = \sqrt{4} \cdot \sqrt{2} - \sqrt{3}\sqrt{4} - \sqrt{2}\sqrt{25} + \sqrt{3}\sqrt{25}$

$A = 2\sqrt{2} - 2\sqrt{3} - 5\sqrt{2} + 5\sqrt{3}$

$A = -3\sqrt{2} + 3\sqrt{3}$.

$B = \sqrt{18} - \sqrt{27} - \sqrt{32} + \sqrt{48}$.

$\sqrt{2} \cdot \sqrt{9} - \sqrt{3} \cdot \sqrt{9} - \sqrt{2} \cdot \sqrt{16} + \sqrt{3} \cdot \sqrt{16}$

$3\sqrt{2} - 3\sqrt{3} - 4\sqrt{2} + 4\sqrt{3} =$

$-1\sqrt{2} + 1\sqrt{3}$ ✓

15. ① $\dfrac{\sqrt{2}}{\sqrt{50} - \sqrt{12}} = \dfrac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{25} - \sqrt{4} \cdot \sqrt{3}} =$

$\dfrac{\sqrt{2}}{5\sqrt{2} - 2\sqrt{3}} = \dfrac{\sqrt{2}\,(5\sqrt{2} + 2\sqrt{3})}{(5\sqrt{2} - 2\sqrt{3})(5\sqrt{2} + 2\sqrt{3})} = \dfrac{\sqrt{2}\,(5\sqrt{2} + 2\sqrt{3})}{(5\sqrt{2})^2 - (2\sqrt{3})^2} =$

$\dfrac{\sqrt{2}\,(5\sqrt{2} + 2\sqrt{3})}{50 - 27} = \dfrac{\sqrt{2}\,(5\sqrt{2} + 2\sqrt{3})}{23}$

17. (B) $\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4$.

$$= \frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})\cdot(\sqrt{5}+\sqrt{3})} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})\cdot(\sqrt{5}-3)} =$$

$$= \frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{\sqrt{5}^2-\sqrt{3}^2}$$

$$= \frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{5-3} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{5-3}$$

$$= \frac{\sqrt{5}(\sqrt{5}+\sqrt{3})}{2} - \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{2}$$

$$= \frac{\sqrt{25}+\cancel{\sqrt{15}}-\cancel{\sqrt{15}}+\sqrt{9}}{2} = \frac{5+3}{2} = 4$$

15. (01) $\frac{7}{2-3\sqrt{2}} = \frac{7(2+3\sqrt{2})}{(2-3\sqrt{2})(2+3\sqrt{2})} =$

$\frac{7(2+3\sqrt{2})}{(4-9\cdot 2)} = \frac{7(2+3\sqrt{2})}{4-18} =$

$\cancel{14+21}$ $7(2+$

16. (B) $\frac{2}{\sqrt{2}-1} - \frac{1}{3-2\sqrt{2}} = -1$.

$= \frac{2\cdot(\sqrt{2}+1)}{(\sqrt{2}-1)\cdot\sqrt{2}+1)} - \frac{1\cdot(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} =$

$= \frac{2\cdot(\sqrt{2}+1)}{\sqrt{2}^2-1^2} - \frac{1(3+2\sqrt{2})}{3^2-(2\sqrt{2})^2} =$

$= \frac{2(\sqrt{2}+1)}{2-1} - \frac{1(3+2\sqrt{2}}{9-4\cdot 4} =$

$= \frac{2(\sqrt{2}+1)}{2-1} - \frac{1(3+2\sqrt{2})}{9-8} = \frac{2\sqrt{2}+2}{1} - \frac{3+2\sqrt{2}}{1} =$

$= 2\cancel{\sqrt{2}}+2-3-2\cancel{\sqrt{2}} = -1$

16. a) $\left(\frac{2}{\sqrt{2}}+\frac{3}{\sqrt{3}}\right)(\sqrt{3}-\sqrt{2})=1 \Leftrightarrow$

$\Leftrightarrow \frac{2\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{2\sqrt{2}}{2}$

$\frac{3\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=\frac{3\sqrt{3}}{3}$

$\left(\frac{3\sqrt{2}}{2}+\frac{3\sqrt{3}}{3}\right)\cdot(\sqrt{3}-\sqrt{2})$

$\left(\frac{9\sqrt{2}}{6}+\frac{6\sqrt{3}}{6}\right)\cdot(\sqrt{3}-\sqrt{2})$

$=(3\sqrt{2}+\sqrt{3})(\sqrt{3}-\sqrt{2})$

$\left(\frac{\cancel{2}\sqrt{2}}{\cancel{2}}+\frac{\cancel{3}\sqrt{3}}{\cancel{3}}\right)(\sqrt{3}-\sqrt{2})=1$

$(\sqrt{2}+\sqrt{3})(\sqrt{3}-\sqrt{2})=1$

$\sqrt{3}^2-\sqrt{2}^2=1$

$3-2=1$

17. (a) $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} - \dfrac{\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5.$

$$\frac{\sqrt{3}(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} - \frac{\sqrt{2}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} =$$

$$\frac{3+\sqrt{6}}{(\sqrt{3})^2-\sqrt{2}^2} - \frac{\sqrt{6}-1}{\sqrt{3}^2-\sqrt{2}^2} = \frac{3+\sqrt{6}}{3-2} - \frac{\sqrt{6}-2}{3-2} = 3+\cancel{\sqrt{6}}-\cancel{\sqrt{6}}+1$$

$\cancel{3\sqrt{6}+4\sqrt{6}=7\sqrt{6}}$

Nadi

$$\frac{\sqrt{3}(\sqrt{3}+\sqrt{2})}{\sqrt{3}^2-\sqrt{2}^2} - \frac{\sqrt{2}(\sqrt{3}-\sqrt{2})}{\sqrt{3}^2-\sqrt{2}^2} =$$

$$= 3+\sqrt{6}-(\sqrt{6}-2)$$

$$= 3+\sqrt{6}-\sqrt{6}+2$$

$$= 5$$

Επομενο Μαθημα

Τεταρτη 13/11 5:30-7.

Σελ 113

21

22

23

24

25

15

16

17

18

19.

Ιδιοτητες

$|x|=\theta \Leftrightarrow x=\theta$ ή $x=-\theta$

$|x|=|y| \Leftrightarrow x=y$ ή $x=-y$

$|x|<\theta \Leftrightarrow -\theta<x<\theta$

$|x|>\theta \Rightarrow x>\theta$ ή $x<-\theta$

$d(x,y)=|x-y|$.

$|x|^2=x^2$

$|x|=|-x|$

Σελ 112

$|x| = |-x|$

(15) (a) $A = |x-3| - |3-x|$

$A = |3-x| - |3-x|$

$A = 0$

(B) $B = \dfrac{|2x-1|}{|1-2x|} - 3\dfrac{|3y+2|}{|-2-3y|} + 2\dfrac{|x-y+z|}{|y-x-z|}$

$B = \dfrac{\cancel{|1-2x|}}{\cancel{|1-2x|}} - 3\dfrac{\cancel{|3y+2|}}{\cancel{|2+3y|}} + 2\dfrac{\cancel{|x-y+z|}}{\cancel{|x-y+z|}}$

$B = 1 - 3 + 2 = 0.$

15. ⓐ $A = |3x-6| - |4-2x| + x.$

$A = 3|x-2| - 2|2-x| + x$

$A = 3|x-2| - 2|x-2| + x$

$A = |\overset{\ominus}{x-2}| + x \quad = 2 - x + x = \underline{\underline{2}}$

• $x < 2 \Rightarrow x - 2 < 0$

Ⓑ. $4\left|\frac{x}{2} - 1\right| + 8\left|\frac{x}{4} - \frac{1}{2}\right| =$

$\left|4\frac{x}{2} - 4\right| + \left|8\frac{x}{4} - 8\frac{1}{2}\right| =$

$= |2x-4| + |2x-4| =$

$= 2|2x-4| = 4|\overset{\ominus}{x-2}| = 4(-x+2)$

$= \underline{\underline{-4x+8}}$

• $x < 2 \Rightarrow x - 2 < 0$

28. (α) $A = |x-1| - 2x + 3$.

1. Αν $x - 1 \geq 0 \Rightarrow x \geq 1$ τότε

$A = |\overset{+}{x-1}| - 2x + 3$

$A = x - 1 - 2x + 3 = -x + 2$

2. Αν $x - 1 < 0 \Rightarrow x < 1$ τότε

$A = |\overset{-}{x-1}| - 2x + 3 = -x + 1 - 2x + 3$

$A = -3x + 4$

$$A = \begin{cases} -x+2, & x \geq 1 \\ -3x+4, & x < 1. \end{cases}$$

25. a) $|x-1|<3$

$-3<x-1<3$

$-2<x<4$

$x\in(-2,4)$

b) $|3x-2|-1<0$

$|3x-2|<1$

$-1<3x-2<1$

$1<3x<3$

$\frac{1}{3}<x<1$

$x\in\left(\frac{1}{3},1\right).$

24. (a) $|x| < 5$

$-5 < x < 5 \qquad x \in (-5, 5).$

25.

(B) $|x-2| \geq 5$

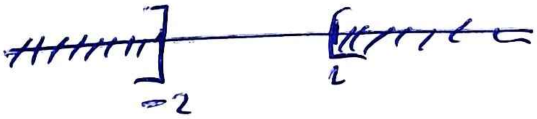
$x-2 \geq 5$ ή $x-2 \leq -5$

$x \geq 7$ ή $x \leq -3$

$x \in (-\infty, -3] \cup [7, +\infty).$

24. (B) $|x| \geq 2 \Rightarrow x \geq 2$ ή $x \leq -2$

$x \in (-\infty, -2] \cup [2, +\infty).$



(γ) $3|x| - 2 < 0 \Rightarrow 3|x| < 2 \Rightarrow |x| < \frac{2}{3}$

$-\frac{2}{3} < x < \frac{2}{3}$

$x \in \left(-\frac{2}{3}, \frac{2}{3}\right).$

(δ) $2 - 3|x| \leq 0 \Rightarrow 2 \leq 3|x| \Rightarrow |x| \geq \frac{2}{3}$

$x \geq \frac{2}{3}$ ή $x \leq -\frac{2}{3}$

$x \in \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{2}{3}, +\infty\right).$

23. (a) $|2x-1| = |x-2|$

$2x-1 = x-2$ ห $2x-1 = -x+2$

$x = -1$

$3x = 3$

$x = 1$

(b) $|3x-2| - |2x-3| = 0$

$|3x-2| = |2x-3|$

$3x-2 = 2x-3$ ห $3x-2 = -2x+3$

$x = -1$

$5x = 5$

$x = 1$

(c) $3|x-2| - 2|x| = 0$

$3|x-2| = 2|x|$

$|3x-6| = |2x|$

$3x-6 = 2x$ ห $3x-6 = -2x$

$x = 6$

$5x = 6$

$x = \frac{6}{5}$

22. (α) $|x-1|=5$

$x-1=5$ ή $x-1=-5$

$x=6$ $x=-4$

(β) $|x-2|-1=0$

$|x-2|=1$

$x-2=1$ ή $x-2=-1$

$x=3$ $x=1$

(γ) $3|x-2|-2|2-x|-5=0$

$3|x-2|-2|x-2|-5=0$

$|x-2|=5$

$x-2=5$ ή $x-2=-5$

$x=7$ $x=-3.$

(δ) $|3x-6|-|4-2x|=1$

$3|x-2|-2|2-x|=1$

$3|x-2|-2|x-2|=1$

$|x-2|=1$

$x-2=1$ ή $x-2=-1$

$x=3$ $x=1$

21. (α) $|x|=1 \Rightarrow x=1$ ή $x=-1$.

(β) $|x|-3=0 \Rightarrow |x|=3 \Rightarrow x=3$ ή $x=-3$

(γ) $3|x|-5=0 \Rightarrow 3|x|=5 \Rightarrow |x|=\frac{5}{3}$

$x=\frac{5}{3}$ ή $x=-\frac{5}{3}$.

(δ) $|x|=-2$ Αδύνατη!.

(ε) $2|x|-5|-x|+6=0$

$2|x|-5|x|+6=0$

$-3|x|=-6$

$|x|=2$

$x=2$ $x=-2$

(ζ) $x^2-2|x|=0$

$|x|^2-2|x|=0$

$|x|(|x|-2)=0$

$|x|=0$ ή $|x|=2$

$x=0$ $x=2$ $x=-2$.

(στ) $|2x|-\left|\frac{x}{2}\right|-1=0$

$2|x|-\frac{|x|}{|2|}-1=0$

$2|x|-\frac{|x|}{2}-1=0$

$4|x|-|x|-2=0$

$3|x|=2$

$|x|=\frac{2}{3}$

$x=\frac{2}{3}$ $x=-\frac{2}{3}$.

28. (8) $\Delta = |x| - |x-1| + |x-2|.$

x		0		1		2	
x	$-$	0	$+$		$+$		$+$
$x-1$	$-$		$-$	0	$+$		$+$
$x-2$	$-$		$-$		$-$	0	$+$

1. Αν $x \le 0$

$$\Delta = |\overset{\ominus}{x}| - |\overset{\ominus}{x-1}| + |\overset{\ominus}{x-2}| = -x - (-x+1) + (-x+2)$$

$$\Delta = -x + x - 1 - x + 2 = -x + 1.$$

2. Αν $0 < x < 1$ τοτε

$$\Delta = |\overset{\oplus}{x}| - |\overset{\ominus}{x-1}| + |\overset{\ominus}{x-2}| = x - (-x+1) + (-x+2)$$

$$\Delta = x + x - 1 - x + 2 = x + 1$$

3. Αν $1 \le x \le 2$

$$\Delta = |\overset{\oplus}{x}| - |\overset{\oplus}{x-1}| + |\overset{\ominus}{x-2}| = x - x + 1 - x + 2 = -x + 3$$

4. Αν $x > 2$ τοτε

$$\Delta = |\overset{\oplus}{x}| - |\overset{\oplus}{x-1}| + |\overset{\oplus}{x-2}| = x - x + 1 + x - 2 = x - 1$$

$|\alpha+\beta| \le |\alpha|+|\beta|$

20.

ⓐ $|\alpha|=2$ $\quad |\beta|=3$ $\quad |\gamma|=4$

Ndo $|\alpha+\beta+\gamma| \le 9$.

Ισχύει ότι $|\alpha+\beta+\gamma| \le |\alpha|+|\beta|+|\gamma|$

$|\alpha+\beta+\gamma| \le 2+3+4$

$|\alpha+\beta+\gamma| \le 9$.

ⓑ Ndo $|\alpha-\beta+3\gamma| \le 17$.

$|\alpha-\beta+3\gamma| = |\alpha+(-\beta)+(3\gamma)|$

$|\alpha+(-\beta)+(3\gamma)| \le |\alpha|+|-\beta|+|3\gamma|$

$|\alpha-\beta+3\gamma| \le 2+|\beta|+3|\gamma|$

$|\alpha-\beta+3\gamma| \le 2+3+3\cdot 4$

$|\alpha-\beta+3\gamma| \le 17$.

40. (B) Nδɔ $|a|+\left|\frac{1}{a}\right| \geq 2$.

$$|a|+\frac{1}{|a|} \geq 2$$

$$|a|^{2}+1 \geq 2|a|$$

$$|a|^{2}-2|a|+1 \geq 0$$

$$(|a|-1)^{2} \geq 0 .$$

41

(B) Nδɔ $\left|\frac{a}{a^{2}+9}\right| \leq \frac{1}{6}$

$$\frac{|a|}{|a^{2}\overset{\oplus}{+}9|} \leq \frac{1}{6}$$

$$\frac{|a|}{a^{2}+9} \leq \frac{1}{6} \quad \Leftrightarrow \quad 6|a| \leq a^{2}+9$$

$$0 \leq a^{2}-6|a|+9$$

$$0 \leq |a|^{2}-6|a|+9$$

$$0 \leq (|a|-3)^{2}$$

16. $A = \left| x - |x| \right| - \left| x + |x| \right|$

- $|x| \geq x$ και $|x| \geq -x$

- $|x| \geq x \Rightarrow 0 \geq x - |x|$
- $|x| \geq -x \Rightarrow |x| + x \geq 0$

$A = -x + |x| - (x + |x|)$

$A = -x + \cancel{|x|} - x - \cancel{|x|}$

$A = -2x$.

17. ⓐ Νδο $(|a| - a)(|a| + a) = 0$

$|a|^2 = a^2$

$|a|^2 + \cancel{|a|a} - \cancel{a|a|} - a^2 = 0$

$a^2 - a^2 = 0$

$0 = 0$,

Ⓑ $|a+1|^2+|a-1|^2=2|a|^2+2$

$(a+1)^2+(a-1)^2=2a^2+2$

$a^2+\cancel{2a}+1+a^2-\cancel{2a}+1=2a^2+2$

$2a^2+2=2a^2+2$

$0=0$.

18. Ⓐ Ndo $|3a|-6\left|\frac{a}{2}\right|=0$

$|aB|=|a||B|$

$\left|\frac{a}{B}\right|=\frac{|a|}{|B|}$

$|3a|-6\left|\frac{a}{2}\right|=0$

$|3||a|-6\frac{|a|}{|2|}=0$

$3|a|-6\frac{|a|}{2}=0$

$3|a|-3|a|=0$

$0=0$,

Ⓑ $|2a-6|-6\left|1-\frac{a}{3}\right|=0$.

$|2(a-3)|-\left|6-6\frac{a}{3}\right|=0$

$2|a-3|-|6-2a|=0$

$2|a-3|-2|3-a|=0$

$2|3-a|-2|3-a|=0$

$0=0$.

ν-οστή Ρίζα

1. $x^2=16 \Rightarrow x=\sqrt{16}$ ή $x=-\sqrt{16}$

$\boxed{x=4}$ $\boxed{x=-4}$

$x^3=8 \Rightarrow x=\sqrt[3]{8} \Rightarrow \underline{\underline{x=2}}$

2. $\sqrt[\nu]{x^\nu}=|x|$, $x\in R$.

3. $\left(\sqrt[\nu]{x}\right)^\nu = x$, $x\geq 0$.

4. $\sqrt[\nu]{x}\,\sqrt[\nu]{y}=\sqrt[\nu]{xy}$

5. $\dfrac{\sqrt[\nu]{x}}{\sqrt[\nu]{y}}=\sqrt[\nu]{\dfrac{x}{y}}$.

6. $\left(\sqrt[\nu]{x}\right)^\mu = x^{\frac{\mu}{\nu}}$

7. $\sqrt[\nu]{\sqrt[\mu]{x}} = \sqrt[\nu\cdot\mu]{x}$ και $\sqrt[\nu\cdot\cancel{\rho}]{a^{\mu\cdot\cancel{\rho}}} = \sqrt[\nu]{a^\mu}$

Εκτός ύλης

21. (B) $B = \sqrt[5]{241 + \sqrt[4]{4 + 3\sqrt[3]{64}}}$

$$B = \sqrt[5]{241 + \sqrt[4]{4 + 3 \cdot 4}}$$

$$B = \sqrt[5]{241 + \sqrt[4]{16}}$$

$$B = \sqrt[5]{241 + 2}$$

$$B = \sqrt[5]{243} = 3.$$

29. (B) Não $\sqrt{3} \cdot \sqrt[3]{3} \cdot \sqrt[6]{3} = 3$

$$3^{\frac{1}{2}} \cdot 3^{\frac{1}{3}} \cdot 3^{\frac{1}{6}} = 3$$

$$3^{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}} = 3$$

$$3^{\frac{3}{6} + \frac{2}{6} + \frac{1}{6}} = 3$$

$$3^{\frac{6}{6}} = 3$$

$$3^{1} = 3$$

$$3 = 3.$$

29. (B) $\sqrt[3]{9}\,\sqrt{3}\,\sqrt[4]{27}\,\sqrt[12]{3} = 9$.

$$9^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} \cdot 27^{\frac{1}{4}} \cdot 3^{\frac{1}{12}} = 9 .$$

$$(3^2)^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} \cdot (3^3)^{\frac{1}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$3^{\frac{2}{3}} \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{3}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$3^{\frac{2}{3}+\frac{1}{2}+\frac{3}{4}+\frac{1}{12}} = 9$$

$$3^{\frac{8}{12}+\frac{6}{12}+\frac{9}{12}+\frac{1}{12}} = 9$$

$$3^{\frac{24}{12}} = 9$$

$$3^2 = 9$$

$$9 = 9 .$$

02 $\dfrac{\sqrt{8}\ \sqrt[3]{4}}{\sqrt[6]{2}} = 4.$

$$\frac{8^{\frac{1}{2}} \cdot 4^{\frac{1}{3}}}{2^{\frac{1}{6}}} = 4$$

$$\frac{(2^3)^{\frac{1}{2}} \cdot (2^2)^{\frac{1}{3}}}{2^{1/6}} = 4$$

$$\frac{2^{\frac{3}{2}} \cdot 2^{\frac{2}{3}}}{2^{\frac{1}{6}}} = 4$$

$$\frac{2^{\frac{3}{2}+\frac{2}{3}}}{2^{\frac{1}{6}}} = 4$$

$$\frac{2^{\frac{9}{6}+\frac{4}{6}}}{2^{\frac{1}{6}}} = 4$$

$$\frac{2^{\frac{13}{6}}}{2^{1/6}} = 4$$

$$2^{\frac{12}{6}} = 4$$

$$2^2 = 4$$

$$4 = 4.$$

25. γ. $\sqrt[3]{2+\sqrt{3}} \cdot \sqrt[3]{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt[3]{2-\sqrt{2+\sqrt{3}}} = 1$

$$\sqrt[3]{(2+\sqrt{3})(2+\sqrt{2+\sqrt{3}})(2-\sqrt{2+\sqrt{3}})} = 1$$

$$\sqrt[3]{(2+\sqrt{3})(2^2-\sqrt{2+\sqrt{3}}^{\,2})} = 1$$

$$\sqrt[3]{(2+\sqrt{3})(4-(2+\sqrt{3}))} = 1$$

$$\sqrt[3]{(2+\sqrt{3})(2-\sqrt{3})} = 1.$$

$$\sqrt[3]{2^2-\sqrt{3}^{\,2}} = 1$$

$$\sqrt[3]{4-3} = 1$$

$$\sqrt[3]{1} = 1.$$

27. σε $\sqrt{5\sqrt[3]{5\sqrt[4]{25}}}$

α' τρόπος

$$\sqrt{5\sqrt[3]{5\cdot 25^{1/4}}} =$$

$$= \sqrt{5\cdot\sqrt[3]{5\cdot(5^2)^{\frac{1}{4}}}} = \sqrt{5\cdot\sqrt[3]{5\cdot 5^{\frac{2}{4}}}}$$

$$= \sqrt{5\cdot\sqrt[3]{5^{1+\frac{1}{2}}}} = \sqrt{5\cdot\sqrt[3]{5^{\frac{3}{2}}}}$$

$$= \sqrt{5\cdot 5^{\frac{\frac{3}{2}}{3}}} = \sqrt{5\cdot 5^{\frac{3}{6}}}$$

$$= \sqrt{5^{1+\frac{1}{2}}} = \sqrt{5^{\frac{3}{2}}} = 5^{\frac{\frac{3}{2}}{2}}$$

$$= 5^{\frac{3}{4}} = \sqrt[4]{5^3}$$

8.50

⑧ ΝΔΟ $[(x-2)(x+2)]^2 + 8(x-1)(x+1) = (x-3)(x+3)(x^2+9) + 89$.

$[x^2-2^2]^2+8(x^2-1^2)=(x^2-3^2)(x^2+9)+89$

$(x^2)^2-2\cdot x^2\cdot 2^2+(2^2)^2+8x^2-8=x^4+\cancel{9x^2}-\cancel{9x^2}-81+89$

$x^4\cancel{-8x^2}+16+\cancel{8x^2}-8=x^4+8$

$\boxed{x^4+8=x^4+8}$

8.47

r) $(a^2+1)^2-(a-2)(a+2)(a^2+4)=$

$= (a^2)^2+2\cdot a^2\cdot 1+1^2-(a^2-2^2)(a^2+4)=$

$= a^4+2a^2+1-(a^4+\cancel{4a^2}-\cancel{4a^2}-16)=$

$= \cancel{a^4}+2a^2+1-\cancel{a^4}+16=$

$= 2a^2+17$

8.48

(δ) $(x-1)^2-(\sqrt{2x}+1)(1-\sqrt{2x})=x^2$

$x^2-2\cdot x\cdot 1+1^2-\left(1^2-\sqrt{2x}^{\,2}\right)=x^2$

$x^2\cancel{-2x}\cancel{+1}\cancel{-1}\cancel{+2x}=x^2$

$\boxed{x^2=x^2}$

(ε) $(x^2-2)^2-x^2(x-2)(x+2)=4.$

$(x^2)^2-2x^2\cdot 2+2^2-x^2\cdot(x^2-2^2)=4$

$\cancel{x^4}\cancel{-4x^2}+4\cancel{-x^4}\cancel{+4x^2}=4$

$\boxed{4=4}$

8.49

(δ) $(x-1)^2-(\sqrt{2x}+1)(1-\sqrt{2x})=x^2$

$x^2-2\cdot x\cdot 1+1^2-(1^2-(\sqrt{2x})^2)=x^2$

$x^2-2x+1-(1-2x)=x^2$

$x^2-\cancel{2x}+\cancel{1}-\cancel{1}+\cancel{2x}=x^2$

$\boxed{x^2=x^2}$

(ε) $(x^2-2)^2-x^2(x-2)(x+2)=4.$

$(x^2)^2-2\cdot x^2\cdot 2+2^2-x^2(x^2-2^2)=4$

$\cancel{x^4}-4\cancel{x^2}+4-\cancel{x^4}+4\cancel{x^2}=4$

$\boxed{4=4}$

8.41

ⓐ $(\sqrt{18}-2\sqrt{3})(3\sqrt{2}+\sqrt{12})=$

$(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})=$

$(3\sqrt{2})^2-(2\sqrt{3})^2=$

$18-12=6$

ⓑ $(\sqrt{8}-1)(2\sqrt{2}+1)+(2\sqrt{3}-3)(\sqrt{12}+3)$.

$(2\sqrt{2}-1)(2\sqrt{2}+1)+(2\sqrt{3}-3)(2\sqrt{3}+3)$

$(2\sqrt{2})^2-1^2+(2\sqrt{3})^2-3^2=$

$8-1+8-9=$

$\cancel{1}-\cancel{1}=$

0

(γ) $$\frac{(\sqrt{5}-\sqrt{48})(4\sqrt{3}+\sqrt{5})}{(\sqrt{32}-\sqrt{75})(5\sqrt{3}+2\sqrt{8})}$$

$$= \frac{(\sqrt{5}-4\sqrt{3})(4\sqrt{3}+\sqrt{5})}{(2\sqrt{8}-5\sqrt{3})(5\sqrt{3}+2\sqrt{8})} =$$

$$= \frac{(\sqrt{5})^2 - (4\sqrt{3})^2}{(2\sqrt{8})^2 - (5\sqrt{3})^2} =$$

$$= \frac{5-48}{32-75} = \frac{-43}{-47} = \frac{43}{47}$$

8.28

(13) $(2\sqrt{3}-2)^2-(\sqrt{6}-2)^2=(2\sqrt{3})^2-2\cdot2\sqrt{3}\cdot2+2^2-(\sqrt{6}^2-2\cdot\sqrt{6}\cdot2+2^2)$

$=4\cdot3-4\sqrt{3}\cdot2+4-(10-4\sqrt{6})=$

$=12-8\sqrt{3}+4-10+4\sqrt{6}=6-8\sqrt{3}+4\sqrt{6}=6-8\sqrt{3}+4\sqrt{2}\sqrt{3}$

(15) $(\sqrt{12}-3)^2+(\sqrt{27}+2)^2=$

$(2\sqrt{3}-3)^2+(3\sqrt{3}+2)^2=$

$=(2\sqrt{3})^2-2\cdot2\sqrt{3}\cdot3+3^2+(3\sqrt{3})^2+2\cdot3\sqrt{3}\cdot2+2^2=$

$=4\cdot3-4\sqrt{3}\cdot3+9+9\cdot3+6\sqrt{3}\cdot2+4=$

$=12\cancel{-12\sqrt{3}}+9+27\cancel{+12\sqrt{3}}+4=$

$=21+27+4=25+27=52$

8.40

ⓐ $\dfrac{(6+2\sqrt{5})(6-2\sqrt{5})}{(2\sqrt{5}+3\sqrt{2})(3\sqrt{2}-2\sqrt{5})} = \dfrac{6^2-(2\sqrt{5})^2}{(3\sqrt{2})^2-(2\sqrt{5})^2}$

$= \dfrac{36-20}{18-20} = \dfrac{16}{-2} = -8$.

ⓑ $(3+\sqrt{7})(3-\sqrt{7}) + (\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$.

$= 3^2-\sqrt{7}^2 + \sqrt{5}^2 - \sqrt{2}^2 =$

$= 9-7+5-2 \quad = 5$.

(8) $\dfrac{(2\sqrt{10}+4\sqrt{2})(4\sqrt{2}-2\sqrt{10})}{(2\sqrt{3}-4)(4+2\sqrt{3})}$.

$$\frac{(4\sqrt{2})^2-(2\sqrt{10})^2}{(2\sqrt{3})^2-4^2}=\frac{32-40}{12-16}=\frac{-8}{-4}=2$$

8.26

$$\text{(61)}\quad x^2 + (2x+5)^2 = (x+4)^2 + (2x+3)^2$$

$$x^2 + (2x)^2 + 2\cdot 2x \cdot 5 + 5^2 = x^2 + 2\cdot x\cdot 4 + 4^2 + (2x)^2 + 2\cdot 2x\cdot 3 + 3^2$$

$$x^2 + 4x^2 + 20x + 25 = x^2 + 8x + 16 + 4x^2 + 12x + 9$$

$$5x^2 + 20x + 25 = 5x^2 + 20x + 25$$

$$\cancel{5x^2} - \cancel{5x^2} + \cancel{20x} - \cancel{20x} + 25 = 25$$

$$\cancel{25} - \cancel{25} = 0$$

$$\text{(13)}\quad (a-3)^2 + (2a-1)^2 = 5(a-1)^2 + 5$$

$$a^2 - 2(a\cdot 3) + 3^2 + (2a)^2 - 2(2a\cdot 1) + 1^2 = 5(a^2 - 2a + 1) + 5$$

$$a^2 - 6a + 9 + 4a^2 - 4a + 1 = 5a^2 - 10a + 5 + 5$$

$$\cancel{5a^2} - \cancel{10a} + \cancel{10} = \cancel{5a^2} - \cancel{10a} + \cancel{10}$$

γ) $(\alpha+\beta)(6\alpha+4\beta)-(3\alpha+2\beta)^2=(\alpha+\beta)^2-(2\alpha+\beta)^2$

$6\alpha^2+4\beta\alpha+6\alpha\beta+4\beta^2-\left((3\alpha)^2+2\cdot3\alpha\cdot2\beta+(2\beta)^2\right)=\alpha^2+2\alpha\beta+\beta^2-(2\alpha)^2+4\alpha\beta+\beta^2$

$6\alpha^2+\boxed{10\alpha\beta}+4\beta^2-9\alpha^2-6\alpha\cdot2\beta-4\beta^2=\alpha^2+\boxed{\cancel{2\alpha\beta}}+\beta^2-4\alpha^2-\cancel{2\alpha\beta}-\beta^2$

$-3\alpha^2+10\alpha\beta+\cancel{4\beta^2}-6\alpha\cdot2\beta-\cancel{4\beta^2}=-3\alpha^2+\beta^2-4\alpha^2-\beta^2$

$\cancel{-3\alpha^2}+10\alpha\beta-12\alpha\beta=\cancel{-3\alpha^2}+\cancel{\beta^2}-4\alpha^2\cancel{-\beta^2}$

$10\alpha\beta-12\alpha\beta=-4\alpha^2$

8.25

$$④\ 4(3x-2)^2 - 9(2x-1)^2 = 7-12x.$$

$$4\cdot\left[(3x)^2-2\cdot 3x\cdot 2+2^2\right] - 9\left((2x)^2-2\cdot 2x\cdot 1+1^2\right) = 7-12x$$

$$4\cdot\left[9x^2-12x+4\right]-9\left[4x^2-4x+1\right]=7-12x$$

$$\cancel{36x^2}-48x+16-\cancel{36x^2}+36x-9=7-12x$$

$$+7-12x=7-12x$$

$$ε\ \left(x+\frac{3}{2}\right)^2-\left(x-\frac{7}{2}\right)^2=10x-10.$$

$$x^2+\cancel{2}\cdot x\ \frac{3}{\cancel{2}}+\left(\frac{3}{2}\right)^2-\left[x^2-2\cdot x\cdot\frac{7}{2}+\left(\frac{7}{2}\right)^2\right]=10x-10$$

$$x^2+3x+\frac{9}{4}-\left[x^2-\frac{\overset{7}{\cancel{14}}x}{\cancel{2}1}+\frac{49}{4}\right]=10x-10$$

$$\cancel{x^2}+3x+\frac{9}{4}-\cancel{x^2}+7x-\frac{49}{4}\ \ 10x-10$$

$$+10x+\underbrace{\frac{9}{4}}_{1}-\underbrace{\frac{49}{4}}_{2}=10x-10$$

$$+10x+\frac{9}{4}--10=10x-10$$

$$+10x--10=10-10\ \checkmark$$

8.24

(β) $9(x+1)^2-(3x-2)^2=$

$9[x^2+2\cdot x\cdot 1+1^2]-[(3x)^2-2\cdot 3x\cdot 2+2^2]=$

$9[x^2+2x+1]-[9x^2-12x+4]=$

$\cancel{9x^2}+18x+9-\cancel{9x^2}+12x-4=$

$+30x+5$

(δ) $(x-2)(x+4)^2-(x-3)^2(x+2)=$

$(x-2)\cdot(x^2+2\cdot x\cdot 4+4^2)-(x^2-2\cdot x\cdot 3+3^2)\cdot(x+2)=$

$(x-2)\cdot(x^2+8x+16)-(x^2-6x+9)\cdot(x+2)=$

$x^3+8x^2+\cancel{16x}-2x^2-\cancel{16x}-32-(x^3+2x^2-6x^2-12x+9x+18)$

$x^3+6x^2-32-(x^3-4x^2-3x+18)$

$\cancel{x^3}+6x^2-32-\cancel{x^3}+4x^2+3x-18$

$+10x^2+3x-50$

8.19

(γ) $(8+w)^2 = 8^2 + 2\cdot 8\cdot w + w^2 = 64 + 16w + w^2$

(στ) $(6-w)^2 = 6^2 - 2\cdot 6\cdot w + w^2 = 36 - 12w + w^2$

8.20

(β) $(5\alpha - 4\beta)^2 = (5\alpha)^2 - 2\cdot 5\alpha\cdot 4\beta + (4\beta)^2 =$

$= 25\alpha^2 - 40\alpha\beta + 16\beta^2$

(ε) $(2xy + x^3y^2)^2 = (2xy)^2 + 2\cdot 2xy\cdot x^3y^2 + (x^3y^2)^2$

$= 4x^2y^2 + 4x^4y^3 + x^6y^4$

(στ) $\left(\frac{x^2}{2} - 3x\right)^2 = \left(\frac{x^2}{2}\right)^2 - \cancel{2}\cdot \frac{x^2}{\cancel{2}}\cdot 3x + (3x)^2$

$= \frac{x^4}{4} - 3x^3 + 9x^2$.

8.22

(B) $(-x^2-2x)^2 = (-x^2)^2 - 2(-x^2)(2x) + (2x)^2$

$= x^4 + 4x^3 + 4x^2$

(8) $(-x^4+3x)^2 = (-x^4)^2 + 2(-x^4)(3x) + (3x)^2 =$

$= x^8 - 6x^5 + 9x^2$

8

Tautotutes

Ταυτότητες

1. $(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

Απόδειξη

$(\alpha+\beta)^2 = (\alpha+\beta)(\alpha+\beta) = \alpha^2+\alpha\beta+\beta\alpha+\beta^2 = \alpha^2+2\alpha\beta+\beta^2$

2. $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

Απόδειξη

$(\alpha-\beta)(\alpha-\beta) = \alpha^2-\alpha\beta-\alpha\beta+\beta^2 = \alpha^2-2\alpha\beta+\beta^2.$

3. $(\alpha-\beta)(\alpha+\beta) = \alpha^2 - \beta^2$

Απόδειξη

$(\alpha-\beta)(\alpha+\beta) = \alpha^2 + \cancel{\alpha\beta} - \cancel{\alpha\beta} - \beta^2 = \alpha^2 - \beta^2.$

Επομενο Μαθημα

8.19 α β δ ε

8.20 α γ δ

8.21 α δ

8.22 α γ

8.24 α γ

8.25 α β γ

8.28 α γ

8.37 ΟΛΗ

8.38 ΟΛΗ

8.39 ΟΛΗ

8.47 α β

8.48 α β

8.49 α β γ.

8.50 α β γ

Νδο $\ln x \leq x-1$

Απόδειξη

$\ln x \leq x-1$

$\ln x - x + 1 \leq 0$

$$\boxed{f(x) = \ln x - x + 1}$$

$$f'(x) = \frac{1}{x} - 1 = \frac{1-x}{x}$$

$\rightarrow \frac{1-x}{x} = 0 \Rightarrow 1-x=0 \Rightarrow \underline{\underline{x=1}}$

x	0		1		$+\infty$
f'		$+$	0	$-$	
f		$\nearrow$		$\searrow$	

$f(x) \leq f(1)$

$\ln x - x + 1 \leq 0$

$\underline{\underline{\ln x \leq x-1}}$

Νδο $e^x \geq x+1$

Απόδειξη

$e^x \geq x+1$

$e^x - x - 1 \geq 0$

$\boxed{f(x) = e^x - x - 1}$

$f'(x) = e^x - 1$

$\rightarrow e^x - 1 = 0$

$e^x = 1$

$\underline{\underline{x = 0}}$

x		0	
f'	$-$	0	$+$
f	$\searrow$		$\nearrow$

$f(x) \geq f(0)$

$f(x) \geq 0$

$e^x - x - 1 \geq 0$

$e^x \geq x+1$ ✓

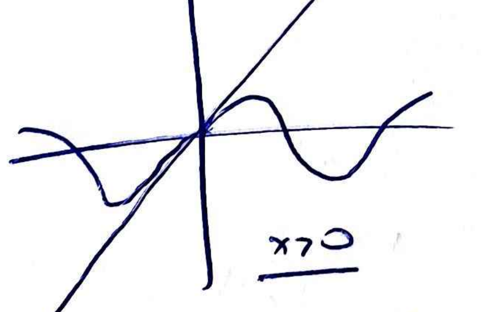
Βασικά Ανισοτικά της ύλης

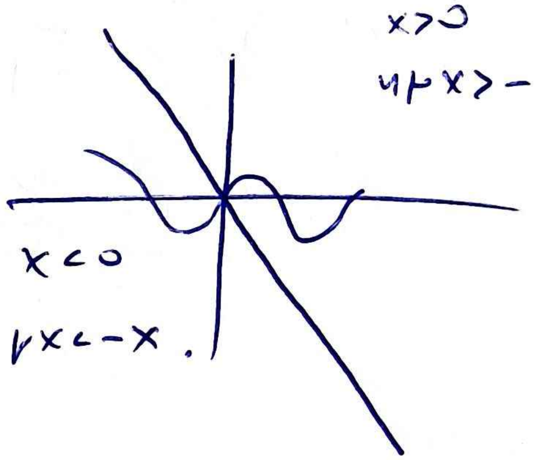
1. $e^x \geq x+1$, $\forall x \in \mathbb{R}$.

2. $\ln x \leq x-1$, $\forall x > 0$.

3. $|ημx| \leq |x|$.

extra





21. $f(x) = 2\sqrt{x} - \dfrac{\ln x}{\sqrt{x}}$ $\qquad D_f = (0, +\infty)$.

$$f'(x) = 2\frac{1}{2\sqrt{x}} - \frac{(\ln x)'\sqrt{x} - \ln x\,(\sqrt{x})'}{\sqrt{x}^{\,2}}$$

$$f'(x) = \frac{1}{\sqrt{x}} - \frac{\frac{1}{x}\sqrt{x} - \ln x\,\frac{1}{2\sqrt{x}}}{x}$$

$$f'(x) = \frac{1}{\sqrt{x}} - \frac{2x - x\ln x}{2x^2\sqrt{x}} \qquad 2x\sqrt{x}$$

$$f'(x) = \frac{1}{\sqrt{x}} - \frac{2 - \ln x}{2x\sqrt{x}}$$

$$f'(x) = \frac{2x}{2x\sqrt{x}} - \frac{2 - \ln x}{2x\sqrt{x}}$$

$$\boxed{f'(x) = \frac{2x - 2 + \ln x}{2x\sqrt{x}}}$$

x		1	
φ'	+		+
φ	$\nearrow$ $-$	0	$+$ $\nearrow$
f'	$-$		+
f	$\searrow$		$\nearrow$

$\varphi(x) = 2x - 2 + \ln x \qquad \varphi(1) = 0$

$\varphi'(x) = 2 + \dfrac{1}{x} > 0$

22. (B) $f(x) = x\ln x - \frac{x^2}{2}.$

$f'(x) = (x)' \cdot \ln x + (\ln x)' \cdot x - \frac{1}{\cancel{2}} \cdot \cancel{2}x$

$f'(x) = \ln x + \frac{1}{\cancel{x}} \cdot \cancel{x} - x = \ln x - x + 1.$

$$\boxed{f'(x) = \ln x - x + 1.}$$

$\cdot\ \ln x \leq x - 1$ $\qquad$ $f'(x) \leq 0$

$\ln x - x + 1 \leq 0$ $\qquad$ $f \downarrow$

22. 01 $f(x) = 2e^x - x^2 - 2x$

$F'(x) = (2e^x)' - (x^2)' - (2x)'$

$$\boxed{F'(x) = 2 \cdot e^x - 2x - 2}$$

$f'(x) = 2(e^x - x - 1) \geq 0$

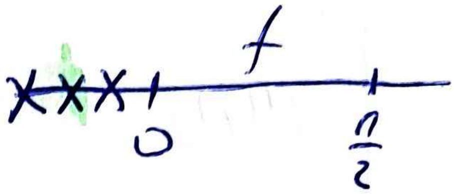
$+$

$f \nearrow$

- $e^x \geq x+1$

$e^x - x - 1 \geq 0$

21. (δ) $f(x)=\begin{cases} \dfrac{1-\sigma\upsilon\nu x}{x}, & x\in(0,\frac{\pi}{2}) \\ 0, & x=0. \end{cases}$



Είναι συνεχής στο 0;

Είναι παρ/μη στο 0;

Μονοτονία $f(x)$.

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{1-\sigma\upsilon\nu x}{x} = 0 \text{ κ' } f(0)=0 \checkmark$$

$$\lim_{x\to0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to0^+} \frac{\frac{1-\sigma\upsilon\nu x}{x}-0}{x}$$

$$= \lim_{x\to0^+} \frac{1-\sigma\upsilon\nu x}{x^2} \underset{DLH}{\overset{\frac{0}{0}}{=}} \lim_{x\to0^+} \frac{\eta\mu x}{2x} =$$

$$= \lim_{x\to0^+} \frac{\sigma\upsilon\nu x}{2} = \frac{1}{2}$$

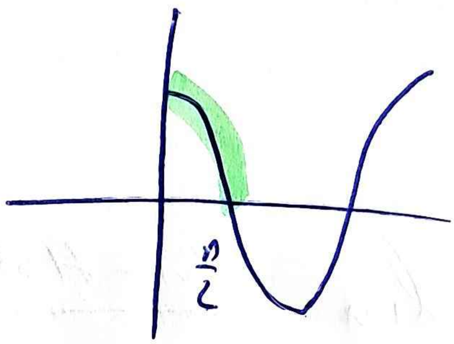
$f'(0)=\frac{1}{2}$.

$$f'(x) = \frac{(1-συνx)' x - (1-συνx)\cdot(x)'}{x^2}$$

$$\boxed{f'(x) = \frac{x\,ημx - 1 + συνx}{x^2}}$$

$$φ(x) = x\,ημx - 1 + συνx$$

$$φ'(x) = \cancel{ημx} + x\,συνx - \cancel{ημx}$$



$$\underline{\underline{φ'(x) = x\,συνx > 0}}$$

x	0	$\frac{π}{2}$
$φ'$		$+$
$φ$	0	$\nearrow$ $+$
f'		$+$
f		$\nearrow$

$x > 0 \Rightarrow φ(x) > φ(0)$

$φ(x) > 0$

21. (B) $f(x) = \dfrac{x^3 - 2\ln x}{2x}$ $\quad D_f = (0, +\infty)$

$$f'(x) = \frac{\left(3x^2 - \frac{2}{x}\right)(2x) - (x^3 - 2\ln x)2}{4x^2}$$

$$f'(x) = \frac{6x^3 - 4 - 2x^3 + 4\ln x}{4x^2}$$

$$f'(x) = \frac{4x^3 + 4\ln x - 4}{4x^2}$$

$$\boxed{f'(x) = \frac{x^3 + \ln x - 1}{x^2}}$$

x	0		1		$+\infty$
φ'		$+$		$+$	
φ		↗ $-$	0	$+$ ↗	
f'		$-$		$+$	
f		↘		↗	

$\varphi(x) = x^3 + \ln x - 1$

$\varphi'(x) = 3x^2 + \dfrac{1}{x} > 0$

22. (γ) $f(x) = \ln^2 x - 2x + 2$

$f'(x) = (\ln x)^2 - 2$

$f'(x) = 2\frac{\ln x}{x} - 2$

$f'(x) = \frac{2\ln x - 2x}{x}$

$D_f = (0, +\infty)$.

$$\boxed{f'(x) = 2\,\frac{\overset{\ominus}{\ln x - x}}{x} < 0}$$

$f \downarrow$.

$(x^2)' = 2x$

$((\ln x)^2)' = 2\ln x\,(\ln x)'$

$= 2\frac{\ln x}{x}$

• $\ln x \leq x - 1$

$\ln x - x \leq -1$

$\ln x - x < 0$

22. 8) $f(x) = x^2 \ln x + \frac{1}{2}x^2 - \frac{2}{3}x^3$

$x > 0$

~~$f(x) = x^2 \left(\ln x + \frac{1}{2} - \frac{2}{3} \right.$~~

$f'(x) = (x^2)' \cdot \ln x + (\ln x)' \cdot x^2 + \frac{1}{2} \cdot (x^2)' - \frac{2}{3}(x^3)'$

$f'(x) = 2x \cdot \ln x + \frac{1}{\cancel{x}} \cdot x^{\cancel{2}} + \frac{1}{\cancel{2}} \cdot \cancel{2}x - \frac{2}{\cancel{3}} \cdot \cancel{3}x^2$

$f'(x) = 2x \cdot \ln x + x + x - 2x^2$

$f'(x) = 2x \cdot \ln x + 2x - 2x^2$

$f'(x) = \underset{+}{2x}(\underset{-}{\ln x - x + 1}) < 0$

$\ln x \leq x - 1$

$\ln x - x + 1 \leq 0$

$f'(x) < 0$

$f(x) \searrow$

23. ① $f(x) = x\ln x - x^2 - x$

$D_f = (0, +\infty)$.

$$f'(x) = (x)'\cdot \ln x + x\cdot(\ln x)' - 2x - 1$$

$$= \ln x + \cancel{1} - 2x - \cancel{1} = \boxed{\ln x - 2x}$$

$$\boxed{f'(x) = \ln x - 2x}$$

$$f''(x) = \frac{1}{x} - 2 = \frac{1-2x}{x}$$

$\rightarrow \frac{1-2x}{x} = 0 \Rightarrow x = \frac{1}{2}$

x	0	$\frac{1}{2}$	$+\infty$
f''	$+$		$-$
f'	$\nearrow$		$\searrow$
f	$\searrow$		$\searrow$

$f'(x) \leq f'\left(\frac{1}{2}\right)$

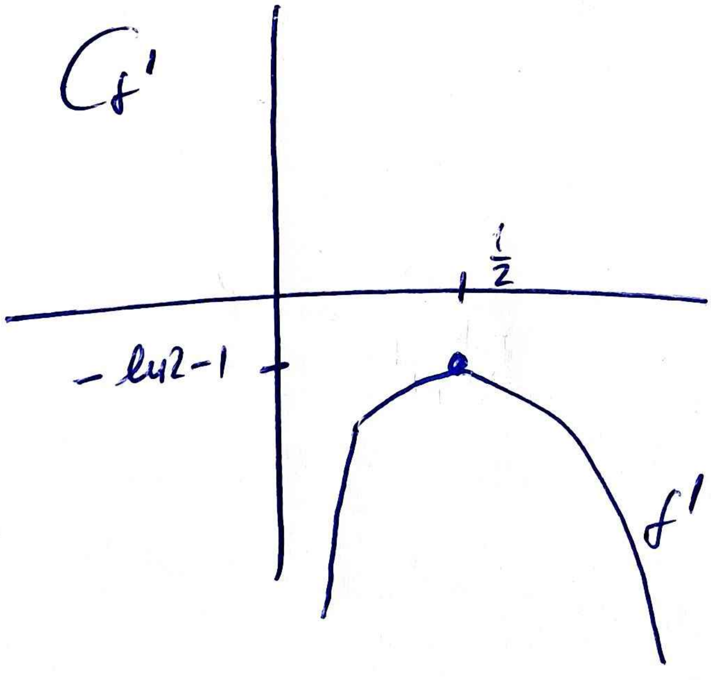
$f'(x) \leq \ln\frac{1}{2} - 1$

$f'(x) \leq \ln 1 - \ln 2 - 1$

$f'(x) \leq -\ln 2 - 1$

$f'(x) < 0$

$C_{f'}$



23. (a) $f(x) = 2e^x - x^2$

$F'(x) = (2\cdot e^x)' - (x^2)'$

$F'(x) = 2\cdot e^x - 2x$

$\boxed{F'(x) = 2(\underbrace{e^x - x}_{(+)})} > 0$

• $e^x \geq x+1$ $\qquad f \nearrow$

$e^x - x \geq 1$

$e^x - x > 0$

(B) $f(x) = \frac{1}{2}x^2 + x - x\ln x$

$F'(x) = \left(\frac{1}{2}x^2\right)' + (x)' - (x)'\cdot \ln x - x\cdot(\ln x)'$

$F'(x) = \frac{1}{2}\cdot 2x + 1 - \ln x - x\cdot\frac{1}{x}$

$f'(x) = x + 1 - \ln x - 1$

$f'(x) = x - \ln x > 0$ $\qquad f \nearrow$

• $\ln x \leq x - 1 \Rightarrow 1 \leq x - \ln x$

$0 < x - \ln x$

$D_f = \mathbb{R}$. και $\underline{x>0}$

24. ① $f(x) = \ln(x - \ln x)$ πρεπει $x - \ln x > 0$

που ειναι!.

$f'(x) = \frac{1}{x-\ln x}\left(1 - \frac{1}{x}\right)$ γιατι $\ln x \leq x-1$

$1 \leq x - \ln x$

$0 < x - \ln x$ ✓

$f'(x) = \frac{1-\frac{1}{x}}{x-\ln x}$

$f'(x) = \frac{x-1}{x-\ln x}$ ⊕

x	0	1	$+\infty$
f'	$-$	$+$	
f	↘	↗	

⑥ $f(x) = \frac{1}{x} - \frac{1}{\varepsilon\varphi x}$ $\quad x \in \left(0, \frac{\pi}{2}\right)$.

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

$f'(x) = -\frac{1}{x^2} + \frac{1}{\varepsilon\varphi^2 x}(\varepsilon\varphi x)'$

$f'(x) = -\frac{1}{x^2} + \frac{1}{\varepsilon\varphi^2 x}\frac{1}{\sigma\upsilon\nu^2 x}$

$f'(x) = -\frac{1}{x^2} + \frac{1}{\frac{\eta\mu^2 x}{\cancel{\sigma\upsilon\nu^2 x}}}\frac{1}{\cancel{\sigma\upsilon\nu^2 x}}$

$\circ\ |\eta\mu x| \leq |x|$

$\eta\mu^2 x \leq x^2$

$x^2 - \eta\mu^2 x \geq 0$

$f'(x) = -\frac{1}{x^2} + \frac{1}{\eta\mu^2 x} = \frac{x^2 - \eta\mu^2 x}{x^2\eta\mu^2 x} > 0$ ⊕

f ↗

24. (α) $f(x) = \ln(e^x - x)$

[illegible] $e^x - x > 0$ [illegible] $D_f = \mathbb{R}.$

$e^x \geq x+1$

$\Rightarrow e^x - x \geq 1$

$e^x - x > 0$ ✓

$$F'(x) = \cancel{\ln} \frac{1}{e^x - x} \cdot (e^x - x)'$$

$$F'(x) = \frac{1}{e^x - x} \cdot (e^x - 1)$$

$$\boxed{f'(x) = \frac{e^x - 1}{e^x - x}}$$

x		0	
f'	$-$	0	$+$
f	↘		↗

$\rightarrow e^x - 1 = 0$

$e^x = 1$

$\underline{x = 0}$

(B) $f(x) = \dfrac{x}{e^x - x}$

[illegible] $e^x - x \neq 0$ [illegible]

$e^x \geq x+1 \Rightarrow e^x - x \geq 1$

$e^x - x \neq 0$

$D_f = \mathbb{R}$.

$$F'(x) = (x)' \cdot (e^x - x) - x \cdot (e^x - x)'$$

$$F'(x) = e^x - x - xe^x - x^2$$

$$\boxed{F'(x) = e^x - x - xe^x - x^2}$$

$$F''(x) = (e^x)' - (x)' - (x)' \cdot e^x - x(e^x)' - (x^2)'$$

$$F''(x) = \cancel{e^x} - 1 - \cancel{e^x} - xe^x - 2x$$

$$F''(x) = -2x - xe^x - 1$$

23. ⑧. $f(x) = (x+1)\ln x + e^x$

$D_f = (0, +\infty)$.

$f'(x) = \ln x + (x+1)\frac{1}{x} + e^x$

$f'(x) = \ln x + 1 + \frac{1}{x} + e^x$

- $e^x \geq x+1$ $\Rightarrow$
- $\ln x \leq x-1$ $\Rightarrow$ $\ln\frac{1}{x} \leq \frac{1}{x} - 1$ $\Rightarrow$

$\ln 1 - \ln x \leq \frac{1}{x} - 1$

$-\ln x \leq \frac{1}{x} - 1$

$\ln x \geq 1 - \frac{1}{x}$

(+)

$e^x + \ln x \geq x+1+1-\frac{1}{x}$

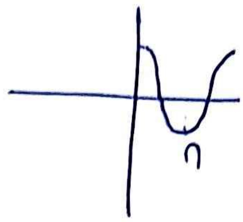
$e^x + \ln x + \frac{1}{x} \geq 2$ $\qquad$ $f'(x) > 0$

$e^x + \ln x + \frac{1}{x} + 1 \geq 3 \Rightarrow f'(x) \geq 3$ f↗

25. (α) $f(x) = 2\eta\mu x - x + 3 \qquad x \in [0, \pi]$.

$f'(x) = 2 \cdot \sigma\upsilon\nu x - 1$

$f'(x) = \underbrace{2\sigma\upsilon\nu x - 1}_{\ominus}$



$-1 \le \sigma\upsilon\nu x \le 1$

$-2 \le \sigma\upsilon\nu x - 1 \le 0$

$-4 \le 2\sigma\upsilon\nu x - 1 \le 0$

$f \searrow$

✓

(β) $f(x) = \frac{1}{2} x^2 + \sigma\upsilon\nu x . \quad x \in [0, \pi]$

$f'(x) = \frac{1}{\cancel{2}} \cdot \cancel{2}x + (-\eta\mu x)$

$f'(x) = x - \eta\mu x > 0$

$f \nearrow$

~~$|\eta\mu x| \le |x|$~~

~~$|x| - |\eta\mu x| \ge 0$~~

?

$\eta\mu x < x , \ x > 0$

$x - \eta\mu x > 0$

24. (B) $f(x) = \dfrac{x}{e^x - x}$ $\quad D_f = \mathbb{R}$.

$$F'(x) = \frac{(x)' \cdot (e^x - x) - x \cdot (e^x - x)'}{(e^x - x)^2}$$

$$F'(x) = \frac{e^x - x - x \cdot (e^x - 1)}{(e^x - x)^2}$$

$$F'(x) = \frac{e^x - \cancel{x} - xe^x + \cancel{x}}{(e^x - x)^2}$$

$$f'(x) = \frac{e^x - xe^x}{(e^x - x)^2}$$

$$f'(x) = \frac{e^x(1 - x)}{(e^x - x)^2}$$

x		1	
f'	$+$	0	$-$
f	$\nearrow$		$\searrow$

(٤) $f(x) = e^{1-x} + \ln x$ $\qquad D_f =]0, +\infty[$.

$f'(x) = -e^{1-x} + \frac{1}{x}$ $\qquad f'(1) = 0$.

$$\boxed{f'(x) = \frac{1 - x e^{1-x}}{x}}$$

$\varphi(x) = 1 - x e^{1-x}$ $\qquad \varphi(1) = 0$

$\varphi'(x) = -e^{1-x} + x e^{1-x}$

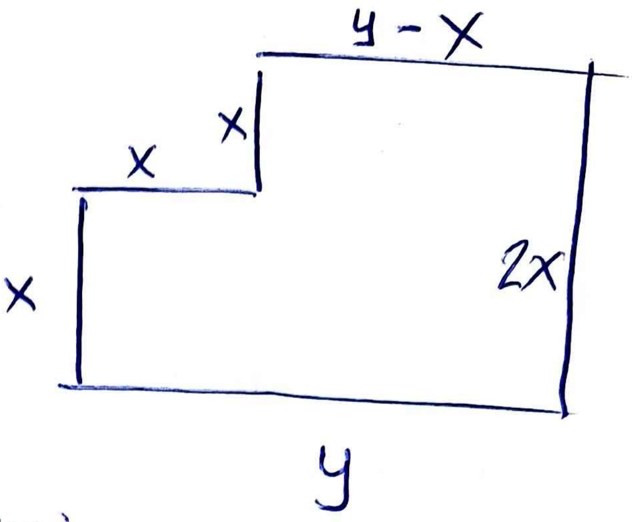
$\varphi'(x) = e^{1-x}(x-1)$.

x		1	
φ'	$-$	0	$+$
φ	$\searrow +$	0	$+ \nearrow$
f'	$+$		$+$
f	$\nearrow$		$\nearrow$

34.

Αν $|x-1|<0,1$

Αν $|y-4|<0,2$



Βρείτε όρια περιμέτρου.

$$\Pi = y + 2x + y - x + x + x + x$$

$$\boxed{\Pi = 2y + 4x}$$

$|x-1| < 0,1$

$-0,1 < x-1 < 0,1$

$\boxed{0,9 < x < 1,1}$

$3,6 < 4x < 4,4$

$|y-4| < 0,2$

$-0,2 < y-4 < 0,2$

$\boxed{3,8 < y < 4,2}$

$7,6 < 2y < 8,4$

(+)

$11,2 < 4x + 2y < 12,8$

$11,2 \leq \Pi < 12,8$

38. (β) $|x-y+1|+x^2=2x-1$

$$|x-y+1| + x^2-2x+1=0$$

$$\boxed{|x-y+1| + (x-1)^2 = 0.}$$

$$\begin{cases} x-y+1=0 \longrightarrow 1-y+1=0 \\ \qquad\qquad\qquad\qquad y=2 \\ x-1=0 \Rightarrow x=1 \end{cases}$$

(γ). $|2x+y-3|+|3x-y-2|=0.$

$$\begin{cases} 2x+y-3=0 \Rightarrow \boxed{y=3-2x} \\ 3x-y-2=0 \end{cases}$$

$$3x-(3-2x)-2=0$$

$$3x-3+2x-2=0 \qquad y=1$$

$$5x=5$$

$$x=1$$

44. B $\left|\frac{2}{x}-3\right|<1$.

$$-1<\frac{2}{x}-3<1$$

$$2<\frac{2}{x}<4$$

$$\frac{1}{2}>\frac{x}{2}>\frac{1}{4}$$

$$2>2x>1$$

$$1>x>\frac{1}{2}$$

$$x\in\left(\frac{1}{2},1\right),$$

46. אם $\left|\frac{3a+1}{a+3}\right| < 1$

אז $|a| < 1$.

$$\frac{|3a+1|}{|a+3|} < 1$$

$$|3a+1| < |a+3|$$

$$|3a+1|^2 < |a+3|^2$$

$$9a^2 + \cancel{6a} + 1 < a^2 + \cancel{6a} + 9$$

$$8a^2 < 8$$

$$a^2 < 1$$

$$a^2 < 1^2$$

$$|a| < |1|$$

$$|a| < 1.$$

$$x^2 < y^2 \iff |x| < |y|$$

42. Αν $-1 \leq x \leq 1$

νδο $|x^3-3x^2+2x-1| \leq 7$.

Αφού $-1 \leq x \leq 1 \Rightarrow \underline{|x| \leq 1}$

Ισχύει: $|\alpha+\beta| \leq |\alpha|+|\beta|$

$|x^3-3x^2+2x-1| = |x^3+(-3x^2)+(2x)+(-1)| \leq$

$\leq |x^3|+|-3x^2|+|2x|+|-1|$

$|x^3-3x^2+2x-1| \leq |x|^3+3|x|^2+2|x|+1 \leq 7$

$|x^3-3x^2+2x-1| \leq 7$

- $|x| \leq 1 \Rightarrow |x|^3 \leq 1^3 \Rightarrow |x|^3 \leq 1$
- $|x| \leq 1 \Rightarrow |x|^2 \leq 1^2 \Rightarrow |x|^2 \leq 1 \Rightarrow 3|x|^2 \leq 3$
- $|x| \leq 1 \Rightarrow 2|x| \leq 2 \Rightarrow 2|x|+1 \leq 3$.

(+)

$$\boxed{|x|^3+3|x|^2+2|x|+1 \leq 7}$$

48. $d(x,2)<3$

(α) νδο $\quad -1<x<5$

$|x-2|<3$

$-3<x-2<3$

$-1<x<5$

(β) i) $A=\dfrac{|\overset{+}{x+1}|+|\overset{-}{x-5}|}{3}=\dfrac{x+1-x+5}{3}=2.$

• $-1<x<5 \Rightarrow 0<x+1<6$

• $-1<x<5 \Rightarrow -6<x-5<0$

ii). $B=\left|\dfrac{x+2}{x-6}\right|=\dfrac{|\overset{+}{x+2}|}{|\overset{-}{x-6}|}=\dfrac{x+2}{6-x}$

• $-1<x<5 \Rightarrow 1<x+2<7$

• $-1<x<5 \Rightarrow -7<x-6<-1$

iii). $\Gamma=|x^2-4x-5|=|(x-5)(x+1)|=|\overset{-}{x-5}||\overset{+}{x+1}|$

$=(5-x)(x+1)$

51. Έστω $A = \dfrac{x^2 - 6|x| + 9}{x^2 - 3|x|}$

(α) Για να ορίζεται η A πρέπει $x^2 - 3|x| \neq 0$

• $x^2 - 3|x| = 0$

$|x|^2 - 3|x| = 0$

$|x|\left(|x| - 3\right) = 0$

$|x| = 0$ ή $|x| - 3 = 0$

$x = 0$ $\qquad$ $|x| = 3$

$x = 3$ $\qquad$ $x = -3$

Η A ορίζεται για $x \neq 0$, $x \neq 3$, $x \neq -3$.

(β). $A = \dfrac{|x|^2 - 6|x| + 9}{|x|^2 - 3|x|} = \dfrac{(|x| - 3)^{\cancel{2}}}{|x|\cancel{(|x| - 3)}} = \dfrac{|x| - 3}{|x|}$

$b)$ i) $|A| = \frac{1}{2}$

$A = \frac{1}{2}$ v $A = -\frac{1}{2}$

$\frac{|x|-3}{|x|} = \frac{1}{2}$

$2|x| - 6 = |x|$

$|x| = 6$

$x = 6$ v $x = -6$

$\frac{|x|-3}{|x|} = -\frac{1}{2}$

$2|x| - 6 = -|x|$

$3|x| = 6$

$|x| = 2$

ii) $|A| < \frac{1}{3}$

$x \neq 0$ $x \neq 3$
$x \neq -3$

$-\frac{1}{3} < A < \frac{1}{3}$

$-\frac{1}{3} < \frac{|x|-3}{|x|} < \frac{1}{3}$

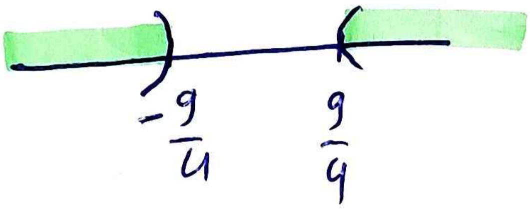
$-\frac{1}{3} < \frac{|x|-3}{|x|}$ <u>και</u> $\frac{|x|-3}{|x|} < \frac{1}{3}$

$-|x| < 3|x|-9$

$9 < 4|x|$

$\frac{9}{4} < |x|$

$x > \frac{9}{4}$ ή $x < -\frac{9}{4}$



$x \in (-\infty, -\frac{9}{4}) \cup (\frac{9}{4}, +\infty)$

$3|x|-9 < |x|$

$2|x| < 9$

$|x| < \frac{9}{2}$

$-\frac{9}{2} < x < \frac{9}{2}$

$x \in (-\frac{9}{2}, \frac{9}{2})$

$$x \in \left(-\frac{9}{2}, -\frac{9}{4}\right) \cup \left(\frac{9}{4}, \frac{9}{2}\right).$$

$$x \in \left(-\frac{9}{2}, -3\right) \cup \left(-3, -\frac{9}{4}\right) \cup \left(\frac{9}{4}, 3\right) \cup \left(3, \frac{9}{2}\right)$$

extra 1

$$A = |x-4| - 2x + 1$$

1. Αν $x - 4 \geq 0 \Rightarrow A = |\overset{+}{x-4}| - 2x + 1 = x - 4 - 2x + 1$

$\hookrightarrow x \geq 4$

$$\boxed{A = -x - 3}$$

2. Αν $x - 4 < 0$

$\hookrightarrow x < 4$

$A = |\overset{-}{x-4}| - 2x + 1 = 4 - x - 2x + 1$

$$\boxed{A = 5 - 3x}$$

$$A = \begin{cases} -x-3, & x \geq 4 \\ 5-3x, & x < 4. \end{cases}$$

extra 2

$$A = |x-1| - |x-2|$$

x		1		2	
$x-1$	$-$	0	$+$		$+$
$x-2$	$-$		$-$	0	$+$

1. A v $x<1$ $\quad A = |\overset{\ominus}{x-1}| - |\overset{\ominus}{x-2}|$

$$A = -x+1-(-x+2)$$

$$\underline{A = -1}$$

2. A v $1 \le x \le 2$ $\quad A = |\overset{\oplus}{x-1}| - |\overset{\ominus}{x-2}|$

$$A = x-1-(-x+2)$$

$$A = x-1+x-2$$

$$\underline{A = 2x-3}$$

3. Αν $x>2$ τότε $A = |\overset{\oplus}{x-1}| - |\overset{\oplus}{x-2}|$

$$A = x-1-(x-2) = x-1-x+2 = 1$$

$$\underline{A=1}$$

$$A = \begin{cases} -1, & x<1 \\ 2x-3, & 1 \le x \le 2 \\ 1, & x>2 \end{cases}$$

Επομενο Μαθημα

Τεταρτη 5:30-7

Τσεκαρετε το μαθημα
13/11/24
16/11/24.

Σελ 114

28 α γ
29
30
32.
33
35
38 α
40 α
41 α.
44 α.

47
49
50
52.

Σελ 114

28 (α) $A = |x-1| - 2x + 3$

1η: Αν $x-1 \geq 0$ δηλαδή $x \geq 1$ τοτε

$$A = |x-1| - 2x + 3$$

$$A = x - 1 - 2x + 3$$

$$\boxed{A = 2 - x}$$

2η: Αν $x-1 < 0$ δηλαδή $x < 1$ τοτε

$$A = |x-1| - 2x + 3$$

$$A = 1 - x - 2x + 3 = 4 - 3x$$

$$A = 4 - 3x$$

γ) $\Gamma = \dfrac{|x+1| + |x-1|}{2}$.

x		-1		1	
$x+1$	$-$	0	$+$		$+$
$x-1$	$-$		$-$	0	$+$

1η: Αν $x < -1$ τότε

$$\Gamma = \frac{|x+1| + |x-1|}{2} = \frac{-x-1-x+1}{2} = \frac{-2x}{2} = -x$$

2η: Αν $-1 \leq x \leq 1$ τότε

$$\Gamma = \frac{|x+1| + |x-1|}{2} = \frac{\cancel{x}+1 \cancel{-x} + 1}{2} = 1$$

3η: Αν $x > 1$ τότε

$$\Gamma = \frac{|x+1| + |x-1|}{2} = \frac{x+\cancel{1} + x \cancel{-1}}{2} = \frac{2x}{2} = x.$$

$$\Gamma = \begin{cases} -x, & x < -1 \\ 1, & -1 \leq x \leq 1 \\ x, & x > 1 \end{cases},$$

30. (α) $|2\alpha-\beta| = |\alpha-2\beta|$.

$2\alpha-\beta = \alpha-2\beta$ ή $2\alpha-\beta = -\alpha+2\beta$

$\boxed{\alpha = -\beta}$ $3\alpha = 3\beta$

α, β αντίθετοι $\boxed{\alpha = \beta}$

ίσοι!.

(β) $|3\alpha-\beta| - |\alpha+2\beta| = 0$.

νδο $\frac{\alpha}{\beta} = \frac{3}{2}$ ή $\frac{\alpha}{\beta} = -\frac{1}{4}$.

$|3\alpha-\beta| = |\alpha+2\beta|$

$3\alpha-\beta = \alpha+2\beta$ ή $3\alpha-\beta = -\alpha-2\beta$

$2\alpha = 3\beta$ $4\alpha = -\beta$

$\alpha = \frac{3}{2}\beta$ $\frac{\alpha}{\beta} = -\frac{1}{4}$

$\frac{\alpha}{\beta} = \frac{3}{2}$

32. $|2x-1|<1$

(α) νδο $0<x<1$

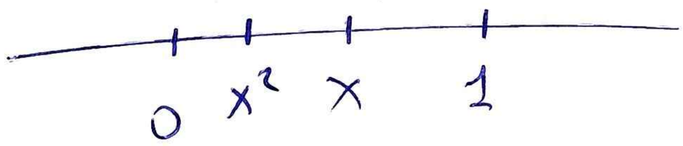
$-1<2x-1<1 \Rightarrow 0<2x<2 \Rightarrow \underline{\underline{0<x<1}}$

(β) i) $\left|\dfrac{x}{x-1}\right| = \dfrac{|x|}{|x-1|} = \dfrac{x}{1-x}$

• $0<x<1 \Rightarrow -1<x-1<0$

ii) $|x^2-x| = |x(x-1)| = |x||x-1| = x(1-x)$

(γ)



Έστω $x^2>x \Rightarrow x^2-x>0 \Rightarrow x(x-1)>0$ Άτοπο!

• $0<x<1 \Rightarrow -1<x-1<0$

Άρα $\underline{\underline{x^2<x}}$

$x^2>0$

35. ⓐ $|x-3|<3$ $\qquad$ $|y-3|<1$

$-3<x-3<3$ $\qquad$ $-1<y-3<1$

$\underline{\underline{0<x<6}}$ $\qquad$ $\underline{\underline{2<y<4}}$

ⓑ . i) $A=|\overset{\ominus}{x-6}|+|\overset{\oplus}{y-2}| = -x+6+y-2 = y-x+4$

• $0<x<6 \Rightarrow -6<x-6<0$

• $2<y<4 \Rightarrow 0<y-2<2$

ii) . $B=\left|\dfrac{x}{y-4}\right| = \dfrac{|\overset{\oplus}{x}|}{|\overset{\ominus}{y-4}|} = \dfrac{x}{4-y}$

• $2<y<4 \Rightarrow -2<y-4<0$

iii) . $|xy-4x| = |x(y-4)| = |\overset{\oplus}{x}|\,|\overset{\ominus}{y-4}| = x|y-4|$

iv) . $|\overset{\ominus}{x-y-4}| = -x+y+4$.

• $0<x<6 \Rightarrow 6>x>0$

• $2<y<4 \Rightarrow -2>-y>-4$

$\oplus$ $\quad 4>x-y>-4$

$0>x-y-4>-8$

vl. $\left|\overset{\ominus}{\frac{x}{y}-3}\right| = -\frac{x}{y}+3$.

- $0<x<6 \Rightarrow 6>x>0$
- $2<y<4 \Rightarrow \frac{1}{2}>\frac{1}{y}>\frac{1}{4}$

$\odot\ 3>\frac{x}{y}>0$

$0>\frac{x}{y}-3>-3$

⑥

- $0<x<6 \Rightarrow 0<2x<12 \Rightarrow 12>2x>0$
- $2<y<4 \Rightarrow -2>-y>-4$

$\oplus$

$10>2x-y>-4$

$7>2x-y-3>-7$

$|2x-y-3|<7$

40. (a) $a^2+B^2 \geq 2|aB|$

$$|a|^2+|B|^2-2|aB| \geq 0$$

$$|a|^2+|B|^2-2|a||B| \geq 0$$

$$(|a|-|B|)^2 \geq 0 .$$

41. (a) $\left|\frac{2a}{a^2+1}\right| \leq 1$

$$\frac{|2a|}{|a^2+1|} \leq 1 \Rightarrow \frac{|2a|}{a^2+1} \leq 1$$

(+)

$$2|a| \leq a^2+1$$

$$0 \leq |a|^2-2|a|+1$$

$$0 \leq (|a|-1)^2 .$$

44. (a) $\left|\frac{1}{x}-2\right|<1$.

$$-1<\frac{1}{x}-2<1$$

$$1<\frac{1}{x}<3$$

$$1>x>\frac{1}{3}$$

47. $d(a,0) < 1$

$|a-0| < 1$

$|a| < 1$

$-1 < a < 1$

(a) $\left|2-|a-1|\right| = |2-(-a+1)| = |2+a-1| =$

$= |1+a| = 1+a.$

- $-1 < a < 1 \Rightarrow -2 < a-1 < 0$
- $-1 < a < 1 \Rightarrow 0 < a+1 < 2$

(B) i) $A = |2a-2| - |3a+4|. = -2a+2-(3a+4)$

$= -2a+2-3a-4 = -5a-2$

- $-1 < a < 1 \Rightarrow -2 < 2a < 2 \Rightarrow -4 < 2a-2 < 0$
- $-1 < a < 1 \Rightarrow -3 < 3a < 3 \Rightarrow 1 < 3a+4 < 7$

ii). $|a^2-1| = |a-1||a+1| = (1-a)(a+1).$

- $-1 < a < 1 \Rightarrow -2 < a-1 < 0$
- $-1 < a < 1 \Rightarrow 0 < a+1 < 2$

49. $A=\dfrac{x^2-9}{|x|-3}$.

(α) Για να ορίζεται η A πρέπει

$$|x|-3\neq 0$$

→ $|x|-3=0 \Rightarrow |x|=3 \Rightarrow x=3$ ή $x=-3$.

Η A ορίζεται αν $x\neq 3, -3$.

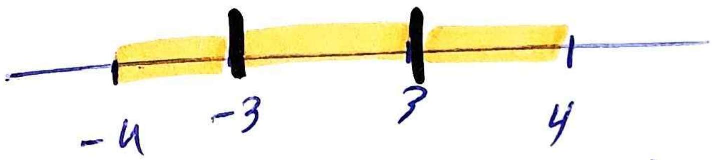
(β) $$A=\frac{|x|^2-9}{|x|-3}=\frac{\cancel{(|x|-3)}(|x|+3)}{\cancel{|x|-3}}=|x|+3.$$

(γ) $A<7$

$$|x|+3<7$$

$$|x|<4$$

$$-4<x<4$$



$x\in(-4,-3)\cup(-3,3)\cup(3,4)$.

50. $A = \dfrac{|x|^3 - 2x^2}{|x| - 2}$

$$A = \frac{|x|^3 - 2|x|^2}{|x| - 2} = \frac{|x|^2 \cancel{(|x|-2)}}{\cancel{|x|-2}}.$$

(α) Για να ορίζεται η A πρέπει $|x| - 2 \neq 0$

$\rightarrow |x| - 2 = 0 \Rightarrow |x| = 2$ $\quad x = 2$ ή $x = -2$

Πρέπει $x \neq 2$, $x \neq -2$.

(β) $A = |x| = x^2$

(γ). $A < 9$

$x^2 < 9$

$|x|^2 < 3^2$

$|x| < |3|$

$|x| < 3$

$-3 < x < 3$.

25. B vdo $\sqrt[3]{49}\cdot\sqrt[3]{5+3\sqrt{2}}\cdot\sqrt[3]{5-3\sqrt{2}}=7.$

$$\sqrt[3]{49\cdot(5+3\sqrt{2})\cdot(5-3\sqrt{2})}=7$$

$$\sqrt[3]{49\cdot(5^2-(3\sqrt{2})^2)}=7$$

$$\sqrt[3]{7^2\cdot(5^2-(3\sqrt{2})^2)}$$

$$\sqrt[3]{49\cdot(25-18)}$$

$$\sqrt[3]{49\cdot 7}=$$

$$\cancel{\sqrt[3]{7^3}}$$

$$7.$$

25. (a) $\sqrt[3]{3} \sqrt[3]{4-\sqrt{7}} \cdot \sqrt[3]{4+\sqrt{7}} = 3$

$$3^{\frac{1}{3}} \cdot (4-\sqrt{7})^{\frac{1}{3}} \cdot (4+\sqrt{7})^{\frac{1}{3}} = 3$$

$$\sqrt[3]{3(4-\sqrt{7})(4+\sqrt{7})} = 3$$

$$\sqrt[3]{3 \cdot (4^2 - \sqrt{7}^2)} = 3$$

$$\sqrt[3]{3 \cdot 9} = 3$$

$$\sqrt[3]{27} = 3$$

$$3 = 3$$

27. (α) $\sqrt{\sqrt[4]{5}} = \sqrt{5^{\frac{1}{4}}}$

$$= 5^{\frac{\frac{1}{4}}{2}} = 5^{\frac{1}{8}}$$

$$\sqrt[\mu]{a^{\nu}} = a^{\frac{\nu}{\mu}}$$

(β) $\sqrt[4]{5^{2}} = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} = \sqrt{5}$

(γ) $\sqrt[3]{3\sqrt{3}} = \sqrt[3]{3 \cdot 3^{1/2}} = \sqrt[3]{3^{3/2}} = 3^{\frac{\frac{3}{2}}{3}}$

$$= 3^{\frac{1}{2}} = \sqrt{3}.$$

(δ) $\dfrac{\sqrt{\sqrt[3]{4}} \cdot \sqrt{2\sqrt{2}}}{\sqrt[12]{2}} = \dfrac{\sqrt{4^{\frac{1}{3}}} \cdot \sqrt{2^{\frac{3}{2}}}}{2^{\frac{1}{12}}} = \dfrac{4^{\frac{\frac{1}{3}}{2}} \cdot 2^{\frac{\frac{3}{2}}{4}}}{2^{\frac{1}{12}}} =$

$$= \frac{4^{\frac{1}{6}} \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{12}}} = \frac{4^{\frac{1}{6}} \cdot \sqrt{2}}{2^{\frac{1}{12}}}$$

24. (01) $\sqrt[3]{2}\,\sqrt[3]{4} + \dfrac{\sqrt[5]{64}}{\sqrt[5]{2}} + \sqrt[3]{\dfrac{125}{27}}$,

~~$A = 3\frac{1}{2}$~~

$A = \sqrt[3]{8} + \sqrt[5]{\dfrac{64}{2}} + \dfrac{5}{3}$

$A = 2 + 2 + \dfrac{5}{3} = 4 + \dfrac{5}{3}$

43. (a) $(\sqrt{2}-1)^3 = (\sqrt{2})^3 - 3\cdot(\sqrt{2})^2\cdot 1 + 3\cdot\sqrt{2}\cdot 1^2 - 1^3$

$\sqrt{8} - 3\cdot 2\cdot 1 + 3\sqrt{2} - 1$

$\sqrt{8} - 6 + 3\sqrt{2} - 1$

$\sqrt{8} - 7 + 3\sqrt{2}$

$\underline{5\sqrt{2} - 7.}$

$(1+\sqrt{2})^3 = (1)^3 + 3\cdot 1^2\cdot\sqrt{2} + 3\cdot 1\cdot(\sqrt{2})^2 + (\sqrt{2})^3$

$1 + 3\sqrt{2} + 3\cdot 2 + \sqrt{8}$

$\cancel{7} + 3\sqrt{2} + \sqrt{8}$

$6 + 3\sqrt{2} + 2\sqrt{2} = \cancel{6}\,7 + 5\sqrt{2}$

(b) $A = \sqrt[3]{5\sqrt{2}-7} - \sqrt[3]{7+5\sqrt{2}}$

$A = \sqrt[3]{(\sqrt{2}-1)^3} - \sqrt[3]{(1+\sqrt{2})^3}$

$A = |\sqrt{2}-1| - |1+\sqrt{2}|$

$A = \cancel{\sqrt{2}} - 1 - 1 - \cancel{\sqrt{2}}$

$\underline{\underline{A = -2}}$

29. (a) $\sqrt{2}\,\sqrt[3]{2}=\sqrt[6]{32}$

$$2^{1/2}\cdot 2^{1/3}=32^{1/6}$$

$$2^{\frac{1}{2}+\frac{1}{3}}=\left(2^{5}\right)^{\frac{1}{6}}$$

$$2^{\frac{5}{6}}=2^{\frac{5}{6}} \checkmark$$

(γ) $\sqrt{5}\,\sqrt[10]{5}\,\sqrt[5]{25}=5$.

$$5^{\frac{1}{2}}\cdot 5^{\frac{1}{10}}\cdot\left(5^{2}\right)^{\frac{1}{5}}=5$$

$$5^{\frac{5}{10}+\frac{1}{10}}\cdot 5^{\frac{2}{5}}=5$$

$$5^{\frac{6}{10}}\cdot 5^{\frac{4}{10}}=5$$

$$5^{\frac{6}{10}+\frac{4}{10}}=5$$

$$5^{\frac{10}{10}}=5$$

$$5^{1}=5$$

(ε) $\sqrt[3]{5} \cdot \sqrt{15} \cdot \sqrt[6]{135} = 15$.

$$5^{\frac{1}{3}} \cdot 3(\sqrt{5}) \cdot \cancel{135}\ 27(\sqrt{5}$$

$$5^{\frac{1}{3}} \cdot 15^{\frac{1}{2}} \cdot 135^{\frac{1}{6}} = 15$$

$$5^{\frac{1}{3}} \cdot 15^{\frac{1}{2}} \cdot 9^{\frac{1}{6}} \cdot 15^{\frac{1}{6}} = 15$$

$$5^{\frac{1}{3}}\ 15^{\frac{2}{3}} \cdot 9^{\frac{1}{6}} = 15$$

$$5^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}\ 5^{\frac{2}{3}}\ 3^{\frac{2}{6}} = 15$$

$$5^{1} \cdot 3^{1} = 15$$

$$15 = 15$$

44. B $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} + \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = 4$

$$\sqrt{\frac{(2-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}} + \sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}} = 4$$

$$\sqrt{\frac{(2-\sqrt{3})^2}{1}} + \sqrt{\frac{(2+\sqrt{3})^2}{1}} = 4$$

$$\overset{+}{|2-\sqrt{3}|} + \overset{+}{|2+\sqrt{3}|} = 4$$

$$2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}} = 4$$

$$4 = 4,$$

(01) $$\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}+\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}=2.$$

$$\sqrt{\frac{(\sqrt{5}+1)(\sqrt{5}+1)}{(\sqrt{5}+1)\cdot(\sqrt{5}-1)}}+\sqrt{\frac{(3-\sqrt{5})\cdot(3-\sqrt{5})}{(3+\sqrt{5})\cdot(3-\sqrt{5})}}=2$$

$$\sqrt{\frac{(\sqrt{5}+1)^2}{4}}+\sqrt{\frac{(3-\sqrt{5})^2}{4}}=2$$

$$\frac{|\sqrt{5}+1|}{2}+\frac{|3-\sqrt{5}|}{2}=2$$

(+)

$$\cancel{\frac{\sqrt{5}+1}{2}}$$

$$\cancel{-\sqrt{5}-1+3+\sqrt{5}=2}$$

$$\frac{\cancel{\sqrt{5}}+1+3-\cancel{\sqrt{5}}}{2}=2$$

$$2=2$$ ✓

5. (B) $(x-2)(x-3)=(x-1)^2$

$x^2-3x-2x+6=x^2-2x+1$

$\cancel{x^2}-3x-\cancel{2x}\cancel{-x^2}\cancel{+2x}=-6+1$

$\frac{-3x}{-3}=\frac{-5}{-3}$ $\boxed{x=\frac{5}{3}}$ ✓

(δ) $1-(2x-1)^2=x-2x(2x-3)$

$1-(4x^2-2x+1)=x-4x^2-6x$

$1-4x^2+2x-1=x-4x^2-6x$

$2x^2+2x-x+4x^2+6x=1-1$

$6x^2+7x=0$

(στ) $x(x-3)^2+8=(x-2)^3$

$x(x^2-2x\cdot3+3^2)+8=x^3-3\cdot x^2\cdot2+3x2^2$

$x(x^2-6x+9)+8=x^3-6x^2+12x$

$x^3-6x^2+9x+8=x^3-6x^2+12x$

$\cancel{x^3}\cancel{-x^3}\cancel{-6x^2}\cancel{+6x^2}+9x-18x=8$

$\frac{-3x}{-3}=\frac{8}{-3}$ $x=\frac{8}{-3}$

1. β) $x - 3 = 3x - 1$,

$x - 3x = 3 - 1$

$\frac{-2x}{-2} = \frac{2}{-2}$

$\boxed{x = -1}$

δ) $x - (3x - 1) = 1 - 2(3x - 2)$.

$x - 3x + 1 = 1 - 6x + 4$

$x - 3x + 6x = 1 - 1 + 4$

$\frac{4x}{4} = \frac{4}{4}$

$\boxed{x = 1}$

3. (B) $\frac{5x}{2} - \frac{x}{6} = x - 1$

$$^{3}\cancel{6}\frac{5x}{\cancel{2}} - \cancel{6}\frac{x}{\cancel{6}} = 6x - 6$$

$$15x - x = 6x - 6$$

$$15x - x - 6x = -6$$

$$8x = -6$$

$$x = -\frac{6}{8}$$

(δ) $\frac{3x-1}{2} - \frac{x-1}{6} = -x + 1$.

$$3(3x-1) - (x-1) = -6x + 6 \Rightarrow$$

$$9x - 3 - x + 1 = -6x + 6 \Rightarrow$$

$$9x - 3 - x + 1 + 6x = 6 \Leftrightarrow$$

$$9x - x + 6x = 6 + 3 - 1 \Leftrightarrow$$

$$14x = 8 \Leftrightarrow$$

$$x = \frac{8}{14} \Rightarrow x = \frac{4}{7}$$

6. β) $3x^2 = 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ ή $x - 2 = 0$

$x = 0$ $\qquad$ $x = 2$

δ) $x^4 = x^3$

$(x^2)^2 = x^3$

$(x^2)^2 - x^3 = 0$

$x^3(x-1) = 0$

$x^3 = 0$ ή $x - 1 = 0$

$x = 0$ $\qquad$ $x = 1$

ζ) $x^3 - 2x^2 + x = 0$

$\cancel{x^3 - (x \cdot x)^2 + x = 0}$

$x(x^2 - 2x + 1) = 0$

$x(x-1)^2 = 0$

$x = 0$ $\qquad$ $x = 1$

3. σζ

$$\frac{2(1-3x)}{5} - \frac{3}{2}(x-1) = -x+2$$

$$4(1-3x) - 15(x-1) = -10x+20$$

$$4-12x-15x+15 = 10x+20$$

$$-12x-10x = -4+15-15+20$$

$$\frac{-22x}{22} = \frac{16}{22}$$

$$x = \frac{8}{11}$$

ν

$$\frac{3x-2}{5} = 2$$

$$3x-2 = 10$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = \frac{12}{3}$$

2. В) $1-(2x-1)=x-2(x-3)$

$1-(2x-1)=x-2x+6$

$\cancel{1}-\cancel{2x}-\cancel{1}-x+\cancel{2x}-6=0$

$-x-6=0$

$-x=6$

$x=-6$

б) $3(x-1)=1-(x+4)$

$3x-3=1-x-4$

$3x-3-1+x-4=0$

$4x-8=0$

$4x=8$

$4x=8$

$x=2$

Επόμενο Μάθημα

Σάββατο 12-1.

1. Σχολικό Βιβλίο

→ 2 αποδείξεις

→ Ορισμοί απόλυτης τιμής.

2. Ασκήσεις μπάρλα

Σελ 111

5, 15, 28 α,β. 48

7. 22 35, 50

8. 27 51.

38

10 25 40

11 26 41.

12. 47

1. Αν $-4<x<-1$ να απλοποιηθεί

$$A = |3x+3| - |2x+8| - |5x|.$$

Λύση

- $-4<x<-1 \Rightarrow -12<3x<-3 \Rightarrow -9<3x+3<0$
- $-4<x<-1 \Rightarrow -8<2x<-2 \Rightarrow 0<2x+8<6$
- $-4<x<-1 \Rightarrow -20<5x<-5$

$$A = |\overset{\ominus}{3x+3}| - |\overset{\oplus}{2x+8}| - |\overset{\ominus}{5x}| = -3x-3-(2x+8)-(-5x)$$

$$A = -\cancel{3x}-3-\cancel{2x}-8+\cancel{5x} = -11.$$

2. Να γραφεί χωρίς απόλυτη τιμή η παράσταση $A = x+1-|2x-6|$.

Λύση

1η. Αν $2x-6\geq 0 \Rightarrow 2x\geq 6 \Rightarrow \underline{x\geq 3}$ $\quad A = x+1-(2x-6)$

$$\boxed{A = -x+7}$$

2η. Αν $2x-6\leq 0 \Rightarrow x<3$

Τότε $A = x+1-(6-2x) = 3x-5$

$$\boxed{A = 3x-5}$$

3. Να απλοποιηθεί η παράσταση.

$$A = \frac{|x|^3 - 3x^2}{2|x| - 6}$$

αφού πρώτα βρείτε τα x ώστε να ορίζεται η A.

Λύση

Για να ορίζεται η A πρέπει

$$2|x| - 6 \neq 0$$

$\longrightarrow$ $2|x| - 6 = 0$

$2|x| = 6 \Rightarrow |x| = 3$

$\underline{\underline{x = 3}}$ ή $\underline{\underline{x = -3}}$

$$A = \frac{|x|^3 - 3x^2}{2|x| - 6} = \frac{|x|^3 - 3|x|^2}{2|x| - 6} = \frac{|x|^2 \cancel{(|x| - 3)}}{2\cancel{(|x| - 3)}}$$

$$= \frac{x^2}{2}.$$

4. (α) Ν.δ.ο. $|\alpha+\beta|^2 + |\alpha-\beta|^2 = 2(|\alpha|^2+|\beta|^2)$

(β) Ν.δ.ο. $|4\alpha\beta| \leq (|\alpha|+|\beta|)^2 \leq 2(\alpha^2+\beta^2)$.

<u>Λυση.</u>

(α) $|\alpha+\beta|^2 + |\alpha-\beta|^2 = 2(|\alpha|^2+|\beta|^2)$.

$(\alpha+\beta)^2 + (\alpha-\beta)^2 = 2\alpha^2+2\beta^2$

$\cancel{\alpha^2}+\cancel{2\alpha\beta}+\cancel{\beta^2}+\cancel{\alpha^2}-\cancel{2\alpha\beta}+\cancel{\beta^2} = \cancel{2\alpha^2}+\cancel{2\beta^2}$

$0=0$

(β) $|4\alpha\beta| \leq (|\alpha|+|\beta|)^2$ και $(|\alpha|+|\beta|)^2 \leq 2(\alpha^2+\beta^2)$

$4|\alpha||\beta| \leq |\alpha|^2+2|\alpha||\beta|+|\beta|^2$ $\qquad$ $|\alpha|^2+2|\alpha||\beta|+|\beta|^2 \leq 2\alpha^2+2\beta^2$

$0 \leq |\alpha|^2-2|\alpha||\beta|+|\beta|^2$ $\qquad$ $0 \leq |\alpha|^2-2|\alpha||\beta|+|\beta|^2$

$0 \leq (|\alpha|-|\beta|)^2$ $\qquad$ $0 \leq (|\alpha|-|\beta|)^2$

5. (α) Έστω, $|\alpha+\beta| < |\alpha-\beta|$. Νδο α, β ετερόσημοι

και στη συνέχεια νδο $\alpha|\beta| + \beta|\alpha| = 0$.
$(\alpha, \beta \neq 0)$

(β) Αν $\left|\dfrac{\alpha+2\beta}{2\alpha+\beta}\right| \leq 1$ νδο $|\alpha| \geq |\beta|$.

(γ) Αν $|x|^3 - 1 \geq x^2 - |x|$ νδο $x \leq -1$ ή $x \geq 1$.

(δ) Αν $|\alpha+2| = \big||\alpha|-2\big|$ νδο $\alpha \leq 0$.

(ε) Αν $|3\alpha-2\beta| = |3\alpha| + |2\beta|$ $\alpha, \beta \neq 0$

νδο α, β ετερόσημοι.

(ζ) νδο $|2\alpha-3\beta| \leq |\alpha-5\beta| + |\alpha+2\beta|$.

(η) Αν $d(2\alpha, 9\beta) = d(\alpha, 0)$.

νδο $\alpha = 9\beta$ ή $\alpha = 3\beta$.

5. α) Έστω $|\alpha+\beta| < |\alpha-\beta|$.

$$|\alpha+\beta|^2 < |\alpha-\beta|^2$$

$$(\alpha+\beta)^2 < (\alpha-\beta)^2$$

$$\cancel{\alpha^2}+2\alpha\beta+\cancel{\beta^2} < \cancel{\alpha^2}-2\alpha\beta+\cancel{\beta^2}$$

$$4\alpha\beta < 0$$

$$\alpha\beta < 0 \Rightarrow \alpha, \beta \text{ ετερόσημοι!}$$

Νδο $\alpha|\beta| + \beta|\alpha| = 0$

$$\alpha|\beta| = -\beta|\alpha|$$

$$\frac{\alpha}{\beta} = -\frac{|\alpha|}{|\beta|}$$

$$-\frac{\alpha}{\beta} = \left|\frac{\alpha}{\beta}\right| \Rightarrow \cancel{-\frac{\alpha}{\beta} = -\frac{\alpha}{\beta}}$$

$$0 = 0 \checkmark$$

(8). $\left|\dfrac{a+2B}{2a+B}\right| \leq 1$ νδο $|a| \geq |B|$.

$$\frac{|a+2B|}{|2a+B|} \leq 1 \Rightarrow |a+2B| \leq |2a+B|$$

$$|a+2B|^2 \leq |2a+B|^2$$

$$a^2 + \cancel{4aB} + 4B^2 \leq 4a^2 + \cancel{4aB} + B^2$$

$$\cancel{3}B^2 \leq \cancel{3}a^2$$

$$|B| \leq |a|$$

(9) Αν $|x|^3 - 1 \geq x^2 - |x|$ νδο $x \leq -1$ ή $x \geq 1$.

$$|x|^3 - 1 - x^2 + |x| \geq 0.$$

$$\underbrace{|x|^3 - 1 - |x|^2}+|x| \geq 0$$

$$|x|^2(|x|-1) + |x|-1 \geq 0$$

$$(|x|-1)(\underset{+}{|x|^2+1}) \geq 0$$

$$\Rightarrow |x|-1 \geq 0 \Rightarrow |x| \geq 1$$

$\checkmark$ $x \geq 1$ ή $x \leq -1$

(5) Αν $|a+2| = ||a|-2|$ ν.δ.ο. $a \le 0$

$a+2 = |a|-2$ ή $a+\cancel{2} = -|a|+\cancel{2}$

$4 = |a| - a$

$\overset{\ominus}{|a|} = -a.$

✓ $a \le 0.$ ✓

Έστω $a>0$.

$4 = a - a$

$4 = 0$

Άτοπο! $\Rightarrow \underline{\underline{a \le 0}}$ ✓

(6). Αν $|3a-2\beta| = |3a| + |2\beta|$

$|3a-2\beta|^2 = (|3a|+|2\beta|)^2$

$\cancel{9a^2} - 12a\beta + \cancel{4\beta^2} = \cancel{9a^2} + 12|a||\beta| + \cancel{4\beta^2}$

$-12a\beta = 12|a||\beta|$

$-a\beta = |a\beta|$. a, β ετερόσημοι

$a\beta < 0$

(5) νδο $|2a-3\beta| \leq |a-5\beta| + |a+2\beta|$

$$|2a-3\beta| = |(a-5\beta)+(a+2\beta)| \leq |a-5\beta| + |a+2\beta|$$

$$|2a-3\beta| \leq |a-5\beta| + |a+2\beta|.$$

(4). Αν $d(2a, 9\beta) = d(a, 0)$, νδο $a = 9\beta$

ή $a = 3\beta$

$$|2a-9\beta| = |a-0|$$

$$|2a-9\beta| = |a|$$

$2a-9\beta = a$ ή $2a-9\beta = -a$

$a = 9\beta$

$3a = 9\beta$

$a = 3\beta$