Τετραγωνική ρίζα

1. Έστω η εξίσωση $x^2 = \alpha$

Προφανώς $\alpha \geq 0$ και $x \in \mathbb{R}$.

$\Leftrightarrow x = \sqrt{\alpha}$ ή $x = -\sqrt{\alpha}$

2. Βασικές ρίζες

$\sqrt{0} = 0$

$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

$\sqrt{100} = 10$

$\sqrt{121} = 11$

$\sqrt{144} = 12$

$\sqrt{169} = 13$

$\sqrt{196} = 14$

$\sqrt{225} = 15$

$\sqrt{256} = 16$

$\sqrt{289} = 17$

$\sqrt{324} = 18$

$\sqrt{361} = 19$

$\sqrt{400} = 20$

3. $\sqrt{x}^2 = x$ при $x \geq 0$

4. $\sqrt{x^2} = |x|$, $x \in \mathbb{R}$.

5. $\sqrt{x}$, $x \geq 0$

6. $\sqrt{x} \geq 0$

7. $\sqrt{x \cdot y} = \sqrt{x} \cdot \sqrt{y}$

8. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

9. $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.

Пробоху

$\sqrt{-5^2} = |-5| = 5$ Лабо

$\sqrt{(-5)^2} = |-5| = 5$ Зшсо

Σελ 136

2. (δ) $\sqrt{8\sqrt{1+\sqrt{2+\sqrt{49}}}} =$

$= \sqrt{8\sqrt{1+\sqrt{2+7}}} =$

$= \sqrt{8\sqrt{1+\sqrt{9}}} =$

$= \sqrt{8\sqrt{1+3}} =$

$= \sqrt{8\cdot\sqrt{4}} = \sqrt{8\cdot 2} = \sqrt{16} = 4.$

3. (β) $\sqrt{(\sqrt{2}-1)^2} + \sqrt{(\sqrt{2}-2)^2}$

$= |\sqrt{2}-1| + |\sqrt{2}-2| = \sqrt{2}-1-\sqrt{2}+2$

- $\sqrt{2} > 1 \Rightarrow \sqrt{2}-1 > 0$
- $\sqrt{2} < 2 \Rightarrow \sqrt{2}-2 < 0$

$= 1$

6. Αν $x = 2\sqrt{3} - 1$ και $y = 2\sqrt{3} + 1$

Βρες $x^2 - xy = (2\sqrt{3} - 1)^2 - \left((2\sqrt{3} - 1)(2\sqrt{3} + 1)\right) =$

$= 4\sqrt{3}^2 + 1 - 4\sqrt{3} - \left(4\sqrt{3}^2 + 2\sqrt{3} - 2\sqrt{3} - 1\right) =$

$= 12 + 1 - 4\sqrt{3} - 12 + 1 = 2 - 4\sqrt{3}$

11. (Β) $B = \dfrac{\sqrt{2^{20} + 4^{12}}}{\sqrt{4^{11} + 8^6}} \Rightarrow$

$\Rightarrow B = \dfrac{\sqrt{2^{20} + 2^{24}}}{\sqrt{2^{22} + 2^{48}}} \Rightarrow$ [illegible]

[illegible]

$\Rightarrow B = \dfrac{\sqrt{2^{20}(1 + 2^4)}}{\sqrt{2^{18}(1 + 2^4)}} = \dfrac{\sqrt{2^{20}}\sqrt{1 + 2^4}}{\sqrt{2^{18}}\sqrt{1 + 2^4}}$

$= \sqrt{\dfrac{2^{20}}{2^{18}}} = \sqrt{2^2} = 2$

(г) $\Gamma = \sqrt{\frac{x^2}{25}} + \sqrt{x^2-2x+1} =$

$= \sqrt{\left(\frac{x}{5}\right)^2} + \sqrt{(x-1)^2} =$

$= \left|\frac{x}{5}\right| + |x-1| = \frac{|x|}{|5|} + |x-1|$

$= \frac{|x|}{5} + |x-1|.$

4. (г) $\left(\sqrt{3}+1\right)^3 = (\sqrt{3})^3 + 3\cdot(\sqrt{3})^2\cdot 1 + 3\cdot\sqrt{3}\cdot 1^2 + 1^3 =$

$= (\sqrt{3})^3 + 9 + 3\sqrt{3} + 1 =$

$= (\sqrt{3})^3 + 3\sqrt{3} + 10 =$

$= 3\sqrt{3} + 3\sqrt{3} + 10 =$

$= \underline{6\sqrt{3}+10}$

12. ⑧ Να δ. $\sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2-\sqrt{2+\sqrt{3}}} = 1$.

$$\sqrt{(2+\sqrt{3})\left(2+\sqrt{2+\sqrt{3}}\right)\left(2-\sqrt{2+\sqrt{3}}\right)} = 1$$

$$\sqrt{(2+\sqrt{3})(4-2-\sqrt{3})} = 1$$

$$\sqrt{(2+\sqrt{3})(2-\sqrt{3})} = 1$$

$$\sqrt{4-3} = 1$$

$$\sqrt{1} = 1$$

$$1 = 1$$

5. (B) Νδο $A^2+B^3 = 33+11\sqrt{3}$

Αν $A = 2-\sqrt{3}$

$B = 2+\sqrt{3}$.

$(2-\sqrt{3})^2 + (2+\sqrt{3})^3 = 4 - 4\sqrt{3} + 3 + 8 + 12\sqrt{3} + 6(\sqrt{3})^2 + (\sqrt{3})^3$

$= 4 - 4\sqrt{3} + 3 + 8 + 12\sqrt{3} + 18 + \sqrt{3}\cdot 3 =$

$= 15 - 4\sqrt{3} + 12\sqrt{3} + 18 + 3\sqrt{3}$

$= 11\sqrt{3} + 33$ ✓

8. (α) $\sqrt{2}\sqrt{8} = \sqrt{16} = 4$

(β) $\sqrt{49 \cdot 81} = \sqrt{49}\cdot\sqrt{81} = 7 \cdot 9 = 54$

(γ) $\frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$

(δ) $\frac{\sqrt{6}\sqrt{18}}{\sqrt{12}} = \frac{\sqrt{6}\sqrt{2}\sqrt{9}}{\sqrt{2}\sqrt{6}} = 3$

(ε) $\frac{\sqrt{18}\cdot\sqrt{48}}{\sqrt{24}} = \frac{\sqrt{3}\sqrt{6}\sqrt{2}\cdot\sqrt{24}}{\sqrt{4}\sqrt{6}} = \frac{\sqrt{6}\sqrt{24}}{2} = \frac{\sqrt{144}}{2}$

$\frac{12}{2} = 6$

(στ) $\frac{\sqrt{8}\sqrt{75}}{\sqrt{150}} = \frac{\sqrt{2}\cdot\sqrt{4}\sqrt{75}}{\sqrt{150}} = \frac{\sqrt{150}^2}{\sqrt{150}} = 2.$

10. $A = \dfrac{\sqrt{20} - 2\sqrt{8} + 3\sqrt{12}}{\sqrt{45} - 2\sqrt{18} + 3\sqrt{27}} =$

$= \dfrac{\sqrt{4}\cdot\sqrt{5} - 2\sqrt{2}\cdot\sqrt{4} + 3\sqrt{3}\cdot\sqrt{4}}{\sqrt{9}\cdot\sqrt{5} - 2\sqrt{2}\cdot\sqrt{9} + 3\sqrt{3}\cdot\sqrt{9}} =$

$= \dfrac{2\sqrt{5} - 2\cdot 2\sqrt{2} + 3\cdot 2\sqrt{3}}{3\sqrt{5} - 2\cdot 3\sqrt{2} + 3\cdot 3\sqrt{3}} =$

$= \dfrac{2\sqrt{5} - 4\sqrt{2} + 6\sqrt{3}}{3\sqrt{5} - 6\sqrt{2} + 9\sqrt{3}} =$

$= \dfrac{2(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})}{3(\sqrt{5} - 2\sqrt{2} + 3\sqrt{3})} =$

$= \dfrac{2}{3}$

Εποχιακά Μαθήματα

Σελ 136

(1) ΟΛΗ

(2) α β γ

(3) α

(4) α β

(5) α

(7)

(12) α β γ.

48. $d(x,2) < 3$

(a) Ndo $-1 < x < 5$

$|x-2| < 3$

$-3 < x-2 < 3$

$-1 < x < 5$ ✓

(b) i) $\dfrac{\overset{(+)}{|x+1|} + \overset{(-)}{|x-5|}}{3} = \dfrac{\cancel{x+1} - \cancel{x+5}}{3} = 2$

- $-1 < x < 5 \Rightarrow 0 < x+1 < 6$
- $-1 < x < 5 \Rightarrow -6 < x-5 < 0$

ii). $B = \left|\dfrac{x+2}{x-6}\right| = \dfrac{\overset{(+)}{|x+2|}}{\overset{(-)}{|x-6|}} = \dfrac{x+2}{6-x}$.

- $-1 < x < 5 \Rightarrow 1 < x+2 < 7$
- $-1 < x < 5 \Rightarrow -7 < x-6 < -1$

iii) $\Gamma = |x^2 - 4x - 5| = \overset{(-)}{|x-5|}\,\overset{(+)}{|x+1|} = (-x+5)(x+1)$.

(13) Αν $|z-2| \leq 3$ και $|w+3| \leq 2$ τότε $|z-w+1| \leq 11$

Απόδειξη

$-3 \leq z-2 \leq 3$

$-1 \leq z \leq 5$

$5 \geq z \geq -1$

$-2 \leq w+3 \leq 2$

$-5 \leq w \leq -1$

$5 \geq -w \geq 1$

$10 \geq z-w \geq 0$

$11 \geq z-w+1 \geq 1$.

$11 \geq z-w+1 \geq -11$

$|z-w+1| \leq 11$

В'тропо ∫

$$|z-w+1| = |(z-2)+(-3-w)+6| \le |z-2|+|3+w|+6$$

$$|z-w+1| \le |z-2|+|w+3| \le 8,$$

- $|z-2| \le 3$
- $|w+3| \le 2$

$$\Big\} |z-2|+|w+3| \le$$

$$|z-2|+|w+3|+6 \le 11.$$

43. ⓐ $|z-1| \le 3$ and $|w-3| \le 1$ } vs $|z+w-2| \le 6$.

$-3 \le z-1 < 3$

$-2 \le z \le 4$

$-1 \le w-3 \le 1$

$2 \le w \le 4$

(+)

$0 \le z+w \le 8$

$-2 \le z+w-2 \le 6$

$-6 \le z+w-2 \le 6$

$|z+w-2| \le 6$ ✓

Β' τρόπος

$$|z+w-2| = |z+1+w-3| \le |z+1|+|w-3|$$

$$|z+w-2| = |(z-1)+(w-3)+2| \le |z-1|+|w-3|+|2|$$

$$\boxed{|z+w-2| \le |z-1|+|w-3|+2 \le 6}$$

- $|z-1| \le 3$
- $|w-3| \le 1$

$\oplus$ $|z-1|+|w-3| \le 4$

$$|z-1|+|w-3|+2 \le 6.$$

37. Αν $|5x-y+1|<4$

$|2x+y-1|<3$.

α) $-4<5x-y+1<4$ $\qquad -3<2x+y-1<3$

(+)

$-7<7x<7$

$-1<x<1$

$|x|<1$ ✓

νδο $|x|<1$.

νδο $-\frac{16}{7}<y<\frac{30}{7}$

$8>-10x+2y-2>-8$ $\qquad -15<10x+5y-5<15$

$-8<-10x+2y-2<8$ (+)

$-23<7y-7<23$

$-15<7y<30 \Rightarrow -\frac{16}{7}<y<\frac{30}{7}$

③ $A = |x+1| + |7y+16|$ (both terms positive)

- $-1 < x < 1 \Rightarrow 0 < |x| < 2$
- $-\frac{16}{7} < y < \frac{30}{7} \Rightarrow -16 < 7y < 30$

$0 < 7y+16 < 46$

$A = x+1+7y+16$

$A = x + 7y + 17$.

$$|\alpha+\gamma| \le |\alpha|+|\gamma|$$

$$|\alpha+\beta+\gamma| \le |\alpha|+|\beta|+|\gamma|$$

ooo

} Ισχύει ✓

20. Εξ 113

Αν $|\alpha| = 2 \quad |\beta| = 3 \quad |\gamma| = 4$

α) τότε $|\alpha+\beta+\gamma| \le 9$.

$$|\alpha+\beta+\gamma| \le |\alpha|+|\beta|+|\gamma|$$

$$|\alpha+\beta+\gamma| \le 2+3+4$$

$$|\alpha+\beta+\gamma| \le 9$$ ✓

(B) Vdo $|a-B+3\gamma| \leq 17$.

$$|a-B+3\gamma| = |a+(-B)+(3\gamma)| \leq |a|+|-B|+|3\gamma|$$

$$|a-B+3\gamma| \leq 2+|B|+3|\delta|$$

$$|a-B+3\gamma| \leq 2+3+12$$

$$|a-B+3\gamma| \leq 17$$ ✓

42. (A) Να δείξετε ότι $(2-\sqrt{3})^2(7+4\sqrt{3})=1$.

$(4-4\sqrt{3}+(\sqrt{3})^2)\cdot(7+4\sqrt{3}) = (4-4\sqrt{3}+3)(7+4\sqrt{3}) =$

$= (7-4\sqrt{3})(7+4\sqrt{3}) = 7^2-(4\sqrt{3})^2 = 49-16\cdot 3$

$= 49-48 = 1$

(B) $(\sqrt{2}+1)^3(5\sqrt{2}-7)=1$.

$(2\sqrt{2}+3(\sqrt{2})^2+3\sqrt{2}+1)(5\sqrt{2}-7) =$

$= (5\sqrt{2}+6+1)(5\sqrt{2}-7) = (5\sqrt{2}+7)(5\sqrt{2}-7) =$

$= -7^2+(5\sqrt{2})^2 = -49+25\cdot 2 = -49+50 = 1$

21. (B) $\sqrt[5]{241+\sqrt[4]{4+3\sqrt[3]{64}}} =$

$= \sqrt[5]{24\angle+\sqrt[4]{4+8\angle 2}} = \sqrt[5]{24\angle+\sqrt[4]{\angle 6}} =$

$= \sqrt[5]{24\angle+2} = \sqrt[5]{243} = 3$

$|\overset{-}{xy}||x\overset{-}{-y}| = (-xy)(-x+y)$

$xy(x-y)$

23. $x<0<y$

(B) $\sqrt{x^4y^2-2x^3y^3+x^2y^4} = x^2y-xy^2$.

$\sqrt{(x^2y-xy^2)^2} = |x^2y-xy^2| =$

- $x<0 \Rightarrow x^2>0 \Rightarrow 0<x^2$ $\Big\}\odot \Rightarrow 0<x^2y$
- $0<y$
- $0<y \Rightarrow 0<y^2$ $\Big\}\odot \Rightarrow 0<y^2x$
- $x<0 \Rightarrow -x>0 \Rightarrow 0<-x$

25. (B) Vsó $\sqrt[3]{49}\ \sqrt[3]{5+3\sqrt{2}}\ \sqrt[3]{5-3\sqrt{2}} = 7$.

$$\sqrt[3]{49(5+3\sqrt{2})(5-3\sqrt{2})} = 7$$

$$\sqrt[3]{49(25-18)} = 7$$

$$\sqrt[3]{49 \cdot 7} = 7$$

$$\sqrt[3]{(7)^3} = 7$$

$$7 = 7$$

11. ⓐ $A = \sqrt{9^{13} + 27^9}$

$$A = \sqrt{(3^2)^{13} + (3^3)^9}$$

$$A = \sqrt{3^{26} + 3^{27}} = \sqrt{3^{26}(1+3)}$$

$$= \sqrt{4 \cdot 3^{26}} = \sqrt{4}\sqrt{3^{26}}$$

$$= 2 \cdot \sqrt{(3^{13})^2}$$

$$= 2 \cdot 3^{13}$$

24. ⓐ $\sqrt[3]{2}\sqrt[3]{4} + \frac{\sqrt[5]{64}}{\sqrt[5]{2}} + \sqrt[3]{\frac{125}{27}} =$

$$= \sqrt[3]{8} + \sqrt[5]{\frac{64}{2}} + \sqrt[3]{\frac{125}{27}} = 2 + \sqrt[5]{32} + \frac{5}{3} =$$

$$= 2 + 2 + \frac{5}{3} = 4 + \frac{5}{3} = \frac{12}{3} + \frac{5}{3} = \frac{17}{3}$$

ⓑ $\sqrt[3]{5}\sqrt[3]{25} + \frac{\sqrt[3]{108}}{\sqrt[3]{4}} + \sqrt[3]{27 \cdot 64} \cdot =$

$$= \sqrt[3]{125} + \sqrt[3]{\frac{108}{4}} + \sqrt[3]{27} \cdot \sqrt[3]{64} =$$

$$= 5 + \sqrt[3]{27} + 3 \cdot \sqrt[3]{64} = 5 + 3 + 3 \cdot 4 = 15 + 12 = 27.$$

Σελ 141

$\sqrt{2}^3 = \sqrt{2}^2\sqrt{2} = 2\sqrt{2}$

43. ⓐ $(\sqrt{2}-1)^3 = 2\sqrt{2} - 3\cdot(\sqrt{2})^2 + 3\cdot\sqrt{2} - 1 =$

$= 2\sqrt{2} - 6 + 3\sqrt{2} - 1 = \underline{5\sqrt{2} - 7}.$

$(1+\sqrt{2})^3 = 1 + 3\sqrt{2} + 3\cdot(\sqrt{2})^2 + \sqrt{2}^3 =$

$= 1 + 3\sqrt{2} + 6 + 2\sqrt{2} = \underline{5\sqrt{2} + 7}$

ⓑ $A = \sqrt[3]{(5\sqrt{2}-7)} - \sqrt[3]{7+5\sqrt{2}} . =$

$= \sqrt[3]{(\sqrt{2}-1)^3} - \sqrt[3]{(1+\sqrt{2})^3}$

$= |\sqrt{2}-1| - |1+\sqrt{2}|$

$= \sqrt{2} - 1 - 1 - \sqrt{2} = -2$

25. (8) NJO $\sqrt[3]{2+\sqrt{3}} \cdot \sqrt[3]{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt[3]{2-\sqrt{2+\sqrt{3}}} = 1$

$\rightarrow \sqrt[3]{2+\sqrt{3}} \cdot \left(\sqrt[3]{(2+\sqrt{2+\sqrt{3}})(2-\sqrt{2+\sqrt{3}})}\right) =$

$= \sqrt[3]{2+\sqrt{3}} \cdot \left(\sqrt[3]{4-(\sqrt{2+\sqrt{3}})^2}\right) =$

$= \sqrt[3]{2+\sqrt{3}} \cdot \sqrt[3]{4-(2+\sqrt{3})}$

$= \sqrt[3]{2+\sqrt{3}} \sqrt[3]{2-\sqrt{3}}$

$= \sqrt[3]{(2+\sqrt{3})(2-\sqrt{3})}$

$= \sqrt[3]{4-3}$

$= \sqrt[3]{1} = 1$

Εποχαο Μαθημα

Σελ 114

28 α γ

32 .

33

34

35

36 .

44

46 .

47

49

52 .

A ✓

46. $\left|\frac{3a+1}{a+3}\right| < 1$ να $|a| < L$.

$$\frac{|3a+1|}{|a+3|} < 1$$

$$|3a+1| < |a+3|$$

$$|3a+1|^2 < |a+3|^2$$

$$(3a+1)^2 < (a+3)^2$$

$$9a^2+6a+1 < a^2+6a+9$$

$$8a^2 < 8$$

$$a^2 < 1$$

$$a^2 < 1^2$$

$$|a| < |1|$$

$$|a| < 1$$ ✓

49. $A = \dfrac{x^2 - 9}{|x| - 3}$

(a) Pour $|x| - 3 \neq 0$

$\longrightarrow |x| - 3 = 0 \Rightarrow |x| = 3$

$x = 3$ ou $x = -3$

// A définie sur $x \neq 3$, $x \neq -3$

(b) $A = \dfrac{|x|^2 - 9}{|x| - 3} = \dfrac{(|x| - 3)(|x| + 3)}{|x| - 3}$

$A = |x| + 3$.

(c) $A < 7 \Rightarrow |x| + 3 < 7$

$|x| < 4$

$-4 < x < 4$

$\Rightarrow x \in (-4; -3) \cup (-3, 3) \cup (3, 4)$.

47. $d(a,0) < 1$.

(01) Nso $|2-|a-1|| = a+1$

$|a-0| < 1 \Rightarrow |a| < 1 \Rightarrow a \in (-1,1)$

$-1 < a < 1$

$|2-\overset{(-)}{|a-1|}| = |2-(-a+1)| = |2+a-1|$

$= |\overset{(+)}{1+a}|$

• $-1 < a < 1 \Rightarrow -2 < a-1 < 0$

• $-1 < a < 1 \Rightarrow 0 < a+1 < 2$

$= a+1$

(B) i) $|\overset{(-)}{2a-2}| - |\overset{(+)}{3a+4}| = -2a+2-3a-4 = -5a-2$

• $-1 < a < 1 \Rightarrow -2 < 2a < 2 \Rightarrow -4 < 2a-2 < 0$

• $-1 < a < 1 \Rightarrow -3 < 3a < 3 \Rightarrow 1 < 3a < 7$.

ii) $|\overset{(-)}{a^2-1}| = 1-a^2$

• $-1 < a < 1 \Rightarrow |a| < 1 \Rightarrow |a|^2 < 1^2 \Rightarrow |a|^2 < 1$

$|a|^2 - 1 < 0$.

44. (A) $\left|\frac{1}{x}-2\right|<1$

$$-1<\frac{1}{x}-2<1$$

$$1<\frac{1}{x}<3$$

$$1>x>\frac{1}{3}$$

(B) $\left|\frac{2}{x}-3\right|<1$

$$-1<\frac{2}{x}-3<1$$

$$2<\frac{2}{x}<4$$

$$\frac{1}{2}>\frac{x}{2}>\frac{1}{4}$$

$$1>x>\frac{1}{2}.$$

42. Αν $-1 \le x \le 1$

να δείξω ότι $|x^3 - 3x^2 + 2x - 1| \le 7$.

- $|x| \le 1 \Rightarrow |x|^3 \le 1^3 \Rightarrow |x|^3 \le 1$
- $|x| \le 1 \Rightarrow |x|^2 \le 1 \Rightarrow 3|x|^2 \le 3$
- $|x| \le 1 \Rightarrow 2|x| \le 2$

(+)

$$|x|^3 + 3|x|^2 + 2|x| \le 6$$

$$|x|^3 + 3|x|^2 + 2|x| + 1 \le 7.$$

$$|x^3 - 3x^2 + 2x - 1| = |x^3 + (-3x^2) + (2x) + (-1)| \le$$

$$\le |x^3| + |-3x^2| + |2x| + |-1|$$

$$\Leftrightarrow |x^3 - 3x^2 + 2x - 1| \le |x|^3 + 3|x|^2 + 2|x| + 1 \le 7$$

36. $|x+y-3|<2 \Rightarrow -2<x+y-3<2$ ⊕

$|x-y-5|<2$. $\quad -2<x-y-5<2$

$$-4<2x-8<4$$

$$4<2x<12$$

$$2<x<6$$

$$x\in(2,6).$$

$$\begin{cases} -2<x+y-3<2 \Rightarrow 2>x+y-3>-2 \\ -2<x-y-5<2 \Rightarrow 2>-x+y+5>-2 \end{cases}$$ ⊕

$$4>2y+2>-4$$

$$2>2y>-6$$

$$1>y>-3$$

$$y\in(-3,1).$$

35. $|x-3|<3$

$|y-3|<1$

(a) Nós $0<x<6$ e $2<y<4$

$-3<x-3<3$

$0<x<6$

$-1<y-3<1$

$2<y<4$

(B) i) $A=\overset{(-)}{|x-6|}+\overset{(+)}{|y-2|}=-x+6+y-2=y-x+4.$

- $0<x<6 \Rightarrow -6<x-6<0$
- $2<y<4 \Rightarrow 0<y-2<2$

ii) $\left|\frac{x}{y-4}\right|=\frac{\overset{(+)}{|x|}}{\overset{(-)}{|y-4|}}=\frac{x}{-y+4}.$

- $0<x<6$
- $2<y<4 \Rightarrow -2<y-4<0$

iii). $|xy - 4x| = |x(y-4)| = \overset{(+)}{|x|}\,\overset{(-)}{|y-4|}$

$= x(-y+4)$.

iv). $\overset{(-)}{|x-y-4|} = -x+y+4$

- $0 < x < 6 \Rightarrow 6 > x > 0$
- $2 < y < 4 \Rightarrow -2 > -y > -4$

$\Big\{ (+)\ 4 > x-y > -4$

$0 > x-y-4 > -8$

v) $\left|\overset{(-)}{\frac{x}{y} - 3}\right| = -\frac{x}{y} + 3$.

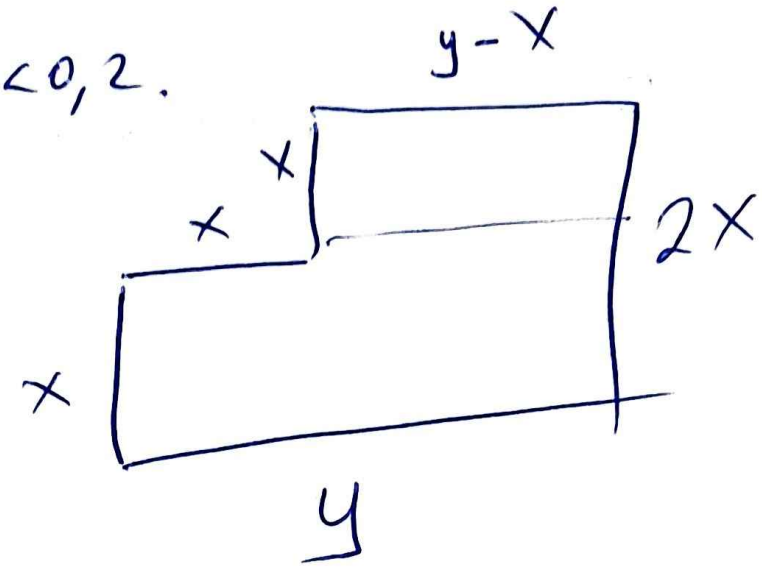
- $0 < x < 6 \Rightarrow 6 > x > 0$
- $2 < y < 4 \Rightarrow \frac{1}{2} > \frac{1}{y} > \frac{1}{4}$

$\Big\{ (-)\ 3 > \frac{x}{y} > 0$

$0 > \frac{x}{y} - 3 > -\frac{1}{4}$

34. • $|x-2|<0,1$

• $|y-4|<0,2$.



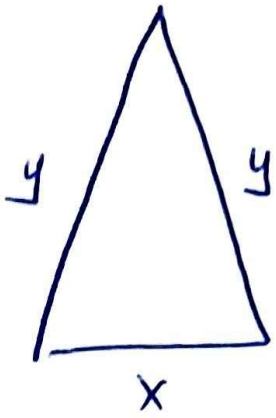
$$P = y + 2x + y - x + 3x$$

$$P = 2y + 4x$$

33.

$|x-1| < 0,2$

$|y-2| < 0,3$



$\prod = x+2y$

$-0,2 < x-1 < 0,2$

$0,8 < x < 1,2$

$-0,3 < y-2 < 0,3$

$1,7 < y < 2,3$

- $0,8 < x < 1,2$
- $1,7 < y < 2,3$

$\oplus$

$3,4 < 2y < 4,6$

$4,2 < x+2y < 5,8$

$4,2 < \prod < 5,8$

32. Αν $|2x-1|<1$

(α) τότε $0<x<1$

$|2x-1|<1 \Leftrightarrow -1<2x-1<1 \Leftrightarrow$

$\Leftrightarrow 0<2x<2 \Leftrightarrow$

$\Leftrightarrow \underline{0<x<1}$

(β) i). $\left|\frac{x}{x-1}\right| = \frac{\overset{(+)}{|x|}}{\underset{(-)}{|x-1|}} = \frac{x}{-x+1} = \frac{x}{1-x}$

• $0<x<1 \Rightarrow -1<x-1<0$

ii). $|x^2-x| = |x(x-1)| = \overset{(+)}{|x|}\,\overset{(-)}{|x-1|} = x(-x+1)$

$= -x^2+x$

0 1, x, x^2 $\quad \underline{0<x^2<x<1}$

Γνωρίζω ότι $0<x<1$

• $0<x<1 \Rightarrow 0<x^2<1$

$0<x^2<x$

Σελ 114

28. (α) $|x-1|-2x+3$

1. Αν $x-1 \geq 0 \Rightarrow x \geq 1$ τότε $|\overset{(+)}{x-1}|-2x+3 =$

$= x-1-2x+3 =$

$= 2-x$.

2. Αν $x-1 < 0 \Rightarrow x < 1$ τότε $|\overset{(-)}{x-1}|-2x+3 =$

$= -x+1-2x+3$

$= -3x+4$.

$$A = \begin{cases} 2-x, & x \geq 1 \\ 4-3x, & x < 1 . \end{cases}$$

(β) $\Gamma = \dfrac{|x+1|+|x-1|}{2}$.

x		-1		1	
$x+1$	$-$	0	$+$		$+$
$x-1$	$-$		$-$	0	$+$

1. Αν $x < -1$ τότε $\Gamma = \dfrac{|\overset{(-)}{x+1}|+|\overset{(-)}{x-1}|}{2} = \dfrac{-x-1-x+1}{2}$

$\Gamma = -x$.

2. Αν $-1 \le x \le 1$ τότε $\Gamma = \frac{|x+1| + |x-1|}{2}$ (+) (−)

$$\Gamma = \frac{x+1 - x + 1}{2} = 1$$

3. Αν $x > 1$ τότε $\Gamma = \frac{|x+1| + |x-1|}{2} = \frac{x+1+x-1}{2}$ (+) (+)

$$\Gamma = \frac{2x}{2} = x$$

$$\Gamma = \begin{cases} -x, & x < -1 \\ 1, & -1 \le x \le 1 \\ x, & x > 1. \end{cases}$$

13. (8) $\frac{3}{\sqrt{8}} = \frac{3\sqrt{8}}{\sqrt{8}\sqrt{8}} = \frac{3\sqrt{8}}{\sqrt{8}^2} = \frac{3\sqrt{8}}{8}$

14. (8) $\frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{(\sqrt{5}-2)(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$= \frac{(\sqrt{5}-2)^2}{\sqrt{5}^2-2^2} = \frac{(\sqrt{5}-2)^2}{1}$,

Сторож Майра

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(13) αB

(14) αBγ

(15) .

(16)

(17).

48. $d(x,2) < 3$

(a) Ndo $-1 < x < 5$.

- $|x-2| < 3 \Rightarrow -3 < x-2 < 3$

$-1 < x < 5$

(B) i) $A = \dfrac{\overset{(+)}{|x+1|} + \overset{(-)}{|x-5|}}{3} = \dfrac{x+1-x+5}{3} = 2.$

- $-1 < x < 5 \Rightarrow 0 < x+1 < 6$
- $-1 < x < 5 \Rightarrow -6 < x-5 < 0$

ii) $B = \left|\dfrac{x+2}{x-6}\right| = \dfrac{\overset{(+)}{|x+2|}}{\overset{(-)}{|x-6|}} = \dfrac{x+2}{6-x}.$

- $-1 < x < 5 \Rightarrow 1 < x+2 < 7$
- $-1 < x < 5 \Rightarrow -7 < x-6 < -1$

iii) $\Gamma = |x^2-4x-5| = |(x-5)(x+1)| = \overset{(-)}{|x-5|}\,\overset{(+)}{|x+1|}$

$(5-x)(x+1).$

46. Av $\left|\frac{3a+1}{a+3}\right| < 1$ vsa $|a| < 1$.

$$\frac{|3a+1|}{|a+3|} < 1 \Rightarrow |3a+1| < |a+3|.$$

$$\Leftrightarrow |3a+1|^2 < |a+3|^2 \quad \Leftrightarrow (3a+1)^2 < (a+3)^2$$

$$9a^2 + 6a + 1 < a^2 + 6a + 9$$

$$8a^2 < 8 \Rightarrow a^2 < 1$$

$$a^2 < 1^2$$

$$|a| < |1|$$

$$|a| < 1$$

44. (B) $\left|\frac{2}{x} - 3\right| < 1$

$$-1 < \frac{2}{x} - 3 < 1$$

$$2 < \frac{2}{x} < 4$$

$$1 < \frac{1}{x} < 2 \Rightarrow \frac{1}{1} > x > \frac{1}{2} \rightarrow$$

$$x \in \left(\frac{1}{2}, 1\right)$$

40. (B) NSO $|a| + \left|\frac{1}{a}\right| \geq 2$.

$$|a| + \frac{|1|}{|a|} \geq 2$$

$$|a| + \frac{1}{|a|} \geq 2$$

$$|a|^2 + 1 \geq 2|a|$$

$$|a|^2 - 2|a| + 1 \geq 0 \quad \Rightarrow (|a|-1)^2 \geq 0 \checkmark$$

91. (B) $\left|\frac{a}{a^2+9}\right| \leq \frac{1}{6}$.

$$\frac{|a|}{\oplus\ |a^2+9|} \leq \frac{1}{6} \quad \rightsquigarrow \frac{|a|}{a^2+9} \leq \frac{1}{6}$$

$$\Leftrightarrow 6|a| \leq a^2+9 \quad \Leftrightarrow 0 \leq a^2 - 6|a| + 9$$

$$0 \leq |a|^2 - 6|a| + 9$$

$$\checkmark \quad 0 \leq (|a|-3)^2 \quad \checkmark$$

51. $A = \dfrac{x^2 - 6|x| + 9}{x^2 - 3|x|}$

(а) Για να ορίζεται η A πρέπει $x^2 - 3|x| \neq 0$.

$\longrightarrow$ $x^2 - 3|x| = 0 \Rightarrow |x|^2 - 3|x| = 0$

$|x|(|x| - 3) = 0$

$|x| = 0$ ή $|x| - 3 = 0$

$|x| = 3$

$x = 0$ $\quad x = 3$ $\quad x = -3$

$x \neq 0$

$x \neq 3$

$x \neq -3$

(β). $A = \dfrac{|x|^2 - 6|x| + 9}{|x|^2 - 3|x|} = \dfrac{(|x| - 3)^2}{|x|(|x| - 3)} = \dfrac{|x| - 3}{|x|}$

(γ) i) $|A| = \dfrac{1}{2} \Rightarrow A = \dfrac{1}{2}$ ή $A = -\dfrac{1}{2}$

$\dfrac{|x| - 3}{|x|} = \dfrac{1}{2}$ $\qquad$ $\dfrac{|x| - 3}{|x|} = \dfrac{-1}{2}$

$2|x| - 6 = |x|$ $\qquad$ $2|x| - 6 = -|x|$

$|x| = 6$ $\qquad$ $3|x| = 6$

$x = 6$ $\quad x = -6$ $\qquad$ $x = 2$ $\quad x = -2$

(B) $|A| < \frac{1}{3}$

$$-\frac{1}{3} < A < \frac{1}{3}$$

$$-\frac{1}{3} < \frac{|x|-3}{|x|} < \frac{1}{3}$$

$-\frac{1}{3} < \frac{|x|-3}{|x|}$ και $\frac{|x|-3}{|x|} < \frac{1}{3}$

$-|x| < 3|x| - 9$

$9 < 4|x|$

$|x| > \frac{9}{4}$

$x > \frac{9}{4}$ ή $x < -\frac{9}{4}$

$x \in (-\infty, -\frac{9}{4}) \cup (\frac{9}{4}, +\infty)$

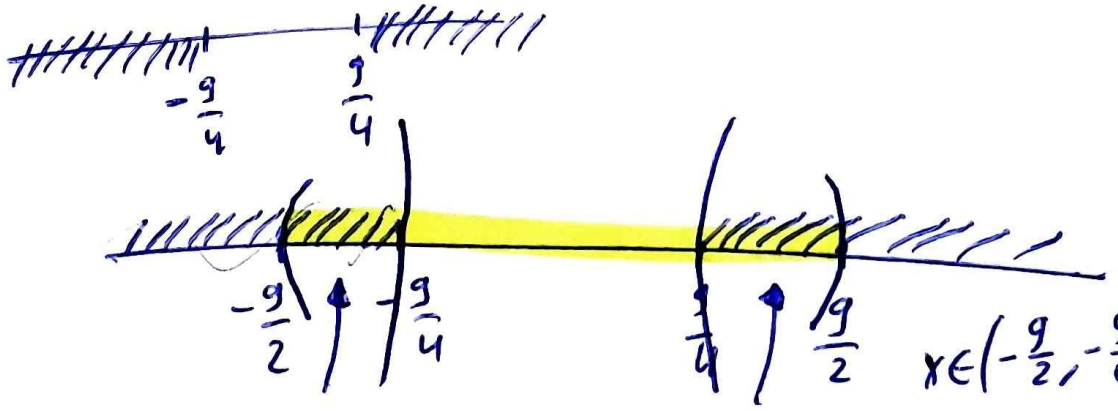
$3|x| - 9 < |x|$

$2|x| < 9$

$|x| < \frac{9}{2}$

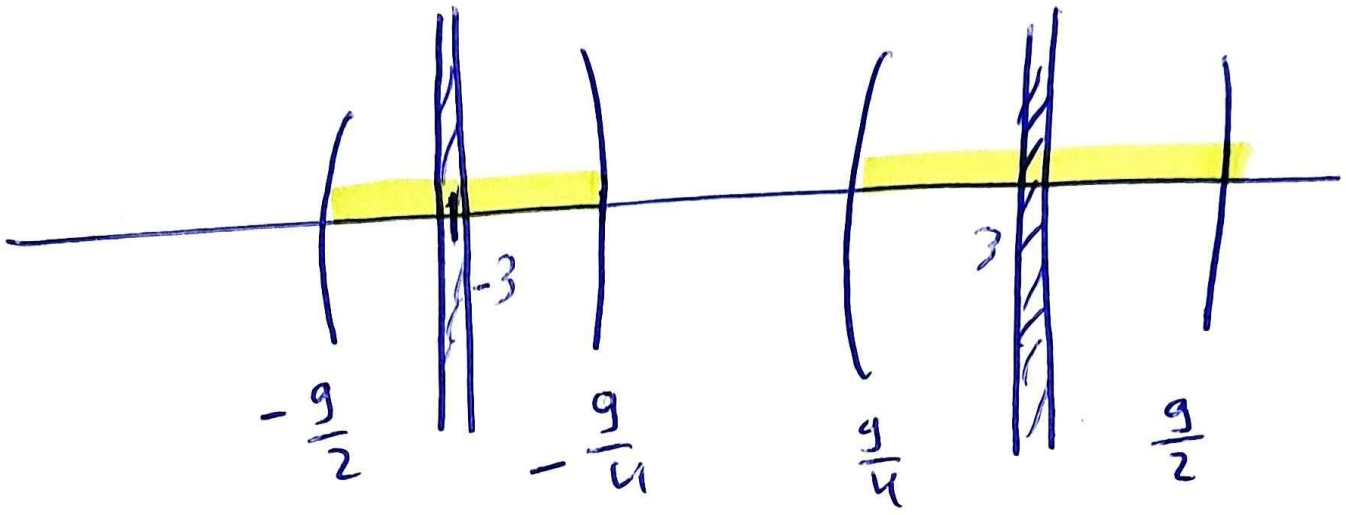
$-\frac{9}{2} < x < \frac{9}{2}$

$x \in (-\frac{9}{2}, \frac{9}{2})$



$x \in (-\frac{9}{2}, -\frac{9}{4}) \cup (\frac{9}{4}, \frac{9}{2})$

Пробни орл.



Отг.

$x \in (-\frac{9}{2}, -3) \cup (-3, -\frac{9}{4}) \cup (\frac{9}{4}, 3) \cup (3, \frac{9}{2})$

42. Av $-1 \le x \le 1$ vsö $|x^3 - 3x^2 + 2x - 1| \le 7$

$|\alpha + \beta + \gamma| \le |\alpha| + |\beta| + |\gamma|$.

- $|x| \le 1 \Rightarrow |x|^3 \le 1$
- $|x| \le 1 \Rightarrow |x|^2 \le 1 \Rightarrow 3|x|^2 \le 3$ } ⊕ .
- $|x| \le 1 \Rightarrow 2|x| \le 2$

$|x|^3 + 3|x|^2 + 2|x| \le 6$

$|x|^3 + 3|x|^2 + 2|x| + 1 \le 7$

$|x^3 - 3x^2 + 2x - 1| = |x^3 + (-3x^2) + (2x) + (-1)| \le$

$\le |x^3| + |-3x^2| + |2x| + |-1|$

$|x^3 - 3x^2 + 2x - 1| \le |x|^3 + 3|x|^2 + 2|x| + 1 . \le 7$

✓ $|x^3 - 3x^2 + 2x - 1| \le 7$ ∟

38. (B) $|x-y+1|+x^2=2x-1$.

$$|x-y+1|+x^2-2x+1=0$$

$$\boxed{|x-y+1|+(x-1)^2=0}$$

$$\begin{cases} x-y+1=0 \Rightarrow 1-y+1=0 \\ x-1=0 \end{cases}$$

$y=2$

$x=1$

(B) $|2x+y-3|+|3x-y-2|=0$

$$\begin{cases} 2x+y-3=0 \\ 3x-y-2=0 \end{cases}$$

$+$ $\quad 5x-5=0$

$x=1$

$3-y-2=0$

$1=y$

28. (D) $B = |x-3| + x-1$

1η. Αν $x-3 \geq 0 \Rightarrow x \geq 3$ $\quad B = |\overset{\oplus}{x-3}| + x-1$

$B = x-3+x-1$

$\underline{B = 2x-4}$

2η. Αν $x-3 < 0$ $\quad$ τότε $\quad B = |\overset{\ominus}{x-3}| + x-1$

$x < 3$

$B = 3-x+x-1$

$\underline{B = 2.}$

$$B = \begin{cases} 2 \, , & x < 3 \\ 2x-4 \, , & x \geq 3 \, . \end{cases}$$

28. ⑧ $\Delta = |x| - |x-1| + |x-2|$

x		0		1		2	
x	−	0	+	+	+		
x−1	−		−	0	+	+	
x−2	−		−		−	0	+

1η. Αν $x < 0$ τότε $\Delta = |\overset{(-)}{x}| - |\overset{(-)}{x-1}| + |\overset{(-)}{x-2}|$

$\Delta = -x - (-x+1) + (-x+2) = -x+x-1-x+2$

$\boxed{\Delta = -x+1}$.

2η. Αν $0 \le x \le 1$ τότε $\Delta = |\overset{(+)}{x}| - |\overset{(-)}{x-1}| + |\overset{(-)}{x-2}|$

$\Delta = x - (-x+1) + (-x+2) = x+x-1-x+2 = x+1$

$\boxed{\Delta = x+1}$

3η. Αν $1<x<2$ τοτε $\Delta = |\overset{(+)}{x}| - |\overset{(+)}{x-1}| + |\overset{(-)}{x-2}|$

$$\Delta = x - (x-1) + (-x+2)$$

$$\Delta = \cancel{x} - \cancel{x} + 1 - x + 2$$

$$\boxed{\Delta = 3 - x}$$

4η. Αν $x \geq 2$ τοτε $\Delta = |\overset{(+)}{x}| - |\overset{(+)}{x-1}| + |\overset{(+)}{x-2}|$

$$\Delta = x - (x-1) + (x-2)$$

$$\Delta = \cancel{x} - \cancel{x} + 1 + x - 2$$

$$\boxed{\Delta = x - 1}$$

$$\Delta = \begin{cases} 1-x, & x<0 \\ x+1, & 0 \leq x \leq 1 \\ 3-x, & 1<x<2 \\ x-1, & x \geq 2. \end{cases}$$

Εφραιμ Μαθητής

Σελ 114

29

30

32.

35.

28 α γ

33

34.

35.

38 α

40 α

41 α

44 α.

47.

49

50

52..

53. ⓐ $\sqrt{7+2\sqrt{10}}$

$$\sqrt{(\sqrt{5}+\sqrt{2})^2} = |\sqrt{5}+\sqrt{2}| = \sqrt{5}+\sqrt{2}.$$

ⓑ $\sqrt{5-2\sqrt{6}} =$

$$\sqrt{(\sqrt{3}-\sqrt{2})^2} = |\sqrt{3}-\sqrt{2}| = \sqrt{3}-\sqrt{2}$$

ⓔ $\sqrt{13-4\sqrt{3}} = \sqrt{(\sqrt{12}-\sqrt{1})^2} = |\sqrt{12}-\sqrt{1}| = \sqrt{12}-\sqrt{1}$

ⓕ $\sqrt{12-4\sqrt{5}} = \sqrt{(\sqrt{10}-\sqrt{2})^2} = |\sqrt{10}-\sqrt{2}| = \sqrt{10}-\sqrt{2}$

53. ⑦ $\sqrt{3-2\sqrt{2}} = \sqrt{(\sqrt{2}-1)^2}$

$\downarrow$

$2\sqrt{2}\cdot 1$

⑧ $\sqrt{7-4\sqrt{3}} = \sqrt{(\sqrt{3}-2)^2} = |\sqrt{3}-2|$

$= 2-\sqrt{3}.$

Χρήσιμη Ιδιότητα

$$\sqrt[\nu]{x^{\mu}} = x^{\frac{\mu}{\nu}}$$

$$\left.\begin{aligned} \sqrt[\nu]{\sqrt[\kappa]{x}} &= \sqrt[\nu\cdot\kappa]{x} \\ \sqrt[\nu\cdot\rho]{x^{\kappa\cdot\rho}} &= \sqrt[\nu]{x^{\kappa}} \end{aligned}\right\} \text{Εκθ. ιδιότ.}$$

54. ㉮ $\sqrt{7+2\sqrt{10}} - \sqrt{7-2\sqrt{10}}$

$2\sqrt{2}\sqrt{5}$

$\sqrt{(\sqrt{2}+\sqrt{5})^2} - \sqrt{(\sqrt{2}-\sqrt{5})^2} = |\sqrt{2}+\sqrt{5}| - |\sqrt{2}-\sqrt{5}|$ (+) (−)

$= \sqrt{2}+\sqrt{5} - (-\sqrt{2}+\sqrt{5}) = 2\sqrt{2}$.

㉯ $\sqrt{6-2\sqrt{5}} - \sqrt{9+4\sqrt{5}}$

$2\sqrt{2}$

$\sqrt{(\sqrt{5}-1)^2} - \sqrt{(\sqrt{5}+2)^2} =$

$= |\sqrt{5}-1| - |\sqrt{5}+2|$ (+) (+)

$\sqrt{5} - 1 - \sqrt{5} + 2 = 1$

54. (B) $B = \sqrt{4-2\sqrt{3}} - \sqrt{4+2\sqrt{3}} \Rightarrow$

$$B = \sqrt{(\sqrt{3}-1)^2} - \sqrt{(\sqrt{3}+1)^2}$$

$$B = |\sqrt{3}-1| - |\sqrt{3}+1|$$

(+) (+)

$$B = \sqrt{3}-1 - (\sqrt{3}+1)$$

$$B = -2.$$

(E) $\Delta = \sqrt{5+2\sqrt{6}} - \sqrt{5-2\sqrt{6}}. \Rightarrow$

$$\Rightarrow \Delta = \sqrt{(\sqrt{2}+\sqrt{3})^2} - \sqrt{(\sqrt{2}-\sqrt{3})^2} \Rightarrow$$

$$\Rightarrow \Delta = |\sqrt{2}+\sqrt{3}| - |\sqrt{2}-\sqrt{3}| \Rightarrow$$

$$\Rightarrow \Delta = \sqrt{2}+\sqrt{3} - (\sqrt{3}-\sqrt{2}) \Rightarrow$$

$$\Rightarrow \Delta = \sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{2} \Rightarrow \Delta = 2\sqrt{2}$$

27. (62) $\sqrt{5\sqrt[3]{5\sqrt[4]{25}}} =$

$= \sqrt{5\sqrt[3]{5 \cdot 25^{1/4}}} =$

$= \sqrt{5\sqrt[3]{5 \cdot (5^2)^{\frac{1}{4}}}} =$

$= \sqrt{5\sqrt[3]{5 \cdot 5^{\frac{1}{2}}}}$

$= \sqrt{5\sqrt[3]{5^{\frac{3}{2}}}}$

$= \sqrt{5\ 5^{\frac{\frac{3}{2}}{3}}} = \sqrt{5 \cdot 5^{\frac{3}{6}}}$

$= \sqrt{5 \cdot 5^{\frac{1}{2}}} = \sqrt[2]{5^{\frac{3}{2}}} =$

$= 5^{\frac{\frac{3}{2}}{2}} = 5^{\frac{3}{4}} = \sqrt[4]{5^3}.$

تمرين محلول

$$\sqrt{5\sqrt[3]{5\sqrt[4]{25}}} =$$

$$= \sqrt{5\sqrt[3]{5\sqrt[4]{5^2}}}$$

$$= \sqrt{5\sqrt[3]{5\sqrt{5}}} = \sqrt{5\sqrt[3]{\sqrt{5^2 \cdot 5}}}$$

$$= \sqrt{5\sqrt[6]{5^3}} = \sqrt{5\sqrt{5}}$$

$$= \sqrt{\sqrt{5^2 \cdot 5}} = \sqrt[4]{5^3}$$

29. ⓔ $\sqrt[3]{5}\ \sqrt{15}\ \sqrt[6]{135} = 15$

$5^{\frac{1}{3}} \cdot 15^{\frac{1}{2}} \cdot \sqrt[6]{3 \cdot 45} = 15$

$5^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot 3^{\frac{1}{6}} \cdot 9^{\frac{1}{6}} \cdot 5^{\frac{1}{6}} = 15$

$5^{\frac{1}{3}} \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{6}} \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{1}{6}} \cdot 3^{\frac{1}{3}} = 15$

$5^{\frac{1}{3}+\frac{1}{2}+\frac{1}{6}} \cdot 3^{\frac{1}{2}+\frac{1}{6}+\frac{1}{3}} = 15$

$5^1 \cdot 3^1 = 15$

$15 = 15$

ⓒ $\dfrac{\sqrt{8}\ \sqrt[3]{4}}{\sqrt[6]{2}} = 4.$

$\dfrac{\sqrt{4} \cdot \sqrt{2} \cdot \sqrt[3]{2^2}}{2^{\frac{1}{6}}} = 4$

$\dfrac{2\sqrt{2} \cdot 2^{\frac{2}{3}}}{2^{\frac{1}{6}}} = 4$

$\dfrac{2 \cdot 2^{\frac{1}{2}} \cdot 2^{\frac{2}{3}}}{2^{\frac{1}{6}}} = 4$

$\dfrac{2^{\frac{2}{2}+\frac{1}{2}+\frac{2}{3}}}{2^{\frac{1}{6}}} = 4$

$\dfrac{2^{\frac{6}{6}+\frac{3}{6}+\frac{4}{6}}}{2^{\frac{1}{6}}} = 4$

$\dfrac{2^2 \cdot 2^{\frac{1}{6}}}{2^{\frac{1}{6}}} = 4$

$4 = 4$

59. $\alpha = \dfrac{\sqrt{8}-\sqrt{50}}{\sqrt{12}-\sqrt{75}} \cdot \dfrac{3}{\sqrt{6}}$ και $\beta = \dfrac{7}{1-2\sqrt{2}} + 2 + 2\sqrt{2}$.

(α) Βρ.Δ α, β.

α. $\dfrac{\sqrt{8}-\sqrt{50}}{\sqrt{12}-\sqrt{75}} = \dfrac{\sqrt{2\cdot 4}-\sqrt{25\cdot 2}}{\sqrt{4\cdot 3}-\sqrt{25\cdot 3}} = \dfrac{2\sqrt{2}-5\sqrt{2}}{2\sqrt{3}-5\sqrt{3}} = \dfrac{-3\sqrt{2}}{-3\sqrt{3}} = \dfrac{\cancel{3}\sqrt{2}}{\cancel{3}\sqrt{3}}$

$\Rightarrow \alpha = \dfrac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{\sqrt{6}}{3} \cdot \dfrac{\cancel{\sqrt{6}}}{\cancel{\sqrt{6}}}$ $\boxed{\alpha = 1}$.

$\beta = \dfrac{7}{1-2\sqrt{2}} + 2 + 2\sqrt{2} = \dfrac{7\cdot(1+2\sqrt{2})}{(1-2\sqrt{2})(1+2\sqrt{2})} + 2 + 2\sqrt{2} =$

$= \dfrac{7+14\sqrt{2}}{1+\cancel{2\sqrt{2}}-\cancel{2\sqrt{2}}-(2\sqrt{2})^2} + 2 + 2\sqrt{2} = \dfrac{7+14\sqrt{2}}{1-8} + 2 + 2\sqrt{2} =$

$= \dfrac{7+14\sqrt{2}}{-7} + 2 + 2\sqrt{2} = \dfrac{1+2\sqrt{2}}{-1} + 2 + 2\sqrt{2} =$

$= -1 - \cancel{2\sqrt{2}} + 2 + \cancel{2\sqrt{2}} \Rightarrow \boxed{\beta = 1}$.

(B) $(a+2\sqrt{2B})^2 = (1+2\sqrt{2})^2 = 1+4\sqrt{2}+(2\sqrt{2})^2 =$

$= 1+4\sqrt{2}+4\cdot 2 = \underline{\underline{9+4\sqrt{2}}}$

$(a-2\sqrt{2B})^2 = (1-2\sqrt{2})^2 = 1-4\sqrt{2}+(2\sqrt{2})^2 =$

$= 1-4\sqrt{2}+4\cdot 2 = \underline{\underline{9-4\sqrt{2}}}$

$\alpha \quad 1+2\sqrt{2}+1-2\sqrt{2} = 2$

$2 = 2$ ✓

Άρα $\sqrt{9+4\sqrt{2B}} - \sqrt{9-4\sqrt{2B}} = 2 \Rightarrow$

$\Rightarrow \sqrt{9+4\sqrt{2}} - \sqrt{9-4\sqrt{2}} = 2 \Rightarrow$

$\Rightarrow$

$\sqrt{(1+2\sqrt{2})^2} - \sqrt{(1-2\sqrt{2})^2} = 2$

$|1+2\sqrt{2}| - |1-2\sqrt{2}| = 2$

$\overset{\oplus}{} \quad \overset{\ominus}{}$

$1+2\sqrt{2} - (-1+2\sqrt{2}) = 2$

29. (B) Να δ. $\sqrt{3} \cdot \sqrt[3]{3} \cdot \sqrt[6]{3} = 3 \Rightarrow$

$\Rightarrow 3^{\frac{1}{2}} \cdot 3^{\frac{1}{3}} \cdot 3^{\frac{1}{6}} = 3 \Rightarrow$

$\Rightarrow 3^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} = 3 \Rightarrow$

$\Rightarrow 3^{\frac{3}{6}+\frac{2}{6}+\frac{1}{6}} = 3 \Rightarrow 3^{\frac{6}{6}} = 3 \Rightarrow 3 = 3$ ✓

29. ⑧ Nδo $\sqrt{5} \cdot \sqrt[10]{5} \cdot \sqrt[5]{25} = 5$

$5^{\frac{1}{2}} \cdot 5^{\frac{1}{10}} \cdot 25^{\frac{1}{5}} = 5$

$5^{\frac{1}{2}} \cdot 5^{\frac{1}{10}} \cdot (5^2)^{\frac{1}{5}} = 5$

$5^{\frac{1}{2}} \cdot 5^{\frac{1}{10}} \cdot 5^{\frac{2}{5}} = 5$

$5^{\frac{1}{2}+\frac{1}{10}+\frac{2}{5}} = 5$ ✓

$5^{\frac{5}{10}+\frac{1}{10}+\frac{4}{10}} = 5$

$5^1 = 5$ ✓

27. ⑨ $\sqrt{\sqrt[4]{5}} = \sqrt[8]{5}$ ✓ $= 5^{1/8}$.

⑧ $\dfrac{\sqrt{\sqrt[3]{4}\ \sqrt{2\sqrt{2}}}}{\sqrt[12]{2}} = \dfrac{\sqrt[6]{4} \cdot \sqrt{2 \cdot 2^{1/2}}}{2^{1/12}} = \dfrac{4^{\frac{1}{6}} \cdot \sqrt{2^{3/2}}}{2^{1/12}}$

$= \dfrac{(2^2)^{\frac{1}{6}} \cdot 2^{\frac{3}{2}\cdot\frac{1}{2}}}{2^{1/12}} = \dfrac{2^{\frac{1}{3}} \cdot 2^{\frac{3}{4}}}{2^{\frac{1}{12}}} = \dfrac{2^{\frac{1}{3}+\frac{3}{4}}}{2^{\frac{1}{12}}}$

$= \dfrac{2^{\frac{7}{12}}}{2^{1/12}} = 2^{6/12} = 2^{1/2} = \sqrt{2}.$

21. ⑧ $\sqrt[3]{9}\,\sqrt{3}\,\sqrt[4]{27}\,\sqrt[12]{3} = 9$.

$$9^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} \cdot 27^{\frac{1}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$(3^2)^{\frac{1}{3}} \cdot 3^{\frac{1}{2}} \cdot (3^3)^{\frac{1}{4}} \, 3^{\frac{1}{12}} = 9$$

$$3^{\frac{2}{3}} \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{3}{4}} \cdot 3^{\frac{1}{12}} = 9$$

$$3^{\frac{2}{3}+\frac{1}{2}+\frac{3}{4}+\frac{1}{12}} = 9$$

$$3^{\frac{8}{12}+\frac{6}{12}+\frac{9}{12}+\frac{1}{12}} = 9$$

$$3^{\frac{24}{12}} = 9$$

$$3^2 = 9$$

✓

29. (α) Ndo $\sqrt{2} \cdot \sqrt[3]{2} = \sqrt[6]{32} \Rightarrow$

$\Rightarrow 2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} = \sqrt[6]{2^5} \Rightarrow$

$\Rightarrow 2^{\frac{1}{2}+\frac{1}{3}} = 2^{\frac{5}{6}} \Rightarrow 2^{\frac{5}{6}} = 2^{\frac{5}{6}}$

27. (β) $\sqrt[4]{5^2} = \sqrt[2]{5^1} = \sqrt{5}$

(γ) $\sqrt[3]{3\sqrt{3}} = \sqrt[3]{3 \cdot 3^{\frac{1}{2}}} = \sqrt[3]{3^1 \cdot 3^{\frac{1}{2}}} = \sqrt[3]{3^{\frac{3}{2}}} =$

$= \left(3^{\frac{3}{2}}\right)^{\frac{1}{3}} = 3^{\frac{3}{6}} = 3^{\frac{1}{2}} = \sqrt{3}$

(ε) $\sqrt{2\sqrt[3]{2\sqrt{2}}} = \sqrt{2\sqrt[3]{2^{\frac{2}{1}} \cdot 2^{\frac{1}{2}}}} =$ ✓

$= \sqrt{2\sqrt[3]{2^{\frac{3}{2}}}} = \sqrt{2 \cdot \left(2^{\frac{3}{2}}\right)^{\frac{1}{3}}} = \sqrt{2 \cdot 2^{\frac{3}{6}}} =$

$= \sqrt{2^{\frac{2}{1}} \cdot 2^{\frac{1}{2}}} = \sqrt{2^{\frac{3}{2}}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{2}} = 2^{\frac{3}{4}} = \sqrt[4]{2^3}$

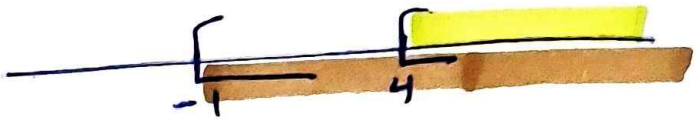
39. $A = \left(\sqrt{x-4} + \sqrt{x+1}\right)\left(\sqrt{x-4} - \sqrt{x+1}\right)$

(α) Βρίσκω τα x ώστε να ορίζεται η Α.

Πρέπει $x-4 \geq 0$ και $x+1 \geq 0$

$x \geq 4$, $x \geq -1$

$x \in [4, +\infty)$



(β) Απλοποιώ

$$A = \left(\sqrt{x-4} + \sqrt{x+1}\right)\left(\sqrt{x-4} - \sqrt{x+1}\right) =$$

$$= \left(\sqrt{x-4}\right)^2 - \left(\sqrt{x+1}\right)^2 = x - 4 - x - 1 = -5$$

36. $A = \dfrac{\sqrt{x^2-2x+1}}{x-1} - \dfrac{\sqrt{x^2-4x+4}}{x-2}$.

(a) $A = \dfrac{\sqrt{(x-1)^2}}{x-1} - \dfrac{\sqrt{(x-2)^2}}{x-2}$

$A = \dfrac{|x-1|}{x-1} - \dfrac{|x-2|}{x-2}$

(b) Αν $1<x<2$ τότε $A=2$.

$A = \overset{(+)}{\dfrac{|x-1|}{x-1}} - \overset{(-)}{\dfrac{|x-2|}{x-2}} \Rightarrow A = \dfrac{x-1}{x-1} - \dfrac{-x+2}{x-2}$

α $1<x<2$

$0<x-2<1$

$A = \dfrac{x-1}{x-1} - \dfrac{-x+2}{x-2} \Rightarrow A = 1 - \dfrac{2-x}{-2+x} \Rightarrow$

• $1<x<2$

$-1<x-2<0$

44. ⓐ $\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}} + \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = 2$

$$\left(\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}} + \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}\right)^2 = 2^2$$

$$\left(\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}\right)^2 + 2\cdot\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}\cdot\sqrt{\frac{-\sqrt{5}+3}{\sqrt{5}+3}} + \left(\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}\right)^2 = 4$$

$$\frac{\sqrt{5}+1}{\sqrt{5}-1} + 2\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}\cdot\frac{3-\sqrt{5}}{\sqrt{5}+3}} + \frac{3-\sqrt{5}}{3+\sqrt{5}} = 4$$

$$\frac{\sqrt{5}+1}{\sqrt{5}-1} + 2\sqrt{\frac{(\sqrt{5}+1)^2}{4}\cdot\frac{(3-\sqrt{5})^2}{4}} + \frac{3-\sqrt{5}}{3+\sqrt{5}} = 4$$

$$\frac{\sqrt{5}+1}{\sqrt{5}-1} + 2\,\frac{|\sqrt{5}+1|\,|3-\sqrt{5}|}{2\cdot 2} + \frac{3-\sqrt{5}}{3+\sqrt{5}} = 4$$

$$\frac{(\sqrt{5}+1)(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} + \frac{(\sqrt{5}+1)(3-\sqrt{5})}{2} + \frac{(3-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = 4$$

$$\frac{(\sqrt{5}+1)^2}{5-1} + \frac{3\sqrt{5}-5+3-\sqrt{5}}{2} + \frac{(3-\sqrt{5})^2}{9-5} = 4$$

$$\frac{(\sqrt{5}+1)^2}{4} + \frac{2\sqrt{5}-2}{2} + \frac{(3-\sqrt{5})^2}{4} = 4$$

$$\frac{(\sqrt{5}+1)^2+(3-\sqrt{5})^2}{4} + \sqrt{5}-1 = 4$$

$$\frac{(\sqrt{5})^2+2\sqrt{5}+1+9-2\cdot 3\sqrt{5}+(\sqrt{5})^2}{4} + \sqrt{5}-1 = 4$$

$$\frac{5+5+1+9+2\sqrt{5}-6\sqrt{5}}{4} + \sqrt{5}-1 = 4$$

$$\frac{20-4\sqrt{5}}{4} + \sqrt{5}-1 = 4$$

$$\frac{4(5-\sqrt{5})}{4} + \sqrt{5}-1 = 4$$

$$5-\sqrt{5}+\sqrt{5}-1 = 4$$

$$4 = 4$$

9. $A = \sqrt{8} - \sqrt{12} - \sqrt{50} + \sqrt{75}$

$A = \sqrt{2 \cdot 4} - \sqrt{3 \cdot 4} - \sqrt{2 \cdot 25} + \sqrt{3 \cdot 25}$

$A = 2\sqrt{2} - 2\sqrt{3} - 5\sqrt{2} + 5\sqrt{3}$

$A = -3\sqrt{2} + 3\sqrt{3}$

$B = \sqrt{18} - \sqrt{27} - \sqrt{32} + \sqrt{48}$.

$B = \sqrt{2 \cdot 9} - \sqrt{3 \cdot 9} - \sqrt{4 \cdot 16} + \sqrt{4 \cdot 12}$

$B = 3\sqrt{2} - 3\sqrt{3} - 2\sqrt{17} + 2\sqrt{12}$

$B = 3\sqrt{2} - 3\sqrt{3} - 2\sqrt{17} + 2\sqrt{34}$

$B = 3\sqrt{2} - 3\sqrt{3} - 2\sqrt{17} + 4\sqrt{3}$

$B = 3\sqrt{2} - 2\sqrt{17} + \sqrt{3}$

37. Av $d(3x_{-}-1)<2$

(01) $A=\sqrt{x^2+2x+1}-\sqrt{1-4x+4x^2} \Rightarrow$

$\Rightarrow A=\sqrt{(x+1)^2}-\sqrt{(2x-1)^2} \Rightarrow$

$A=|x+1|-|2x-1| = x+1+2x-1 = 3x$ (+) (−)

$d(3x_{-}-1)$

$|3x+1|<2$

$-2<3x+1<2$

$-3<3x<1$

$-1<x<\frac{1}{3}$

- $-1<x<\frac{1}{3} \Rightarrow 0<x+1<\frac{4}{3}$
- $-1<x<\frac{1}{3} \Rightarrow -2<2x<\frac{2}{3}$

$-3<2x-1<-\frac{1}{1}$

(B) $\dfrac{\sqrt{x^2+4x+4}}{x+2}-\dfrac{9x^2-1}{\sqrt{1-6x+9x^2}}=$

$=\dfrac{\sqrt{(x+2)^2}}{x+2}-\dfrac{(3x-1)(3x+1)}{\sqrt{(3x-1)^2}}=$

$=\dfrac{|x+2|}{x+2}-\dfrac{(3x-1)(3x+1)}{|3x-1|}=\dfrac{x+2}{x+2}-\dfrac{(3x-1)(3x+1)}{(1-3x)}=$

$=1$

$=1+\dfrac{(1-3x)(3x+1)}{1-3x}$

- $-1<x<\frac{1}{3} \Rightarrow 1<x<\frac{1}{3}$
- $-1<x<\frac{1}{3} \Rightarrow -3<3x<1 \Rightarrow -4<3x-1<0$

$2+3x$.

44. ⓐ Νδο $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} + \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = 4$.$\Rightarrow$

$\Rightarrow \left(\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} + \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\right)^2 = 16 \Rightarrow$

$\Rightarrow \cancel{\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}^2} + \cancel{\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}^2} + 2\left(\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\right)\left(\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\right) = 16 \Rightarrow$

$\Rightarrow \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{2+\sqrt{3}}{2-\sqrt{3}} + 2\sqrt{\frac{\cancel{2-\sqrt{3}}}{\cancel{2+\sqrt{3}}} \cdot \frac{\cancel{2+\sqrt{3}}}{\cancel{2-\sqrt{3}}}} = 16 \Rightarrow$

$\Rightarrow \frac{(2-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} + \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} + 2 = 16 \Rightarrow$

$\Rightarrow \frac{4-2\sqrt{3}-2\sqrt{3}+\cancel{\sqrt{3}^2}}{4-\cancel{\sqrt{3}^2}} + \frac{4+2\sqrt{3}+2\sqrt{3}+\cancel{\sqrt{3}^2}}{4-\cancel{\sqrt{3}^2}} + 2 = 16 \Rightarrow$

$\Rightarrow 4 - \cancel{4\sqrt{3}} + 3 + 4 + \cancel{4\sqrt{3}} + 3 + 2 = 16 \Rightarrow$

$\Rightarrow 16 = 16$

41. (a) $(1+2\sqrt{5})^2 = 1 + 4\sqrt{5} + (2\sqrt{5})^2 = 1+4\sqrt{5}+4\cdot 5 =$

$= 1+4\sqrt{5}+20 = 21+4\sqrt{5}$

$(1-2\sqrt{5})^2 = 1 - 4\sqrt{5} + (2\sqrt{5})^2 = 1-4\sqrt{5}+4\cdot 5 = 1-4\sqrt{5}+20 =$

$= 21-4\sqrt{5}$

(β) $A = \sqrt{21+4\sqrt{5}} - \sqrt{21-4\sqrt{5}} =$

$= \sqrt{(1+2\sqrt{5})^2} - \sqrt{(1-2\sqrt{5})^2} =$

$= |\overset{(+)}{1+2\sqrt{5}}| - |\overset{(-)}{1-2\sqrt{5}}|$

$= 1+2\sqrt{5} - (-1+2\sqrt{5})$

$= 1+2\sqrt{5}+1-2\sqrt{5}$

$= 2.$

25. ⓐ $\sqrt[3]{3}\sqrt[3]{4-\sqrt{7}}\sqrt[3]{4+\sqrt{7}}=3$.

$$\sqrt[3]{3\cdot(4-\sqrt{7})(4+\sqrt{7})}=3 \Rightarrow$$

$$\Rightarrow \sqrt[3]{3\cdot(4^2-\sqrt{7}^2)}=3$$

$$\sqrt[3]{3\cdot(16-7)}=3$$

$$\sqrt[3]{3\cdot 9}=3$$

$$\sqrt[3]{27}=3$$

$$3=3.$$

Επομενο Μαθημα

Τρίτη 5:30-6:30

Γραφουμε Τους Απολυτα.

1. Θεωρια

Βιβλιο

Σελ 62 : Ορισμοι απολυτων αριθ.

Σελ 63 : Αποδείξεις.

+ Ιδιοτητες για Σ-Λ.

2. Ασκησεις

Μπαρδας

5 9 15 20 25 35 46.

7 11 17 22 26 38 48.

8 13 19 23 28 Β. 40 51,

41.

Η εξίσωση $ax+\beta=0$

$$ax = -\beta$$

1η περίπτωση: Αν $a \neq 0$ τότε $\frac{ax}{a} = \frac{-\beta}{a}$

Η εξίσωση έχει μοναδική λύση.

$$\boxed{x = -\frac{\beta}{a}}$$

2η περίπτωση: Αν $a = 0$ τότε $0x = \beta$

i) Αν $\beta = 0$ τότε $0x = 0$ Αόριστη ή Ταυτότητα.

ii) Αν $\beta \neq 0$ τότε $0x = \beta$

Αδύνατη.

1. (в) $x-3=3x-1 \Rightarrow$

$\Rightarrow x-3-3x+1=0 \Rightarrow -2x-2=0 \Rightarrow \frac{-2x}{2}=\frac{2}{2} \Rightarrow x=-1$

(δ) $x-(3x-1)=1-2(3x-2) \Rightarrow$

$\Rightarrow x-3x+1=1-6x+4 \Rightarrow$

$\Rightarrow x-3x+1-1+6x-4=0 \Rightarrow$

$\Rightarrow 4x-4=0 \Rightarrow \frac{4x}{4}=\frac{4}{4} \Rightarrow x=1$

(δ2) $0,2\cdot x-0,3\cdot[2x-(3x-1)]=-0,7 \Rightarrow$

$\Rightarrow 0,2x-0,3\cdot[2x-3x+1]=-0,7 \Rightarrow$

$\Rightarrow 0,2x-0,3[-x+1]=-0,7 \Rightarrow$

$\Rightarrow 0,2x+0,3x-0,3=-0,7 \Rightarrow$

$\Rightarrow 0,5x-0,3+0,7=0 \Rightarrow$

$\Rightarrow 0,5x+0,4=0 \Rightarrow 0,5x=-0,4$

6. (r) $x^3 = 2x^2$

$x^3 - 2x^2 = 0 \Rightarrow x^2(x-2) = 0$

$x = 0$ ni $x - 2 = 0$

$x = 2$

(20) $-x^2 + x = 0$

$x - x^2 = 0$

$x(-x+1) = 0$ $x = 0$ ni $-x + 1 = 0$

$-x = -2$

$x = 1$.

(s) $x^3 - 2x^2 + x = 0$.

$x(x^2 - 2x + 1) = 0$

$x(x-1)^2 = 0$

$x = 0$ ni $x - 1 = 0$

$x = 1$

2. (B) $1-(2x-1)=x-2(x-3) \Rightarrow$

$\Rightarrow 1-2x+1=x-2x+6 \Rightarrow$

$\Rightarrow$ [illegible] $\boxed{x=-4}$

(δ) $3(x-1)=1-(x+4) \Rightarrow$

$\Rightarrow 3x-3=1-x-4 \Rightarrow$

$\Rightarrow \frac{4x}{4}=\frac{0}{4} \Rightarrow \boxed{x=0}$

✓

3. (B) $\frac{3 \cdot 5x}{2} - \frac{x}{6} = 6x - 6 \cdot 1$

$15x - x = 6x - 6$

$15x - 6x - x = -6$

$8x = -6.$

$x = -\frac{3}{4}$

(δ) $\frac{3(3x-1)}{2} - \frac{x-1}{6} = -6x + 6 \cdot 1.$

$3(3x-1) - (x-1) = -6x + 6$

$9x - 3 - x + 1 + 6x = 6$

$9x - x + 6x = 3 + 6 - 1$

$\frac{14x}{14} = \frac{8}{14}$

$x = \frac{4}{7}$

4. (B) Αν η εξίσωση έχει ρίζα το -1

Βρείτε το α $\quad \frac{x-\alpha}{2} - \frac{3\alpha - x}{3} = -x + \alpha$

$$\frac{-1-\alpha}{2} - \frac{-3\alpha+1}{3} = 1+\alpha \Rightarrow$$

$$\Rightarrow 2(-1-\alpha) - 3(-3\alpha+1) = 1+\alpha \Rightarrow$$

$$\Rightarrow -2-2\alpha+9\alpha-3 = 1+\alpha \Rightarrow$$

$$\Rightarrow -5+7\alpha+ = 1+\alpha \Rightarrow -\alpha+7\alpha = 6 \Rightarrow \frac{6\alpha}{6} = \frac{6}{6} \Rightarrow \alpha = 1$$

5. (β) $(x-2)(x-3) = (x-1)^2$

$x^2 - 3x - 2x + 6 = x^2 - 2x + 1$

$x^2 - 5x - x^2 + 2x = -6$

$-3x = -6$

$x = 2$

(δ) $1 - (2x-1)^2 = x - 2x(2x-3)$

$1 - (4x^2 - 4x + 1) = x - 4x^2 + 6x$

$1 - 4x^2 + 4x - 1 = x - 4x^2 + 6x$

$4x - x - 6x = 0$

$-3x = 0$

$x = 0$

(ε) $x(x-3)^2 + 8 = (x-2)^3$

$x(x^2 - 6x + 9) + 8 = x^3 - 3x^2 \cdot 2 + 3 \cdot 1^2 \cdot x - 2^3$

$x^3 - 6x^2 + 9x + 8 = x^3 - 6x^2 + 12x - 8$

$9x + 8 = 12x - 8$

$9x - 12x = -8 - 8$

$-3x = -16$

$x = \frac{16}{3}$.

9. (52) $\frac{x}{x^2-1} = \frac{1}{x+1}$ $\Rightarrow$

$\Rightarrow \frac{x}{(x-1)(x+1)} = \frac{1}{x+1}$ $\Rightarrow$

$\Rightarrow \quad x = x-1 \Rightarrow 0x = -1 \Rightarrow$ ΑΔΥΝΑΤΗ

- $x^2-1 = 0 \Rightarrow (x-1)(x+1) = 0 \Rightarrow$ $x-1=0 \Rightarrow \boxed{x=1}$ ή $x+1=0 \Rightarrow \boxed{x=-1}$
- $x+1=0 \Rightarrow x=-1$

$x \neq 1$ και $x \neq -1$

10. (B) $\frac{9x-1}{9x^2-1} + \frac{1}{1-3x} = \frac{2}{3x+1}$.

πρπ $x \neq \frac{1}{3}, -\frac{1}{3}$

- $9x^2-1 = 3^2x^2-1 = (3x-1)(3x+1) = 0$
- $1-3x = 1/3x = 3x-1 = 0$
- $3x+1 = 3x+1 = 0$

$3x-1=0$ ή $3x+1=0$

$3x=1$ $3x=-1$

$x=\frac{1}{3}$ ή $x=-\frac{1}{3}$

Εκπ $= (3x-1)(3x+1)$

$$\frac{9x-1}{(3x-1)(3x+1)} - \frac{1}{3x-1} = \frac{2}{3x+1}.$$

$$(3x-1)(3x+1)\frac{9x-1}{(3x-1)(3x+1)} - (3x-1)(3x+1)\frac{1}{3x-1} = (3x-1)(3x+1)\frac{2}{3x+1}$$

$9x-1-3x-1 = 6x-2 \Rightarrow 6x-2 = 6x-2 \Rightarrow 6x = 2(3x-1)$

$0=0$ Αορίστη

Λύση

$x \in \mathbb{R} - \left\{-\frac{1}{3}, \frac{1}{3}\right\}$.

3. (02) $\frac{2(1-3x)}{5} - \frac{3}{2}(x-1) = -x+2$

$10 \cdot \frac{2(1-3x)}{5} - 10 \cdot \frac{3}{2}(x-1) = 10(2-x)$

$4(1-3x) - 15(x-1) = 20 - 10x$

$4 - 12x - 15x + 15 = 20 - 10x$

$4 - 12x - 15x + 15 - 20 + 10x = 0$

$-12x - 15x + 10x = -4 - 15 + 20$

$-17x = 1 \Rightarrow \boxed{x = \frac{1}{17}}$

(11) $\frac{3x-2}{5} = 2$.

$5 \cdot \frac{3x-2}{5} = 10 \Rightarrow 3x-2 = 10 \Rightarrow 3x = 12 \Rightarrow \boxed{x = 4}$

9. (B) $\frac{1}{x+2} - \frac{4}{x^2-4} = \frac{2}{x-2}$.

1. Παραγοντοποιώ τους παρονομαστές

$$\frac{1}{x+2} - \frac{4}{(x-2)(x+2)} = \frac{2}{x-2}$$

2. Περιορισμοί.

Πρπ $x \neq 2$
$x \neq -2$

→ $x+2=0 \Rightarrow \boxed{x=-2}$

→ $(x-2)(x+2)=0$ $x-2=0$ ή $x+2=0$ $x=2$ $x=-2$

→ $x-2=0$ $(x=2)$

3. ΕΚΠ : $(x-2)(x+2)$

$$(x-2)(x+2)\frac{1}{x+2} - (x-2)(x+2)\frac{4}{(x-2)(x+2)} = (x-2)(x+2)\frac{2}{x-2}$$

$$x-2-4 = 2(x+2)$$

$$x-6 = 2x+4$$

$$x-2x = 4+6$$

$$-x = 10$$

$$\boxed{x=-10}$$ ✓

7. (B) $x^2 - 2x(x-1) = x \Rightarrow$

$\Rightarrow x^2 - 2x^2 + 2x = x \Rightarrow$

$\Rightarrow -x^2 = -x \Rightarrow x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ ή $x - 1 = 0$

$x = 1$.

(δ) $(x-2)^2 - (2x-1)(x-4) = 0 \Rightarrow$

$\Rightarrow x^2 - 4x + 4 - 2x^2 + 8x + x - 4 = 0 \Rightarrow$

$\Rightarrow -x^2 + 5x = 0 \Rightarrow x^2 = 5x \Rightarrow$

$\Rightarrow x^2 - 5x = 0 \Rightarrow x(x-5) = 0 \Rightarrow$

$\Rightarrow x = 0$ ή $x - 5 = 0 \Rightarrow$

$\Rightarrow x = 5$

9. (δ) $\frac{x-1}{x} - \frac{1}{x+1} + \frac{1}{x^2+x} = 0 . \Rightarrow$

$$\Rightarrow \frac{x-1}{x} - \frac{1}{x+1} + \frac{1}{x(x+1)} = 0$$

→ $x = 0$

→ $x+1 = 0 \Rightarrow x = -1$

→ $x = 0$ ή $x+1 = 0 \Rightarrow x = -1$

$x \neq 0$

$x \neq -1$

ΕΚΠ $x(x+1)$.

$$x(x+1)\frac{x-1}{x} - x(x+1)\frac{1}{x+1} + x(x+1)\frac{1}{x(x+1)} = 0 \Rightarrow$$

$\Rightarrow (x+1)(x-1) - x + 1 = 0 \Rightarrow$

$\Rightarrow x^2 - x + x - 1 - x + 1 = 0 \Rightarrow$

$\Rightarrow x^2 - x = 0 \Rightarrow$

$\Rightarrow x(x-1) = 0 \Rightarrow$

$x = 0$ ή $x - 1 = 0$

$x = 1$ ✓

Εξίσωση

$$3(x+3) - \lambda^2(1-x) = -3x(\lambda-1)$$

$$3x + 9 - \lambda^2 + \lambda^2 x = -3x\lambda + 3x$$

$$3x + \lambda^2 x + 3x\lambda - 3x = -9 + \lambda^2$$

$$(\lambda^2 + 3\lambda)x = \lambda^2 - 3^2$$

$$\boxed{\lambda(\lambda+3)x = (\lambda-3)(\lambda+3)}$$

Αν $\lambda = 0 \longrightarrow 0x = -9$ Αδύνατη

Αν $\lambda + 3 = 0 \Rightarrow \lambda = -3 \longrightarrow 0x = 0$ Ταυτότητα

Αν $\lambda \neq 0, \lambda \neq -3$ τότε

$$x = \frac{(\lambda-3)(\lambda+3)}{\lambda(\lambda+3)}$$

Εξίσωση

$$\lambda^2 x - \lambda^2 = 4x - 3\lambda + 2$$

~~$\lambda^2(x-1) = 4x - 3\lambda + 3x$~~

$$\lambda^2 x = 4x = \lambda^2 - 3\lambda + 2$$

$$x(\lambda^2 - 4) = \lambda^2 - 3\lambda + 2$$

$$x(\lambda - 2)(\lambda + 2) = \lambda^2 - 3\lambda + 2$$

$\Delta = b^2 - 4ac$

$\Delta = 9 - 8$

$\Delta = 1$

$a = 1$

$b = -3$

$\gamma = 2$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{3 \pm 1}{2}$$

$\frac{4}{2} = 2$

$\frac{2}{2} = 1$

$$\boxed{(\lambda - 2)(\lambda + 2)\, x = (\lambda - 2)(\lambda - 1)}$$

1η) Αν $\lambda - 2 = 0 \Rightarrow \lambda = 2$ $\quad 0x = 0$ Αόριστη

2η) Αν $\lambda + 2 = 0 \Rightarrow \lambda = -2$ $\quad 0x = 12$ ✓

3η) Αν $\lambda \neq 2, \lambda \neq -2$ τότε $\quad x = \frac{(\lambda - 2)(\lambda - 1)}{(\lambda - 2)(\lambda + 2)}$

ΕΠΙΛΟΓΗ

$\lambda^2(x-1) - 2x + 5\lambda = 2(x+3) \Rightarrow$

$\Rightarrow \lambda^2 x - \lambda^2 - 2x + 5\lambda = 2x + 6 \Rightarrow$

$\Rightarrow \lambda^2 x - 4x = 6 + \lambda^2 - 5\lambda \Rightarrow$

$\Rightarrow x(\lambda^2 - 4) = (\lambda - 2)(\lambda - 3) \Rightarrow$

$\Rightarrow \boxed{x(\lambda-2)(\lambda+2) = (\lambda-2)(\lambda-3)}$

$0x = 0 \Rightarrow$ ΑΟΡΙΣΤΗ

- Αν $\lambda = 2$
- Αν $\lambda = -2$

$0x = 20 \Rightarrow$ ΑΔΥΝΑΤΗ

- Αν $\lambda \neq 2, \lambda \neq -2$ τότε $x = \frac{(\lambda-2)(\lambda-3)}{(\lambda-2)(\lambda+2)} \Rightarrow$

$\Rightarrow x = \frac{\lambda-3}{\lambda+2}$

Εξίσωση

$$\lambda^3 x + 2 = (4x+1)\cdot\lambda \Rightarrow$$

$$\Rightarrow \lambda^3 x + 2 = 4\lambda x + \lambda \Rightarrow$$

$$\Rightarrow \lambda^3 x - 4\lambda x = -2 + \lambda \Rightarrow$$

$$\Rightarrow \lambda x(\lambda^2 - 4) = -2 + \lambda \Rightarrow$$

$$\Rightarrow \lambda x(\lambda-2)(\lambda+2) = -2 + \lambda$$

$$\boxed{\lambda(\lambda-2)(\lambda+1)\ x = \lambda - 2}$$

1. Αν $\lambda = 0 \Rightarrow 0x = -2$ Αδύνατη
2. Αν $\lambda = 1 \Rightarrow 0x = 0$ Αόριστη
3. Αν $\lambda = -2 \Rightarrow 0x = -4$ Αδύνατη

4. Αν $\lambda \neq 0, \lambda \neq 2, \lambda \neq -2 \Rightarrow$

$$\Rightarrow x = \frac{\lambda - 2}{\lambda(\lambda-2)(\lambda+2)} = \frac{1}{\lambda(\lambda+2)}$$

15.

$$3(\lambda+\mu)x - 8 = (x-1)(2\lambda+3\mu)$$

$$3\lambda x + 3\mu x - 8 = 2\lambda x + 3\mu x - 2\lambda - 3\mu$$

$$3\lambda x + 3\mu x - 2\lambda x - 3\mu x = 8 - 2\lambda - 3\mu$$

$$\boxed{\lambda x = 8 - 2\lambda - 3\mu}$$

1. Αν $\lambda = 0$ τότε $0x = 8 - 3\mu$

i) Αν $8 - 3\mu = 0 \Rightarrow 8 = 3\mu$

$\mu = \frac{8}{3}$

Αορισ.

ii). Αν $8 - 3\mu \neq 0 \Rightarrow \mu \neq \frac{8}{3}$

Αδυνατη.

2. Αν $\lambda \neq 0$

$$x = \frac{8 - 2\lambda - 3\mu}{\lambda}.$$

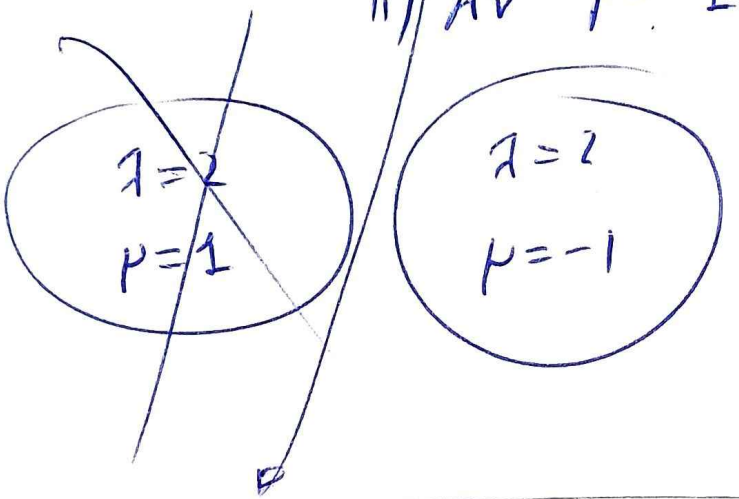
17. $(\lambda-2)x = \mu^2-1$. Τακτοποιώ

$\lambda(\lambda-2)x = \mu^2-\mu$ Αδιάστατη .

Αν $\lambda-2=0 \Rightarrow \lambda=2$ τότε $0x=\mu^2-1$

i) Αν $\mu=1$ τότε $0x=0$ Ταυτότητα

ii) Αν $\mu=-1$ τότε $0x=0$ Ταυτότητα



Αν $\lambda=0$ τότε $0x=\mu^2-\mu$

για να είναι αδύνατη $\mu^2-\mu\neq 0$

$\mu(\mu-1)\neq 0$

$\mu\neq 0 \quad \mu\neq 1$

(Τ Τόμπα Την πήρα)

 Av $\lambda = 2$ zoli $\alpha_x = \mu^2 - \mu$

preoli $\mu^2 - \mu \neq 0$

$\mu(\mu - 1) \neq 0$

$\mu \neq 0 \quad \mu \neq 1$.

$\lambda = 2$
$\mu \neq 0 \quad \mu \neq 1$

$\lambda = 2 \qquad \mu = -1$

10. (δ) $\dfrac{x^2-1}{x^2-x} = 1 + \dfrac{1}{x}$

$\dfrac{(x+1)(x-1)}{x(x-1)} = 1 + \dfrac{1}{x}$

$\longrightarrow$ $x \neq 0$ και $x-1 \neq 0 \Rightarrow x \neq 1$

$\dfrac{x+1}{x} = 1 + \dfrac{1}{x}$

$\dfrac{x+1}{x} = \dfrac{x+1}{x} \Rightarrow 0 = 0$

Δορωα!

$\dfrac{x+1}{x} - \dfrac{x+1}{x} = 0$

$\dfrac{x-x+1-1}{x} = 0$

$\dfrac{0}{x} = 0$

Λύση

$x \in \mathbb{R} - \{0, 1\}$.

12. (Β) $\lambda^2(x+1)=2(2-\lambda x)$.

Σχολή: ( )( ) x = ( )( )

$\lambda^2 x+\lambda^2=4-2\lambda x$.

$\lambda^2 x+2\lambda x=4-\lambda^2$

$(\lambda^2+2\lambda)x=4-\lambda^2$

$\boxed{\lambda(\lambda+2)x=(2-\lambda)(2+\lambda)}$

1. Αν $\lambda=0 \Rightarrow 0x=4$ Αδύνατη.
2. Αν $\lambda=-2 \Rightarrow 0x=0$ Αόριστη.
3. Αν $\lambda\neq 0, \lambda\neq -2 \Rightarrow x=\dfrac{(2-\lambda)(2+\lambda)}{\lambda(\lambda+2)}=\dfrac{\lambda-2}{\lambda}$.

(Δ) $\lambda^2(x-1)+\lambda=4x-2$.

$\lambda^2 x-\lambda^2+\lambda=4x-2$

$\lambda^2 x-4x=\lambda^2-\lambda-2$

$(\lambda^2-4)x=\lambda^2-\lambda-2$

$\boxed{(\lambda-2)(\lambda+2)x=(\lambda-2)(\lambda+1)}$

1. Αν $\lambda=2$ $0x=0$ Αόριστη
2. Αν $\lambda=-2$ $0x=4$ Αδύνατη.
3. Αν $\lambda\neq 2$ και $\lambda\neq -2$ $x=\dfrac{(\lambda-2)(\lambda+1)}{(\lambda-2)(\lambda+2)}$

Εα 167

①

(β) $2|x|-3=0$

$2|x|=3$

$|x|=\frac{3}{2}$

$x=\frac{3}{2}$ $x=-\frac{3}{2}$

(δ) $2(|x|-1)=|x|-2$.

$2|x|-2=|x|-2$

$2|x|-x-|x|+2=0$

$|x|=0$ $x=0$

(ϛ) $\frac{|x|-2}{3}+|x|=-1$.

$|x|-2+3|x|=-3$

$4|x|=-1$ Αδύνατη

4. (B) $|2x-3| = |3x-2|$

ili

$2x-3 = 3x-2$

$-x = 1$

$x = -1$

ili

$2x-3 = -3x+2$

$5x = 5$

$x = 1$

(8) $|x-4| - 3|x| = 0$

$|x-4| = 3|x|$

$\boxed{|x-4| = |3x|}$

ili $x-4 = 3x$

$-2x = 4$

$x = -2$

ili $x-4 = -3x$

$4x = 4$

$x = 1$

(02) $\dfrac{|2x-1|}{3} = \dfrac{|x-2|}{2}$

$\dfrac{2\,|2x-1|}{6} = \dfrac{3\,|x-2|}{6}$

$2|2x-1| = 3|x-2|$

$|4x-2| = |3x-6|$

ili $4x-2 = 3x-6$

$x = -4$

ili $4x-2 = -3x+6$

$7x = 8$

$x = \dfrac{8}{7}$

2. (β) $|5x-1|-4=0 \Rightarrow$

$\Rightarrow |5x-1|=4 \Rightarrow 5x-1=4$ ή $5x-1=-4$

$5x=5$ $\quad$ $5x=-3$

$x=1$ $\quad$ $x=-\frac{3}{5}$

(δ) $2(|x-3|-1)=3|x-3|-2 \Rightarrow$

$\Rightarrow 2|x-3|-2=3|x-3|-2 \Rightarrow$

$\Rightarrow 2|x-3|-3|x-3|=0 \Rightarrow$

$\Rightarrow -|x-3|=0 \Rightarrow -x+3=0 \Rightarrow -x=-3 \Rightarrow x=3$

(82) $\frac{|x-1|}{2} - \frac{|x-1|-1}{4} = 1 \Rightarrow$

$\Rightarrow 4\frac{|x-1|}{2} - 4.\frac{|x-1|-1}{4} = 1 \Rightarrow$

$\Rightarrow 2|x-1| - (|x-1|-1) = 4$

$2|x-1| - |x-1| + 1 = 4$

$|x-1| = 3$

$x-1 = 3$ ή $x-1 = -3$

$x = 4$

$x = -2$

3. (B) $\left|\frac{2x}{3}\right| - |-2x| = -1 \Rightarrow$

$\Rightarrow \frac{2|x|}{3} - 3|2x| = -1 \Rightarrow$

$\Rightarrow 2|x| - 6|x| = -3 \Rightarrow$

$\Rightarrow -4|x| = -3 \Rightarrow$

$\Rightarrow |x| = \frac{3}{4} \Rightarrow$ $x = \frac{3}{4}$ or $x = -\frac{3}{4}$

(B) $|6x-2| - |3-9x| = -1 \Rightarrow$

$\Rightarrow 2|3x-1| - 3|1-3x| = -1 \Rightarrow$

$\Rightarrow 2|3x-1| - 3|3x-1| = -1 \Rightarrow$

$\Rightarrow -|3x-1| = -1 \Rightarrow$

$\Rightarrow |3x-1| = 1 \Rightarrow$ $3x-1=1 \Rightarrow \frac{3x}{3} = \frac{2}{3} \Rightarrow x = \frac{2}{3}$ or $3x-1=-1 \Rightarrow \frac{3x}{3} = \frac{0}{3} \Rightarrow x = 0$

(52) $\frac{|3x-6|}{4} - 4\left|1-\frac{x}{2}\right| = 1 \Rightarrow$

$\Rightarrow |3x-6| - 4\left|1-\frac{x}{2}\right| = 4 \Rightarrow$

$\Rightarrow |3x-6| - |4-2x| = 4 \Rightarrow$

$\Rightarrow 3|x-2| - 2|2-x| = 4 \Rightarrow$

$\Rightarrow 3|x-2| - 2|x-2| = 4 \Rightarrow$

$\Rightarrow |x-2| = 4 \Rightarrow$ $\begin{cases} x-2=4 \Rightarrow x=6 \\ x-2=-4 \Rightarrow x=-2 \end{cases}$

Εποραο Μοδηρα

Σελ 156

1. αιγε
2. αγ
3. αγε7
4. α
5. αγε
6. αιβδε
7. αγ
9. αγε
10. αγ.
11.
12. αδ
13.
14.
16.

24. (β) $(3x-2)^2 = (3x-2)(x-1)$.

$$(3x-2)^2 - (3x-2)(x-1) = 0$$

$$(3x-2)(3x-2-x+1) = 0$$

$$(3x-2)(2x-1) = 0$$

ή $2x-1=0$

$3x-2=0$

$x = \frac{2}{3}$

$x = \frac{1}{2}$

(δ) $x(x-2) - (x+1)(2-x) = 0$.

$$x(x-2) + (x+1)(x-2) = 0$$

$$(x-2)(x+x+1) = 0$$

$$(x-2)(2x+1) = 0$$

ή $x-2=0$

$x = 2$

ή $2x+1=0$

$2x = -1$

$x = -\frac{1}{2}$

26. (B) $\dfrac{1}{1-\frac{1}{x}} - \dfrac{1}{x} = \dfrac{1}{x^2-x}$

$\longrightarrow 1 - \frac{1}{x} = 0 \iff 1 = \frac{1}{x} \iff \boxed{x = 1}$

$\longrightarrow \boxed{x = 0}$

$\longrightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \quad \boxed{x=0}$ ή $\boxed{x=1}$.

Πρπ $x \neq 0,\ x \neq 1$.

$$\frac{1}{\frac{x-1}{x}} - \frac{1}{x} = \frac{1}{x(x-1)}$$

$$\frac{x}{x-1} - \frac{1}{x} = \frac{1}{x(x-1)}$$

$x^2 - (x-1) = 1$

$x^2 - x + 1 = 1$

$x^2 - x = 0$

$x(x-1) = 0$

~~$x \neq 0$~~ ~~$x \neq 1$~~

Αδύνατη.

29. $\dfrac{x-2a}{x+2a} = \dfrac{x^2}{x^2-4a^2}$

$$\frac{x-2a}{x+2a} = \frac{x^2}{(x-2a)(x+2a)}$$

- $x+2a \neq 0$
- $(x-2a)(x+2a) \neq 0$

$\longrightarrow$ $x+2a=0 \Rightarrow \boxed{x=-2a}$

$\rightarrow (x-2a)(x+2a)=0 \Rightarrow x-2a=0$ ή $x+2a=0$

$x=2a$ $\qquad$ $x=-2a$

Πρ $x \neq -2a$, $x \neq 2a$.

$(x-2a)^2 = x^2$

$|x-2a| = |x|$

$x-2a = x$ ή $x-2a = -x$

Αδύνατη

$2x = 2a$

$x = a$

30. Θ. $\frac{x-3\alpha}{\beta} = \frac{x-3\beta}{\alpha}$ εχει λυση $\forall \alpha, \beta \in \mathbb{R}^*$

$$\alpha(x-3\alpha) = \beta(x-3\beta)$$

$$\alpha x - 3\alpha^2 = \beta x - 3\beta^2$$

$$\alpha x - \beta x = 3\alpha^2 - 3\beta^2$$

$$(\alpha-\beta)x = 3(\alpha-\beta)(\alpha+\beta)$$

1. Αν $\alpha=\beta$ τοτε $0x=0$ Αορισα

2. Αν $\alpha \neq \beta$ τοτε εχει μοναδικη λυση

$$x = \frac{3(\alpha-\beta)(\alpha+\beta)}{\alpha-\beta}$$

$$x = 3(\alpha+\beta)$$

Σε καθε περιπτωση εχω λυση.

25. (B) $8x^3 + (x-2)^3 - (3x-2)^3 = 0$.

$(2x)^3 + (x-2)^3 + (2-3x)^3 = 0$

$2x + x - 2 + 2 - 3x = 0$

$0 = 0$

$3(2x)(x-2)(2-3x) = 0$

$6x(x-2)(2-3x) = 0$

$x = 0$

$x - 2 = 0$

$x = 2$

$2 - 3x = 0$

$-3x = -2$

$x = \frac{2}{3}$

28. Βρείτε όλους τους πραγματικούς αριθμούς λ για τους οποίους η εξίσωση

$\lambda(x-\lambda) = x-1$ να έχει μοναδική λύση το 1.

$$\lambda(x-\lambda) = x-1$$
$$\lambda x - \lambda^2 = x-1$$
$$\lambda x - x = \lambda^2 - 1$$
$$\boxed{(\lambda-1)x = \lambda^2-1}$$

πρέπει $\lambda - 1 \neq 0 \Rightarrow \underline{\lambda \neq 1}$ ώστε να έχει μοναδική λύση

άρα $\dfrac{(\lambda-1)x}{(\lambda-1)} = \dfrac{(\lambda-1)(\lambda+1)}{(\lambda-1)}$

$$\boxed{x = \lambda+1}$$
$$-\lambda = 1-x$$
$$-\lambda = 1-1$$
$$\underline{\lambda = 0}$$

31. Αν η εξίσωση $(\lambda+1)x=\mu-2$ έχει τουλάχιστον δύο λύσεις και η εξίσωση $(\mu a^2-\lambda)x=\lambda-1$ έχει μοναδική λύση $\forall a \in \mathbb{R}$.

Αν $(\lambda+1)x=\mu-2$ αόριστη

$\lambda=-1 \quad \mu=2$

$(2a^2+1)x=-2$.

$2a^2+1 \neq 0$

$x=\dfrac{-2}{2a^2+1}$ μοναδική λύση !.

4. (α) $2\alpha x - 3(x-\alpha) = x(\alpha-1)$ εχει ρίζα το -2.

$$2\alpha(-2) - 3(-2-\alpha) = -2(\alpha-1)$$

$$-4\alpha + 6 + 3\alpha = -2\alpha + 2$$

$$-\alpha + 6 = -2\alpha + 2$$

$$\boxed{\alpha = -4}$$

12. (α) $\lambda^2(x-1) = 5(5x-\lambda)$.

Συνολ: $(\ )(\ )x = (\ )(\ )$

$$\lambda^2 x - \lambda^2 = 25x - 5\lambda$$

$$\lambda^2 x - 25x = \lambda^2 - 5\lambda$$

$$(\lambda^2 - 25)x = \lambda^2 - 5\lambda$$

$$\boxed{(\lambda-5)(\lambda+5)x = \lambda(\lambda-5)}$$

αν $\lambda \neq 5$, $\lambda \neq -5$ τοτε $x = \frac{\lambda(\lambda-5)}{(\lambda+5)(\lambda-5)}$

$$x = \frac{\lambda}{\lambda+5}.$$

1. Αν $\lambda = 5$ τοτε $0x = 0$ Ταυτοτητα
2. Αν $\lambda = -5$ τοτε $0x = 50$ Αδυνατη

12. (β) $\lambda^3(x-1) = 4\lambda x - 8 \Rightarrow$

$\Rightarrow \lambda^3 x - \lambda^3 = 4\lambda x - 8 \Rightarrow$

$\Rightarrow \lambda^3 x - 4\lambda x = \lambda^3 - 8 \Rightarrow$

$\Rightarrow \lambda x(\lambda^2 - 4) = \lambda^3 - 8 \Rightarrow$

$\Rightarrow \lambda x(\lambda-2)(\lambda+2) = \lambda^3 - 2^3 \Rightarrow$

$$\boxed{\lambda(\lambda-2)(\lambda+2)\,x = (\lambda-2)(\lambda^2+2\lambda+4)}$$

$(-2-2)(-2+2) = 4-4 \neq 4-4$

1. Αν $\lambda = 0$, τότε $0x = -8$ αδύνατη.

2. Αν $\lambda = 2$, τότε $0x = 0$ Αόριστη ή ταυτότητα

3. Αν $\lambda = -2$, τότε $0x = -16$ αδύνατη.

4. Αν $\lambda \neq 2$, $\lambda \neq -2$ $x = \dfrac{(\lambda-2)(\lambda^2+2\lambda+4)}{(\lambda-2)(\lambda+2)\cdot\lambda} = \dfrac{(\lambda^2+2\lambda+4)}{(\lambda+2)\cdot\lambda}$

24. (σε) $(x+1)^2 + x^2 - 1 = 0 \Rightarrow$

$\Rightarrow (x+1)^2 + (x-1)(x+1) = 0 \Rightarrow$

$\Rightarrow (x+1)(x+1+x-1) = 0 \Rightarrow$

$\Rightarrow 2x(x+1) = 0 \Rightarrow$

$\Rightarrow 2x = 0 \Rightarrow$ ή $x+1 = 0 \Rightarrow$

$\Rightarrow \frac{2x}{2} = \frac{0}{2}$ $\Rightarrow \boxed{x = -1}$

$\Rightarrow \boxed{x = 0}$

(η) $x^3 - x^2 - x + 1 = 0 . \Rightarrow$

$\Rightarrow x^2(x-1) - (x-1) = 0 \Rightarrow$

$\Rightarrow (x-1)(x^2-1) = 0 \Rightarrow$

$\Rightarrow (x-1)(x-1)(x+1) = 0 \Rightarrow$

$\Rightarrow (x-1)^2(x+1) = 0 \Rightarrow$

$\Rightarrow x-1 = 0 \Rightarrow$ ή $x+1 = 0 \Rightarrow$

$\Rightarrow \boxed{x = 1}$ $\Rightarrow \boxed{x = -1}$

(i) $x^3 - 7x + 6 = 0 \Rightarrow$

$\Rightarrow x^3 - x - 6x + 6 = 0 \Rightarrow$

$\Rightarrow x(x^2 - 1) - 6(x - 1) = 0 \Rightarrow$

$\Rightarrow x(x-1)(x+1) - 6(x-1) = 0 \Rightarrow$

$\Rightarrow (x-1)(x(x+1) - 6) = 0 \Rightarrow$

$\Rightarrow (x-1)(x^2 + x - 6) = 0 \Rightarrow$

$\Rightarrow (x-1)(x-2)(x+3) = 0 \Rightarrow$

$\Rightarrow x - 1 = 0 \Rightarrow$ or $x - 2 = 0 \Rightarrow$ or $x + 3 = 0 \Rightarrow$

$\Rightarrow \boxed{x = 1}$ $\Rightarrow \boxed{x = 2}$ $\Rightarrow \boxed{x = -3}$

7. Ⓑ $|x-2| - 3|2x-1| = x$

αν $x-2 \geq 0$ και $2x-1 \geq 0$ τότε

$$x-2-3(2x-1) = x$$

$$x-2-6x+3 = x$$

$$-6x = -1$$

$$6x = 1$$

$$x = \frac{1}{6}$$

αν $x-2 \geq 0$ και $2x-1 < 0$ τότε

$$x-2+3(2x-1) = x$$

$$-2+6x-3 = 0$$

$$6x = 5$$

$$x = \frac{5}{6}$$

αν $x-2 < 0$ και $2x-1 \geq 0$ τότε

$$-x+2-3(2x-1) = x$$

$$-2x+2-6x+3 = 0$$

$$-8x = -5$$

$$x = \frac{5}{8}$$ ✓

αν $x-2 < 0$ και $2x-1 < 0$ τότε

$$-x+2+3(2x-1) = x$$

$$-2x+2+6x-3 = 0$$

$$4x-1 = 0$$

$$x = \frac{1}{4}$$ ✓

(δ). $d(x,1) - d(x,-2) = x$

$|x-1| - |x+2| = x$

x		-2		1	
x-1	−		−	0	+
x+2	−	0	+		+

1. Αν $x < -2$ τότε

$-x+1-(-x-2) = x$

$-x+1+x+2 = x$

$-x = -3$ ~~$x = 3$~~

2. Αν $-2 \leq x \leq 1$ τότε.

$-x+1-(x+2) = x$

$-x+1-x-2 = x$

$-1 = 3x$

$x = -\frac{1}{3}$ ✓

3. Αν $x > 1$ τότε

$x-1-x-2 = x$ ~~$x = -3$~~

6. (B) $d(3x, -1) = 3 + 5x \Rightarrow$ Αν $3 + 5x \geq 0$

$\Rightarrow |3x + 4| = 3 + 5x \Rightarrow$ ή $3x + 4 = -3 - 5x \Rightarrow$

$\Rightarrow 3x + 4 = 3 + 5x \Rightarrow$ $\Rightarrow \frac{8x}{8} = \frac{-2}{8} \Rightarrow$

$\Rightarrow \frac{-2}{2} = \frac{2x}{2} \Rightarrow$ $\Rightarrow \boxed{x = -\frac{1}{4}}$ ✓

$\Rightarrow \boxed{x = -1}$ ✗

(Γ) $d(x, 1) = x + 2 \Rightarrow$

$\Rightarrow |x - 4| = x + 2$

Αν $x + 2 \geq 0 \Rightarrow x \geq -2$ τότε

$x - 1 = x + 2$ ✗ ή $x - 1 = -x - 2$

Αδύνατη

$2x = 1$ ✓

$\boxed{x = \frac{1}{2}}$

5. (B) $\sqrt{x^2-6x+9} = 1 \Rightarrow$

$\Rightarrow \sqrt{(x-3)^2} = 1 \Rightarrow$

$\Rightarrow |x-3| = 1 \Rightarrow \quad x-3=1 \quad$ ή $\quad x-3=-1$

$x=4 \qquad\qquad x=2$

(8) $\sqrt{25x^2-10x+1} - |3x-5| = 0$

$\sqrt{(5x-1)^2} = |3x-5|$

$|5x-1| = |3x-5|$

$5x-1 = 3x-5 \quad$ ή $\quad 5x-1 = -3x+5$

$2x = -4 \qquad\qquad 8x = 6$

$x = -2 \qquad\qquad x = \frac{6}{8}$.

(29) $\sqrt{4x^2+1} = |2x-1|. \Rightarrow$

$\Rightarrow |4x+1| = |2x-1| \Rightarrow$

$\Rightarrow 4x+1 = 2x-1 \quad$ ή $\quad 4x+1 = -2x+1$

$2x = -2 \qquad\qquad 6x = 0$

$x = -1 \qquad\qquad x = 0$

$x = 0$

Επαναληψη Μαθημα

Σελ 160

(24) αγεπθ.

(25) αι

(26) αι

(27)

(30) αβ.

Σελ 167

(1) αγε

(2) αγε

(3) αγε

(4) αγε

(5) αγε.

8. (β) $d(x,5) - 5 = -x$.

$$|x-5| - 5 = -x$$

$$|x-5| = 5-x$$

αν $x-5 \geq 0$ τότε

$$x-5 = 5-x$$

$$2x = 10$$

$$x = 1$$

αν $x-5 < 0$ τότε $\Rightarrow x < 5$

$$-x+5 = 5-x$$

$$0x = 0$$

Αόριστη η Ταυτότητα

Λύση

$x \in [-\infty, 5]$.

11. (δ) $\big||2x-1|-5\big| = 5 \Rightarrow$

$|2x-1|-5 = 5$ ή $|2x-1|-5 = -5$

$|2x-1| = 10$ $\quad$ $|2x-1| = 0$

$2x-1 = 10$ ή $2x-1 = -10$ $\quad$ $2x = 1$

$2x = 11$ $\quad$ $2x = -9$ $\quad$ $x = \frac{1}{2}$ ✓

$x = \frac{11}{2}$ $\quad$ $x = -\frac{9}{2}$

(ε) $\big||x|-2\big| - 3\big| = 1 \Rightarrow$

$\Rightarrow |x|-2|-3 = 1$ ή $|x|-2|-3 = -1$

$|x|-2| = 4$ $\quad$ $|x|-2| = 2$

$|x|-2 = 2$ ή $|x|-2 = -2$

$|x| = 4$ $\quad$ $|x| = 0$

$x = 4$ $\quad$ $x = -4$ $\quad$ $x = 0$

$|x|-2 = 4$ ή $|x|-2 = -4$

$|x| = 6$ $\quad$ $|x| = -2$

$x = 6$ $\quad$ $x = -6$ $\quad$ Αδύνατη

17. ⓐ $||x-1|-2| = x-3$.

Πρεπει $x-3 \geq 0 \Rightarrow x \geq 3$ ωστε

$|x-1|-2 = x-3$ ή $|x-1|-2 = -x+3$

$|x-1| = x-1$ $\quad\quad$ $|x-1| = 5-x$

πρεπει $x-1 \geq 0$ $\quad\quad$ πρεπει $5-x \geq 0$

$x \geq 1$ $\quad\quad$ $x \leq 5$

$x-1 = x-1$ ή $x-1 = -x+1$ $\quad\quad$ $x-1 = 5-x$ ή $x-1 = -5+x$

Αορίστη $\quad\quad$ $2x = 2$ $\quad\quad$ $2x = 6$ $\quad\quad$ Αδύνατη

Δεκτή όλη $\quad\quad$ $x = 1$ $\quad\quad$ $x = 3$

τα $x \in [1, +\infty)$

Λύση

οριμ $x \geq 3$

Δεκτών

$x \in [3, +\infty)$ $\quad\quad$ $x \in [3, +\infty)$

(3) $|\,|2x-1|-1\,| = x-1$

Πρεπι $x-1 \geq 0 \Rightarrow x \geq 1$

$|2x-1|-1 = x-1$ ή $|2x-1|-1 = 1-x$

$|2x-1| = x$ $|2x-1| = 2-x$

Πρέπ $x \geq 0$ πρέπ $2-x \geq 0$

$2 \geq x$

$2x-1 = x$ ή $2x-1 = -x$ $2x-1 = 2-x$ ή $2x-1 = x-2$

$x = 1$ ✓ $3x = 1$ $3x = 3$ $x = -1$

$x = \frac{1}{3}$ ✗ $x = 1$ ✓

$x = 1$

18. (α) $|x-2|+1=\alpha$.

$$|x-2|=\alpha-1.$$

• Αν $\alpha-1>0$

$x-2=\alpha-1$ ή $x-2=-\alpha+1$

$x=\alpha-1$ $x=3-\alpha$

2 ρίζες

• Αν $\alpha-1=0$ τότε $|x-2|=0$

$\alpha=1$ $x=2$

1 ρίζα

• Αν $\alpha-1<0$

καμία ρίζα

(13) $|x-1|+2=a$

$|x-1| = a-2$

αν $a-2>0 \Rightarrow a>2$ τότε

ή $x-1=a-2$ ή $x-1=-a+2$

$x=a-1$ $x=3-a$

2 είδες

αν $a-2=0 \Rightarrow a=2$ τότε

$|x-1|=0 \Rightarrow x-1=0 \Rightarrow x=1$

1 είδα

αν $a-2<0 \Rightarrow a<2$ τότε

$|x-1| = a-2$ άτοπο ΑΣΙΝΑΤΟ

οπότε 3 είδες.

16. $|x|+2 = \dfrac{2}{\sqrt{x^2+1}}$

$$|x| = \frac{2}{\sqrt{x^2+1}} - 2$$

$$|x| = \frac{2 - 2\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$|x| = \frac{2\left(1 - \sqrt{x^2+1}\right)}{\sqrt{x^2+1}}$$

$$|x| - \frac{2\left(1 - \sqrt{x^2+1}\right)}{\sqrt{x^2+1}} = 0$$

$$|x| + \frac{2\left(\sqrt{x^2+1} - 1\right)}{\sqrt{x^2+1}} = 0$$

- $x^2 \geq 0 \Rightarrow x^2+1 \geq 1 \Rightarrow \sqrt{x^2+1} \geq \sqrt{1} \Rightarrow \sqrt{x^2+1} \geq 1$

$\sqrt{x^2+1} - 1 \geq 0$

- $\sqrt{x^2+1} - 1 = 0$

$\sqrt{x^2+1} = 1$

$x^2+1 = 1$

$x^2 = 0$

$x = 0$

- $|x| = 0$

$x = 0$

$x = 0$

15. ⓐ $|x^2-2x|+|x^2-4|=0 \Rightarrow$

$\Rightarrow |x(x-2)|+|(x-2)(x+2)|=0 \Rightarrow$

$\Rightarrow |x||x-2|+|x-2||x+2|=0$

$|x-2|\left(|x|+|x+2|\right)=0$

$|x-2|=0$ и $|x|+|x+2|=0$.

$x=2$ ✓

$\begin{cases} x=0 \\ x+2=0 \Rightarrow x=-2 \end{cases}$ А дальше!

ⓑ. $|x-1|+(x-y-2)^2+\sqrt{x+2y-2}=0 \Rightarrow$

$\Rightarrow$

$\begin{cases} |x-1|=0 \Rightarrow x=1 \\ (x-y-2)^2=0 \Rightarrow x-y-2=0 \Rightarrow x-y=2. \\ \sqrt{x+2y-2}=0 \end{cases}$

$1-y=2$

$-1=y$

$1+2y-2=0$

$1-2-2=0$ $z=-1$

14. (σε) $\sqrt{x^2-4x+4} = (x-2)^2 \Rightarrow$

$\Rightarrow \sqrt{(x-2)^2} = (x-2)^2 \Rightarrow$

$\Rightarrow |x-2| = (x-2)^2 \Rightarrow |x-2| = (|x-2|)^2 \Rightarrow$

$\Rightarrow |x-2| - |x-2|^2 = 0$

$|x-2|(1-|x-2|) = 0$

$|x-2| = 0$ ή $1-|x-2| = 0$

$x = 2$ $\quad 1 = |x-2|$

$x-2 = 1$ ή $x-2 = -1$

$x = 3$ $\quad x = 1$

13. (B) $|x^2-1|-|x-1|=0 \Rightarrow$

$\Rightarrow |x^2-1|=|x-1| \Rightarrow$

$\Rightarrow x^2-1=x-1 \Rightarrow$ ή $x^2-1=1-x \Rightarrow$

$\Rightarrow x^2-x=0 \Rightarrow$ $\Rightarrow x^2+x-2=0 \Rightarrow$

$\Rightarrow x(x-1)=0 \Rightarrow$ $\Rightarrow (x-1)(x+2)=0 \Rightarrow$

$\Rightarrow \boxed{x=0}$ ή $x-1=0 \Rightarrow$ $\Rightarrow x-1=0 \Rightarrow$ ή $x+2=0 \Rightarrow$

$\Rightarrow \boxed{x=1}$ $\boxed{x=1}$ $\Rightarrow \boxed{x=-2}$

(Δ) $|3x^2-2x|=|4-6x| \Rightarrow$

$\Rightarrow$ $3x^2-2x=4-6x \Rightarrow$ ή $3x^2-2x=6x-4 \Rightarrow$

$\Rightarrow 3x^2+4x-4=0 \Rightarrow$ $\Rightarrow 3x^2-8x+4=0 \Rightarrow$

$\Rightarrow \Delta=16+48=64$ $\Rightarrow \Delta=64-48=16$

$\Rightarrow x=\dfrac{-4\pm 8}{6}$ $x=\dfrac{8\pm 4}{6} \Rightarrow \begin{cases} x=\frac{12}{6}=2 \\ x=\frac{4}{6}=\frac{2}{3} \end{cases}$

$\boxed{x=\frac{2}{3}}$

$\boxed{x=-2}$

14. (B) $(x-1)^2 - 3|x-1| = 0$

$$|x-1|^2 - 3|x-1| = 0$$

$$|x-1|\left(|x-1|-3\right) = 0$$

$|x-1| = 0$ atau $|x-1| - 3 = 0$

$x - 1 = 0$ $\qquad$ $|x-1| = 3$

$x = 1$ $\qquad$ $x-1 = 3$ atau $x-1 = -3$

$x = 4$ $\qquad$ $x = -2$

(E) $x^2 - 6|x| + 9 = 0$

$|x|^2 - 6|x| + |3|^2 = 0$

$(|x|-3)^2 = 0$

$|x| - 3 = 0$

$|x| = 3$

$x = 3$ $\qquad$ $x = -3$

12. (B) $\frac{|2x+1|}{|1-2x|} = 3 \Rightarrow$

$\Rightarrow |2x+1| = 3|1-2x| \Rightarrow$

$\Rightarrow 2x+1 = 3-6x \Rightarrow$ ή $2x+1 = 6x-3 \Rightarrow$

$\Rightarrow \frac{8x}{8} = \frac{2}{8} \Rightarrow x = \frac{1}{4}$ $\Rightarrow 4x = 4 \Rightarrow$

$\Rightarrow \boxed{x=1}$

14. (B) $(x-1)^2 - 3|x-1| = 0.$

$3|x-1| = (x-1)^2$

ή $3(x-1) = (x-1)^2$

$3(x-1) - (x-1)^2 = 0$

$(x-1)(3-(x-1)) = 0$

$(x-1)(3-x+1) = 0$

$(x-1)(4-x) = 0$

ή $\boxed{x=1}$ ή $\boxed{x=4}$

ή $3(x-1) = -(x-1)^2$

$3(x-1) + (x-1)^2 = 0$

$(x-1)(3+x-1) = 0$

$(x-1)(2+x) = 0$

ή $\boxed{x=1}$ ή $\boxed{x=-2}$

II. ⑦ $\left|\,|\,|x-1|-2\,|-1\,\right| = 2$.

1) $||x-1|-2|-1 = 2$

$||x-1|-2| = 3$

2) $|x-1|-2 = 3$

$|x-1| = 5$

3) $x-1 = 5$

$x = 6$

3) $x-1 = -5$

$x = -4$

1) $||x-1|-2|-1 = -2$

$||x-1|-2| = -1$

Abszolút érték nem lehet negatív.

2) $|x-1|-2 = -3$

$|x-1| = -1$

3) Abszolút érték nem lehet negatív

~~$x = 1$~~

II. ⑦ $\left|\,|\,|x-1|-2\,|-1\,\right| = 2$.

$|\,|x-1|-2\,|-1 = 2$

$|\,|x-1|-2\,| = 3$

$|x-1|-2 = 3$

$|x-1| = 5$

$x-1 = 5$

$x = 6$

$x-1 = -5$

$x = -4$

$|x-1|-2 = -3$

$|x-1| = -1$

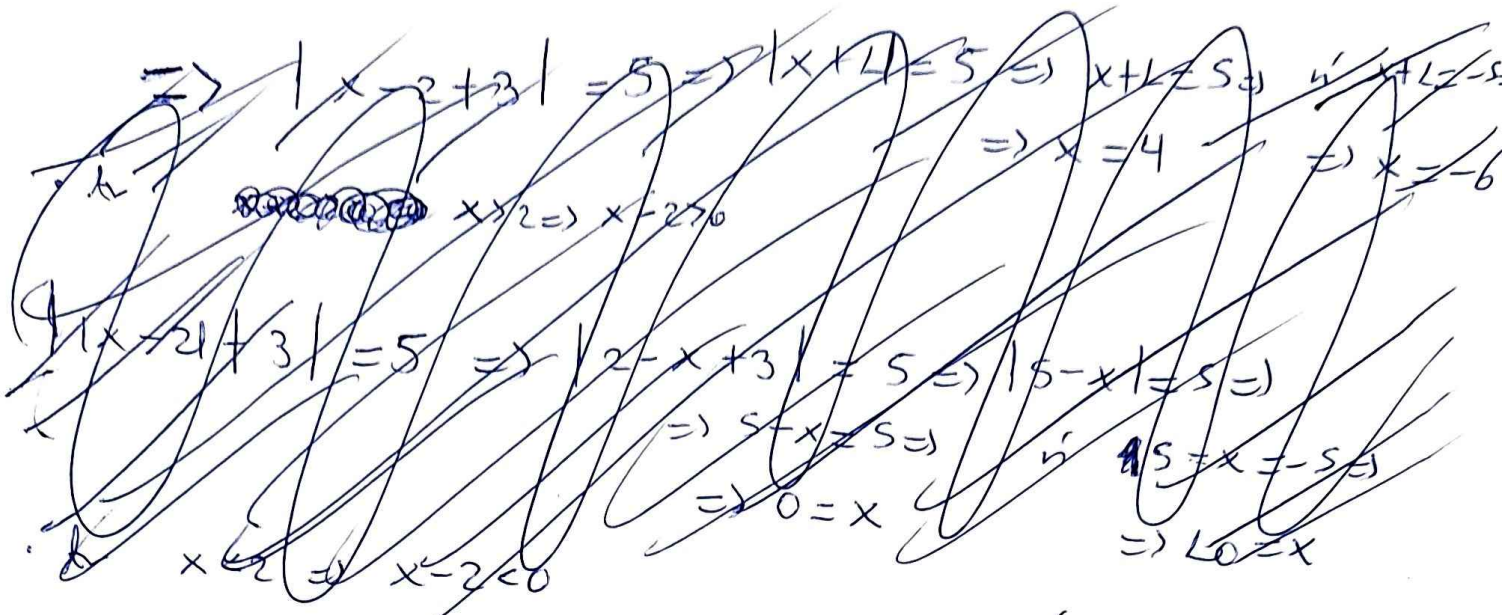
Absolut ayni $|x-1| \neq$ manfiy son.

$|\,|x-1|-2\,|-1 = -2$

$|\,|x-1|-2\,| = -1$

Absolut ayni $|\,|x-1|-2\,| \neq$ manfiy son.

10. (B) . $\left||x-2|+3\right| = 5$. $\Rightarrow$



$\Rightarrow |x-2|+3 = 5 \Rightarrow |x-2| = 2 \Rightarrow x-2 = 2 \Rightarrow$ ή $x-2 = -2 \Rightarrow$

$\Rightarrow x = 4$ $\Rightarrow x = 0$

(δ) $\left||x| - x\right| + \left||x| + x\right| = 2$.

$|x| - x + |x| + x = 2 \Rightarrow 2|x| = 2 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$

- $|x| \geq x \Rightarrow |x| - x \geq 0$
- $|x| \geq -x \Rightarrow |x| + x \geq 0$.

11. (B) $|3|x| - 2| = 2$.

αν $x \geq 0$ τότε

η' $3x - 2 = 2$

$3x = 4$

$x = \frac{4}{3}$

η' $3x - 2 = -2$

$3x = 0$

$x = 0$

αν $x < 0$ τότε

η' $-3x - 2 = 2$

$-3x = 4$

$x = -\frac{4}{3}$

η' $-3x - 2 = -2$

$-3x = 0$

$x = 0$

οι 4 τροφή

(3) $|3|x|-2| = 2$

Β' τρόπος

$3|x|-2 = 2$ ή $3|x|-2 = -2$

$3|x| = 4$ $3|x| = 0$

$|x| = \frac{4}{3}$ $x = 0$

$x = \frac{4}{3}$ ή $x = -\frac{4}{3}$

9. $d(2x,3) = 3-2x$

(α) Νδο $x \le \frac{3}{2}$

$|\overset{(-)}{2x-3}| = 3-2x \Rightarrow 2x-3 < 0$

$2x < 3$

$x < \frac{3}{2}$

(β) Αν $x \le \frac{3}{2}$ νδο $K = |\overset{(-)}{2x-3}| - 2|\overset{(+)}{3-x}|$

είναι ανεξάρτητη του x.

- $x \le \frac{3}{2} \Rightarrow 2x \le 3 \Rightarrow \boxed{2x-3 \le 0}$

- $x \le \frac{3}{2} \Rightarrow -x \ge -\frac{3}{2} \Rightarrow 3-x \ge 3-\frac{3}{2}$

$3-x \ge \frac{3}{2}$

$K = -2x+3-2(3-x) = -2x+3-6+2x = -3$

30. (α) $\alpha(x-\alpha) = \beta(x-\beta)$

$$\alpha x - \alpha^2 = \beta x - \beta^2$$

$$\alpha x - \beta x = \alpha^2 - \beta^2$$

$$x(\alpha-\beta) = (\alpha-\beta)(\alpha+\beta)$$

1. Αν $\alpha=\beta$ τότε $0x = 0$

2. Αν $\alpha \neq \beta \neq 0$ τότε $x = \dfrac{\beta^2+\alpha^2}{\alpha-\beta}$.

Έχει πολλές λύσεις

~~Στην οπ $\alpha=\beta$ και $\alpha \neq 0$ τότε~~

~~Αδύνατη.~~

(B) $(x+2\alpha)^2 - (x-2\beta)^2 = 8\alpha(\alpha+\beta)$

$x^2+4\alpha x+4\alpha^2-(x^2-4\beta x+4\beta^2) = 8\alpha^2+8\alpha\beta$

$x^2+4\alpha x+4\alpha^2-x^2+4\beta x-4\beta^2 = 8\alpha^2+8\alpha\beta$

$(4\alpha+4\beta)x = 4\alpha^2+4\beta^2+8\alpha\beta.$

$(\alpha+\beta)x = \alpha^2+2\alpha\beta+\beta^2$

$(\alpha+\beta)x = (\alpha+\beta)^2$

1. Av $\alpha+\beta=0 \Rightarrow \alpha=-\beta$ ωzc $0x=0$

2. Av $\alpha+\beta\neq 0$ ωzc $x=\alpha+\beta$.

$\Leftarrow$ xu λύση $\forall\alpha,\beta\in\mathbb{R}$

27. $\lambda(\lambda-1)x = 2-2\lambda$

$$\lambda(\lambda-1)x = -2(\lambda-1)$$

1. Αν $\lambda=0$ τότε $0x=2$ Αδύνατη

2. Αν $\lambda=1$ τότε $0x=0$ Αόριστη

3. Αν $\lambda\neq 0, \lambda\neq 1$ τότε

$$x = \frac{-2(\lambda-1)}{\lambda(\lambda-1)}$$

$$x = -\frac{2}{\lambda}$$

(β) Αν $\lambda=-1$ έχει μοναδική ρίζα το 2.

(α) Για να έχει ρίζα το 2 πρέπει

$$2\lambda(\lambda-1) - 2 + 2\lambda = 0$$
$$2\lambda(\lambda-1) + 2(\lambda-1) = 0$$
$$2(\lambda-1)(\lambda+1) = 0$$

$\lambda=1$, $\lambda=-1$

26. ⓐ $\dfrac{x-2}{1-\frac{2}{x}} = -1$

При $x \neq 0$ и $1 - \frac{2}{x} \neq 0$

$1 \neq \frac{2}{x}$

$x \neq 2$

$x - 2 = -\left(1 - \frac{2}{x}\right)$

$x - 2 = -1 + \frac{2}{x}$

$x^2 - 2x = -x + 2$

$x^2 - x - 2 = 0.$

~~$x = 2$~~ $x = -1$ ✓

24. ⑤ $(x^2-9)(x-1) = (x^2-1)(x-3)$

$(x-3)(x+3)(x-1) - (x-1)(x+1)(x-3) = 0$

$(x-3)(x-1)(x+3-x-1) = 0$

$(x-3)(x-1)\cdot 2 = 0$

$x-3=0$ 또는 $x-1=0$

$x=3$ $x=1$

① $x(x-1)^2 = x^2-2x+1$

$x(x-1)^2 = (x-1)^2$

$x(x-1)^2 - (x-1)^2 = 0$

$(x-1)^2(x-1) = 0$

$(x-1)^3 = 0$ $\Rightarrow x-1=0$ $x=1$

(8) $x^3 - 3x^2 = (2x-1)(x-3)$

$x^2(x-3) - (2x-1)(x-3) = 0$

$(x-3)(x^2-2x+1) = 0$

$(x-3)(x-1)^2 = 0$

$x-3=0$ or $(x-1)^2 = 0$

$x=3$ $x=1$

Εποχαο Μαθημ

Σελ 167

(1) αγε
(2) αγε
(3) αγε
(4) αγε
(5) αγε.

} παλιά.

Σελ 168

(6) αγ
(7) αγ.
(8) αβ
(10) αγ
(11) αγε
(12) α
(13) αγ
(14) αγε.