31 $f(x) = x^5 + 2x - 2$

(α) • $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$

• $x_1 < x_2 \Rightarrow 2x_1 - 2 < 2x_2 - 2$

$\Big\}$ (+) $f\nearrow$ άρα f 1-1

άρα αντιστρ.

→ $f^{-1}(x) = x \Leftrightarrow f(x) = x \Leftrightarrow x^5 + 2x - 2 = x$

$\underbrace{x^5 + x - 2}_{\varphi(x)} = 0$

$\varphi(x) = 0$

$\varphi(x) = \varphi(1)$

φ 1-1

$x = 1$

• $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$ (+).

• $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2$

$\varphi\nearrow \Rightarrow \varphi$ 1-1

(β). $f^{-1}(x) = 0$

$f(f^{-1}(x)) = f(0)$

$x = -2$.

x		-2	
f^{-1}	$-$ $\nearrow$	0	$+$ $\nearrow$

Η $f(x)$ και $f^{-1}(x)$ έχουν την ίδια μονοτονία πάντα!

Έστω $f^{-1}(x_1) < f^{-1}(x_2)$

$\overset{f\nearrow}{\Rightarrow}$

$f(f^{-1}(x_1)) < f(f^{-1}(x_2)) \Rightarrow x_1 < x_2 \Rightarrow f^{-1}\nearrow$

$x < -2 \Rightarrow f^{-1}(x) < f^{-1}(-2) \Rightarrow f^{-1}(x) < 0$

$x > -2 \Rightarrow f^{-1}(x) > f^{-1}(-2) \Rightarrow f^{-1}(x) > 0$

(γ). $\left(f^{-1}(x)\right)^5 + 2f^{-1}(x) = 0$

$$\underbrace{\left(f^{-1}(x)\right)^5 + 2f^{-1}(x) - 2} = -2$$

$$f\left(f^{-1}(x)\right) = -2$$

$$\underline{\underline{x = -2}}$$

(δ). $f^{-1}(x-2) + x^5 < -2x$

$$f^{-1}(x-2) + x^5 + 2x < 0$$

$$f^{-1}(x-2) + x^5 + 2x - 2 < -2$$

$$f^{-1}(x-2) + f(x) + 2 < 0.$$

Θεωρώ $\varphi(x) = f^{-1}(x-2) + f(x) + 2$

• $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2 \Rightarrow f^{-1}(x_1 - 2) < f^{-1}(x_2 - 2)$

(+)

• $x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \rightarrow f(x_1) + 2 < f(x_2) + 2$

$\varphi(x_1) < \varphi(x_2)$.

$\varphi(x) \nearrow$

Άρα $f^{-1}(x-2) + f(x) + 2 < 0$

$\varphi(x) < 0$

$\varphi(x) < \varphi(0)$

$\varphi \nearrow$

$x < 0$.

$\varphi(0) = f^{-1}(-2) + f(0) + 2 = 0 - 2 + 2 = 0$.

$f(0) = -2$

$f^{-1}(-2) = 0$

30 $f^2(x) + 1 = e^{x - f(x)}$ $\forall x \in \mathbb{R}$.

(a) , $f^2(x) + 1 = e^{x} \cdot e^{-f(x)}$

$f^2(x) + 1 = e^{x} \frac{1}{e^{f(x)}}$

$$\boxed{e^{f(x)}\left(f^2(x) + 1\right) = e^{x}}$$

- $f(x_1) = f(x_2) \Rightarrow \boxed{e^{f(x_1)} = e^{f(x_2)}}$
- $f(x_1) = f(x_2) \Rightarrow f^2(x_1) = f^2(x_2)$

$\odot$

$$\boxed{f^2(x_1) + 1 = f^2(x_2) + 1}$$

$$\underbrace{e^{f(x_1)}\left(f^2(x_1) + 1\right)} = \underbrace{e^{f(x_2)}\left(f^2(x_2) + 1\right)}$$

$$e^{x_1} = e^{x_2}$$

$x_1 = x_2$ f 1-1.

(β) Αφού f 1-1 αντιστρέφεται. $f(\mathbb{R})=\mathbb{R}$

$$e^{f(x)}\left(f^2(x)+1\right)=e^x$$

Έστω $f(x)=y$ και $x=f^{-1}(y)$

$$e^y\left(y^2+1\right)=e^{f^{-1}(y)}$$

$$\ln\left(e^y\cdot(y^2+1)\right)=f^{-1}(y)$$

$$f^{-1}(y)=\ln e^y+\ln(y^2+1)$$

$$f^{-1}(y)=y+\ln(y^2+1).$$

$$\boxed{f^{-1}(x)=x+\ln(x^2+1)}$$

$D_{f^{-1}}=\mathbb{R}.$

(γ) $f(x)=x \iff f^{-1}(x)=x \iff x+\ln(x^2+1)=x$

$\ln(x^2+1)=0 \iff x^2+1=1 \iff x^2=0 \iff \underline{x=0}$

(δ) $f(x^2+1)-f(\sigma\upsilon\nu x)=0$

f 1-1.

$x^2+1=\sigma\upsilon\nu x \to \underbrace{x^2}_{+}+\underbrace{1-\sigma\upsilon\nu x}_{+}=0$

$\downarrow$ $\underline{x=0}$ $\quad\downarrow$ $\underline{x=0}$

1

29 $f(x) = \dfrac{ax-1}{x-2}$ $D_f = \mathbb{R} - \{2\}$

$\longrightarrow f(3) = 2 \iff \dfrac{3a-1}{3-2} = 2 \iff 3a-1 = 2$

$3a = 3$

$\underline{\underline{a = 1}}$

$$\boxed{\begin{array}{l} f(x) = \dfrac{x-1}{x-2} \\ D_f = \mathbb{R} - \{2\} \end{array}}$$

$$\boxed{\begin{array}{l} g(x) = e^x + 1 \\ D_g = \mathbb{R} \end{array}}$$

a

$(f \circ g)(x) = f(g(x)) = \dfrac{e^x + 1 - 1}{e^x + 1 - 2} = \dfrac{e^x}{e^x - 1}$

$x \in D_g$ και $g(x) \in D_f$ $D_{f \circ g} = \mathbb{R}^*$

$x \in \mathbb{R}$ $e^x + 1 \neq 2$

$e^x \neq 1$

$\underline{\underline{x \neq 0}}$

(β) $h(x) = \dfrac{e^x}{e^x - 1}$

$h(x_1) = h(x_2) \Leftrightarrow \dfrac{e^{x_1}}{e^{x_1} - 1} = \dfrac{e^{x_2}}{e^{x_2} - 1}$

$$e^{x_1}(e^{x_2} - 1) = e^{x_2}(e^{x_1} - 1)$$

$$\cancel{e^{x_1} e^{x_2}} - e^{x_1} = \cancel{e^{x_2} e^{x_1}} - e^{x_2}$$

$$e^{x_1} = e^{x_2}$$

$$x_1 = x_2$$

h 1-1.

(γ) Αφού h 1-1 αντιστρέφεται.

Θέτω $h(x) = y \Rightarrow y = \dfrac{e^x}{e^x - 1}$

$y(e^x - 1) = e^x \Leftrightarrow ye^x - y = e^x$

$ye^x - e^x = y \Leftrightarrow e^x(y-1) = y$

$e^x = \dfrac{y}{y-1}$ $y \neq 1$ και $\dfrac{y}{y-1} > 0$

y		0		1	
y	−	0	+		+
$y-1$	−		−	0	+
$\frac{y}{y-1}$	+		−	‖	+

$x = \ln \dfrac{y}{y-1}$

$y \in (-\infty, 0) \cup (1, +\infty)$

$f^{-1}(x) = \ln\left(\dfrac{x}{x-1}\right)$

Τελος

$x \neq 0 \Rightarrow \ln\left(\frac{y}{y-1}\right) \neq 0$

$$\frac{y}{y-1} \neq 1$$

$$\cancel{y} \neq \cancel{y} - 1$$

$0 \neq -1$ που ισχύει.

(δ) $h^{-1}(x) > \ln 2$ $\qquad x \in (-\infty, 0) \cup (1, +\infty)$

$$\ln\left(\frac{x}{x-1}\right) > \ln 2$$

$$\frac{x}{x-1} > 2$$

$$\frac{x}{x-1} - 2 > 0 \Rightarrow \frac{x}{x-1} - \frac{2(x-1)}{x-1} > 0$$

$$\frac{x-2x+2}{x-1} > 0 \Leftrightarrow \frac{2-x}{x-1} > 0.$$

x		1		2	
$2-x$	+		+	0	−
$x-1$	−	0	+		+
$\frac{2-x}{x-1}$	−		+		−

$x \in (1, 2)$.

28 $f(x) = e^x + \ln(x+1) - 1$ $D_f = (-1, +\infty)$

(a) $\bullet\ x_1 < x_2 \Rightarrow \ln(x_1+1) - 1 < \ln(x_2+1) - 1$

$\bullet\ x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$

$\Big\}\ (+)$

$f \nearrow$

(B) $e^{x^2} + \ln(x^2+1) > 1$

$\underbrace{e^{x^2} + \ln(x^2+1) - 1}_{} > 0$

$f(x^2) > f(0)$

$f \nearrow$

$x^2 > 0$

$x \in \mathbb{R}^*$

γ) $e^{x^2} - e^{x+2} > \ln\dfrac{x+3}{x^2+1}$.

$$e^{x^2} - e^{x+2} > \ln(x+3) - \ln(x^2+1)$$

$$\underbrace{e^{x^2} + \ln(x^2+1) - 1}_{f(x^2)} > \underbrace{e^{x+2} + \ln(x+3) - 1}_{f(x+2)}.$$

$f \uparrow$

$x^2 > x+2 \Rightarrow x^2 - x - 2 > 0$

x	-1		2	
x^2-x-2	$+$ 0	$-$	0	$+$

$x \in (-\infty, -1) \cup (2, +\infty)$

δ) $\underbrace{e^{x^3+x-1} + \ln(x^3+x) - 1} - e = \ln\dfrac{2}{e}$

$$f(x^3+x-1) = \ln 2 - \ln e + e$$

$$f(x^3+x-1) = \ln 2 = f(1)$$

f 1-1

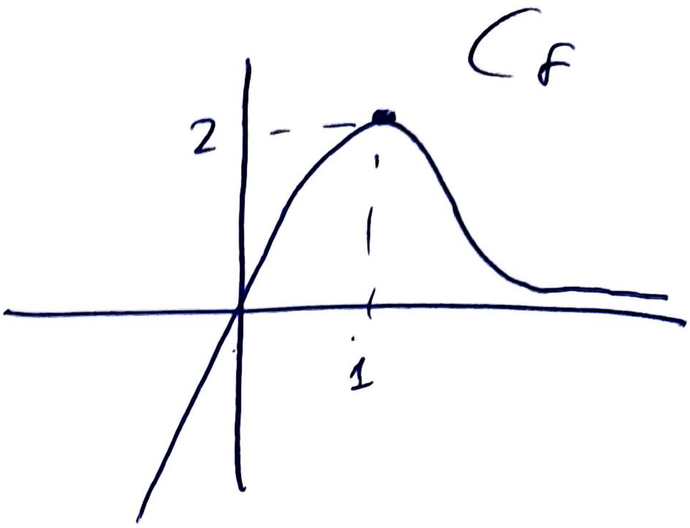
1	0	1	-2	1
↓	1	1	2	
1	1	2	0	

$x^3+x-1 = 1$

$x^3+x-2 = 0 \longrightarrow (x-1)(x^2+x-2) = 0$ $\quad x=1$

$\Delta < 0$

27



(α) Το $\frac{1}{e}$ και το $\frac{1}{\pi}$ ανήκουν στο $(-\infty, 1]$ στο οποίο $f \nearrow$

$$\frac{1}{e} > \frac{1}{\pi}$$

$$f \nearrow$$

$$f\left(\frac{1}{e}\right) > f\left(\frac{1}{\pi}\right).$$

(β) $f(2x^2+1) > f(x^2+2)$

$$\left.\begin{array}{l} \bullet\ 2x^2+1 > 1 \\ \bullet\ x^2+2 > 1 \end{array}\right\} f \searrow$$

$$2x^2+1 < x^2+2$$

$$x^2 < 1$$

$$x^2 < 1^2$$

$$|x| < 1$$

$$x \in (-1, 1).$$

(γ) $f(x) = (x-1)^4 + 2$

$\underbrace{f(x) - 2}_{\ominus} \underbrace{- (x-1)^2}_{\ominus} = 0$

Γνωρίζω ότι $f(x) \leq 2 \Rightarrow f(x) - 2 \leq 0$

Το "=" για $x = 1$.

Το $-(x-1)^2 \leq 0$

Το "=" για $x = 1$.

Μοναδική λύση το $\underline{\underline{x = 1}}$

(δ) $e^{-a} f(x) = 1$.

$f(x) = \frac{1}{e^{-a}} = e^{a} \Leftrightarrow \boxed{f(x) = e^{a}}$

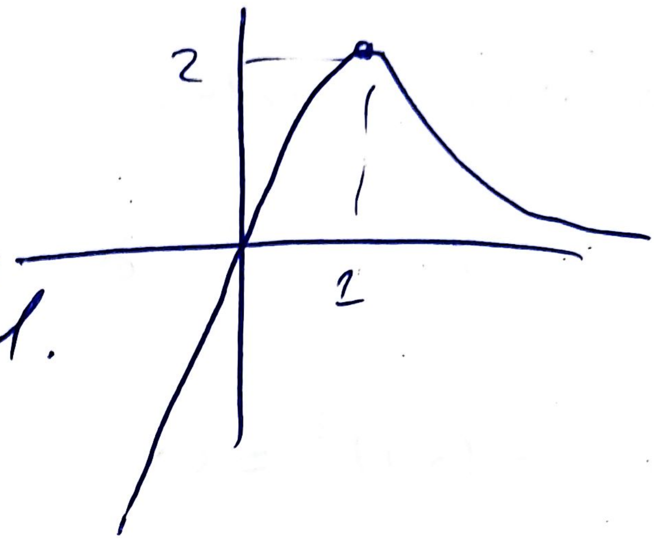
Θέτω $f(x) = \lambda$.

$f(x) = \lambda$

1. Αν $\lambda \leq 0$ τοτε 1 ρίζα

$e^a \leq 0$

Αδυνατο!



2. Αν $0<\lambda<2$ τοτε 2 ρίζες.

$0 \leq e^a < 2$

$0 \leq e^a$ και $e^a < 2$

Ισχυει!. $a < \ln 2$

3. Αν $\lambda = 2$ τοτε 1 ρίζα

$e^a = 2$

$a = \ln 2$

4. Αν $\lambda > 2$ τοτε καμια ρίζα

$e^a > 2$

$a > \ln 2$

26

$f(x) = 1 - \ln x \quad , x > 0 \qquad D_f = (0, +\infty).$

$g(x) = \ln\left(\frac{e}{x}\right) \quad , \frac{e}{x} > 0 \qquad D_g = (0, +\infty).$

$x > 0$

$h(x) = \sqrt{x} \qquad D_h = [0, +\infty).$

(α) Έχουν ίδιο πεδίο ορισμού αφού

$D_f = D_g = (0, +\infty)$

$g(x) = \ln\left(\frac{e}{x}\right) = \ln e - \ln x = 1 - \ln x = f(x)$

Ίσες!

(β) $f_1 = \frac{1}{f}$

$f_1(x) = \left(\frac{1}{f}\right)(x) = \frac{1}{f(x)} = \frac{1}{1 - \ln x}$

$x \in D_f$ και $f(x) \neq 0$

$x > 0 \qquad 1 - \ln x \neq 0$

$\ln x \neq 1$

$D_{f_1} = (0, e) \cup (e, +\infty). \qquad x \neq e$

(γ) $\varphi(x) = (h \circ f)(x) = h(f(x)) = \sqrt{1 - \ln x}$

$x \in D_f$ και $f(x) \in D_h$

$x > 0$

$1 - \ln x \geq 0$

$\ln x \leq 1$

$x \leq e$.

$D_\varphi = (0, e]$.

(δ) $\varphi(x) > 1 \iff \sqrt{1 - \ln x} > 1$

$1 - \ln x > 1$

$-\ln x > 0$

$\ln x < 0$

$\underline{\underline{x < 1}}$

Λογω D_φ

κρατάω τα $x \in (0, 1)$.

Επόμενο Μάθημα

Δευτέρα 8/7/2024

1. Σελ 147

2	3	8	9	10	11	12	13	14	19	20
		α	α	α	α	α	α	α	α	α
		β	β	β			β	β	γ	γ.
			δ				δ			
			ε				ε			
							στ			
							ζ			
							η			
							θ			

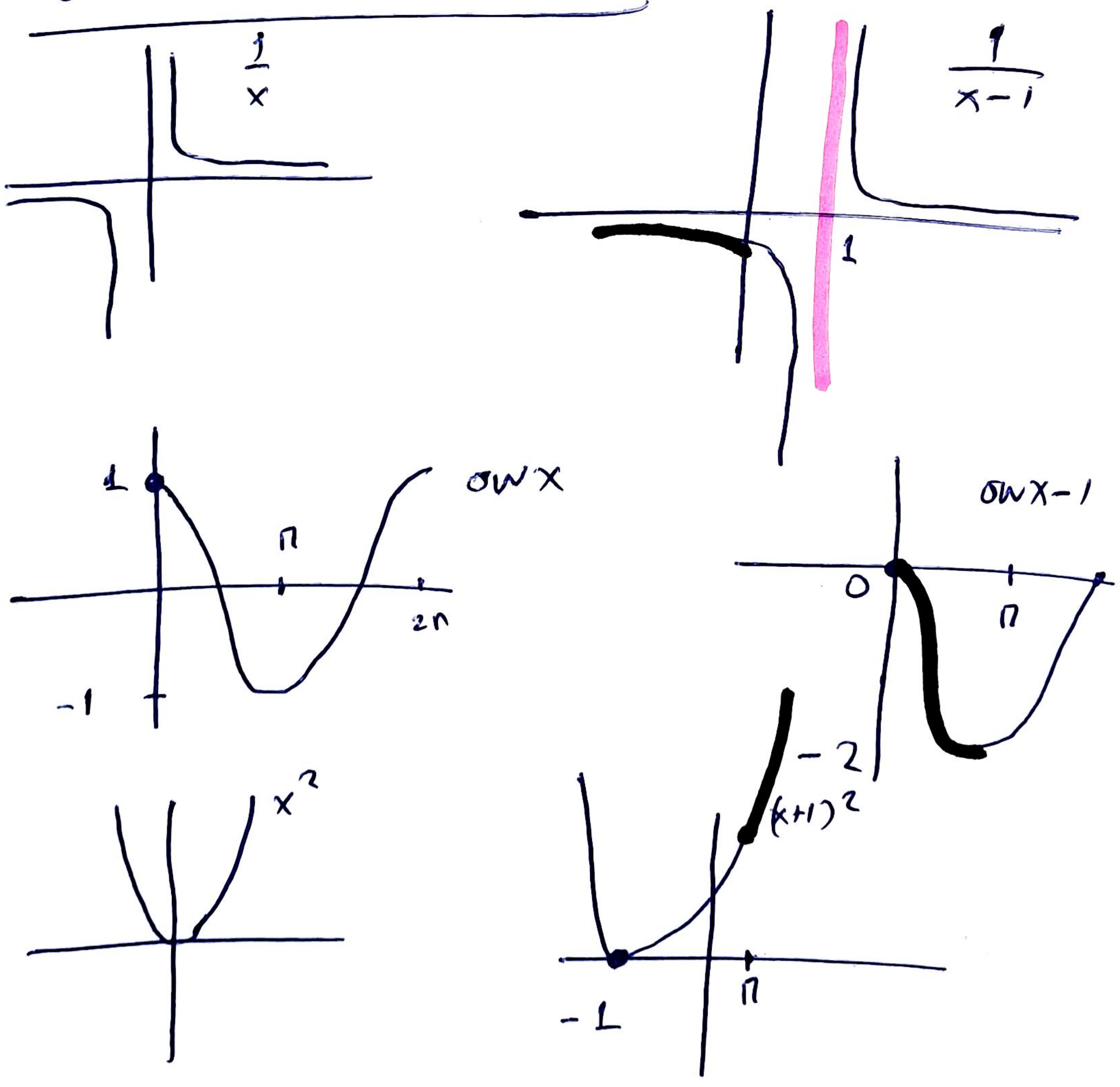
2. Θεωρία

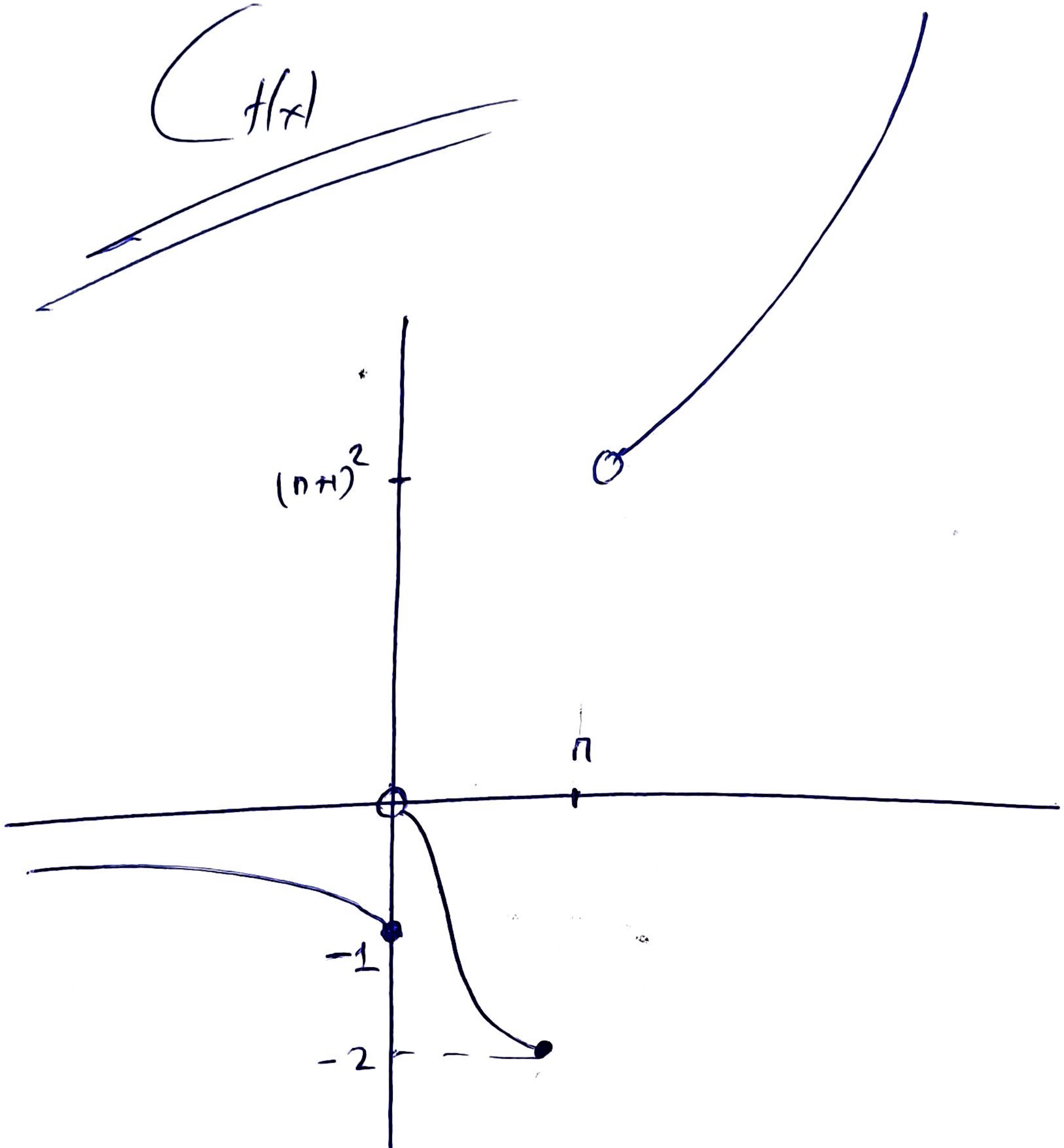
1 5 9 10 11

Άσκηση

$$f(x) = \begin{cases} \dfrac{1}{x-1}, & x \le 0 \\ \sigma\upsilon\nu x - 1, & 0 < x \le \pi \\ (x+1)^2, & x > \pi. \end{cases}$$

Βασικές συναρτήσεις





$f(0) = -1$

$\lim\limits_{x \to 0^-} f(x) = -1$

$\lim\limits_{x \to 0^+} f(x) = 0$

$\lim\limits_{x \to 0} f(x) = X$

$\lim\limits_{x \to n^-} f(x) = -2$

$\lim\limits_{x \to n^+} f(x) = (n+1)^2$

$\lim\limits_{x \to n} f(x) = X$

$f(n) = -2$

12 ⓐ $$\lim_{x \to -1} \left(\frac{1}{x+1} + \frac{2}{x^2-1} \right) =$$

$$= \lim_{x \to -1} \left(\frac{1}{x+1} + \frac{2}{(x-1)(x+1)} \right) =$$

$$= \lim_{x \to -1} \left(\frac{x-1}{(x-1)(x+1)} + \frac{2}{(x-1)(x+1)} \right)$$

$$= \lim_{x \to -1} \left(\frac{x+1}{(x-1)(x+1)} \right) = \lim_{x \to -1} \frac{1}{x-1} = \frac{1}{-2}.$$

13 ⓐ $$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x \to 1} \frac{(x-1)(\sqrt{x}+1)}{\sqrt{x}^2 - 1^2} = \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{x-1}} =$$

$$= \lim_{x \to 1} (\sqrt{x}+1) = 2.$$

(β) $$\lim_{x\to 5} \frac{\sqrt{x}-\sqrt{5}}{5-x} = \lim_{x\to 5} \frac{(\sqrt{x}-\sqrt{5})(\sqrt{x}+\sqrt{5})}{(5-x)(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x\to 5} \frac{\sqrt{x}^2-\sqrt{5}^2}{-(x-5)(\sqrt{x}+5)} = \lim_{x\to 5} \frac{\cancel{x-5}}{-\cancel{(x-5)}(\sqrt{x}+5)}$$

$$= \lim_{x\to 5} \frac{1}{-(\sqrt{x}+5)} = \frac{1}{-(\sqrt{5}+5)}$$

(δ) $$\lim_{x\to 1} \left(\frac{\sqrt{x+3}-2}{x^2-1}\right) = \lim_{x\to 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(x+1)(\sqrt{x+3}+2)}$$

$$= \lim_{x\to 1} \frac{x+3-4}{(x-1)(x+1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)(\sqrt{x+3}+2)}$$

$$= \frac{1}{2\cdot 4} = \frac{1}{8}.$$

(8) $\lim\limits_{x \to 2} \dfrac{x^2-2x}{\sqrt{x^2+5}-3} =$

$= \lim\limits_{x \to 2} \dfrac{x(x-2)\left(\sqrt{x^2+5}+3\right)}{\left(\sqrt{x^2+5}-3\right)\left(\sqrt{x^2+5}+3\right)} =$

$= \lim\limits_{x \to 2} \dfrac{x(x-2)\left(\sqrt{x^2+5}+3\right)}{\sqrt{x^2+5}^{\,2}-3^2}$

$= \lim\limits_{x \to 2} \dfrac{x\cancel{(x-2)}\left(\sqrt{x^2+5}+3\right)}{\cancel{(x-2)}(x+2)} = \dfrac{2 \cdot 6}{4} = 3.$

(9). $\lim\limits_{x \to 1} \dfrac{\sqrt{3x^2+1}-2}{x^2-5x+4} =$

$= \lim\limits_{x \to 1} \dfrac{\left(\sqrt{3x^2+1}-2\right)\left(\sqrt{3x^2+1}+2\right)}{(x-1)(x-4)\left(\sqrt{3x^2+1}+2\right)} =$

$= \lim\limits_{x \to 1} \dfrac{3\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-4)\left(\sqrt{3x^2+1}+2\right)} = \dfrac{6}{-3 \cdot 4} = -\dfrac{1}{2}.$

(n) $$\lim_{x\to 0} \frac{x}{1-\sqrt{3x+1}} = \lim_{x\to 0} \frac{x\left(1+\sqrt{3x+1}\right)}{\left(1-\sqrt{3x+1}\right)\left(1+\sqrt{3x+1}\right)}$$

$$= \lim_{x\to 0} \frac{x\left(1+\sqrt{3x+1}\right)}{1^2-\sqrt{3x+1}^{\,2}} = \lim_{x\to 0} \frac{x\left(1+\sqrt{3x+1}\right)}{\cancel{1}-3x-\cancel{1}}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\left(1+\sqrt{3x+1}\right)}{-3\cancel{x}} = \frac{2}{-3}.$$

(14) (a) $$\lim_{x\to -1} \frac{\sqrt{1-3x}+2x}{x^2-1} =$$

$$= \lim_{x\to -1} \frac{\sqrt{1-3x}^{\,2}-(2x)^2}{(x-1)(x+1)\left(\sqrt{1-3x}-2x\right)}$$

$$= \lim_{x\to -1} \frac{1-3x-4x^2}{(x-1)(x+1)\left(\sqrt{1-3x}-2x\right)} = \lim_{x\to -1} \frac{-4\cancel{(x+1)}\left(x-\frac{1}{4}\right)}{(x-1)\cancel{(x+1)}\left(\sqrt{1-3x}-2x\right)}$$

$$= \frac{-4\left(1-\frac{1}{4}\right)}{-2\left(2+2\right)} = \frac{-4+1}{-8} = \frac{3}{8}.$$

(B) $\lim_{x\to -1}\left(\dfrac{\sqrt{x^2+3}-2}{\sqrt{x+2}-\sqrt{2x+3}}\right) =$

$$= \lim_{x\to -1} \frac{\left(\sqrt{x^2+3}-2\right)\left(\sqrt{x^2+3}+2\right)\left(\sqrt{x+2}+\sqrt{2x+3}\right)}{\left(\sqrt{x+2}-\sqrt{2x+3}\right)\left(\sqrt{x+2}+\sqrt{2x+3}\right)\left(\sqrt{x^2+3}+2\right)}$$

$$= \lim_{x\to -1} \frac{(x-1)\cancel{(x+1)}\left(\sqrt{x+2}+\sqrt{2x+3}\right)}{-\cancel{(x+1)}\left(\sqrt{x+2}+\sqrt{2x+3}\right)}$$

$$= \lim_{x\to -1} \frac{(x-1)\left(\sqrt{x+2}+\sqrt{2x+3}\right)}{-\left(\sqrt{x+2}+\sqrt{2x+3}\right)}$$

$$= \frac{-2\cancel{(1+1)}}{-\cancel{(1+1)}} = 2.$$

19 (α)

$$\lim_{x\to 2} \frac{|\overset{\oplus}{x-1}| - |\overset{\ominus}{x-3}|}{x^2-4} =$$

$$= \lim_{x\to 2} \frac{x-1-(-x+3)}{x^2-4} =$$

$$= \lim_{x\to 2} \frac{x-1+x-3}{x^2-4} =$$

$$= \lim_{x\to 2} \frac{2x-4}{x^2-4} = \lim_{x\to 2} \frac{2\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \frac{2}{4}$$

(β) $$\lim_{x\to -1^+} \frac{|\overset{0}{x+1}| + x^2-x-2}{x^3+1} =$$

x		-1	
$x+1$	$-$	0	$+$

$$= \lim_{x\to -1^+} \frac{|\overset{\oplus}{x+1}| + x^2-x-2}{x^3+1} = \lim_{x\to -1^+} \frac{x+1+x^2-x-2}{x^3+1}$$

$$= \lim_{x\to -1^+} \frac{x^2-1}{x^3+1} = \lim_{x\to -1^+} \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)}.$$

$$= \frac{-2}{3}.$$

20 (a) $$\lim_{x\to 1} \frac{|x-1| + x^2-2x+1}{x^5-1}$$

x	1
$x-1$	$-\ 0\ +$

Δεν υπαρχει

$$\to \lim_{x\to 1^-} \frac{|x-1| + x^2-2x+1}{x^5-1} = \lim_{x\to 1^-} \frac{-x+1+x^2-2x+1}{x^5-1}$$

$$= \lim_{x\to 1^-} \frac{x^2-3x+2}{x^5-1} = \lim_{x\to 1^-} \frac{(x-2)\cancel{(x-1)}}{\cancel{(x-1)}(x^4+x^3+x^2+x+1)}$$

1	0	0	0	0	-1	1
	1	1	1	1	1	
1	1	1	1	1	0	

$$\frac{-1}{5}$$

$$\to \lim_{x\to 1^+} \frac{|x-1| + x^2-2x+1}{x^5-1} = \lim_{x\to 1^+} \frac{x^2-x}{x^5-1}$$

$$= \lim_{x\to 1^+} \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x^4+x^3+x^2+x+1)} = \frac{1}{5}$$

(γ). $\lim_{x\to 3} \dfrac{x^2-9-\sqrt{x^2-6x+9}}{\overset{+}{|x-1|}-2}$,

$= \lim_{x\to 3} \dfrac{x^2-9-\sqrt{(x-3)^2}}{x-3} =$

$= \lim_{x\to 3} \dfrac{x^2-9-|\overset{0}{x-3}|}{x-3}$

x	3
$x-3$	$-\ \phi\ +$

$\to \lim_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)+\cancel{(x-3)}}{\cancel{x-3}} = 6+1 = 7$

$\to \lim_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)-\cancel{(x-3)}}{\cancel{x-3}} = 5.$

Δεν υπαρχει

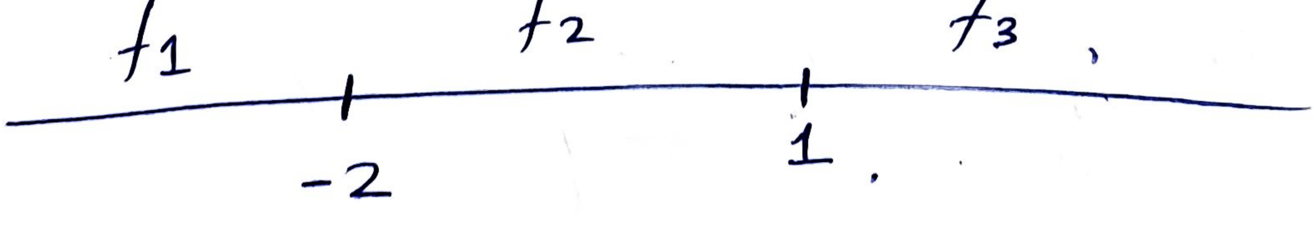
15 8 $f(x) = \begin{cases} x^3-2x, & x<-2 \\ 3x+2, & -2\le x<1 \\ x-1, & x\geq 1 \end{cases}$.

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} (x^3-2x) = -8+4 = -4$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} (3x+2) = -4$$

Άρα $\boxed{\lim_{x\to -2} f(x) = -4}$.

$$\left.\begin{aligned} &\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3x+2) = 5 \\ &\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x-1) = 0 \end{aligned}\right\}$$ Το όριο δεν υπάρχει.



(18) $f(x) = \begin{cases} 3x-a, & x \le -1 \\ x^2-ax+\beta, & -1 < x < 1 \\ x^3-ax^2+\gamma, & x \geq 1 \end{cases}$

Πληροφορία

Το $\lim_{x \to -1} f(x)$ υπαρχει στο $\mathbb{R} \Rightarrow \lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$\lim_{x \to 1} f(x) = 1 \Rightarrow \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 1$.

Αρα

$\boxed{\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)} \Leftrightarrow \lim_{x \to -1^-}(3x+a) = \lim_{x \to -1^+}(x^2-ax+\beta)$

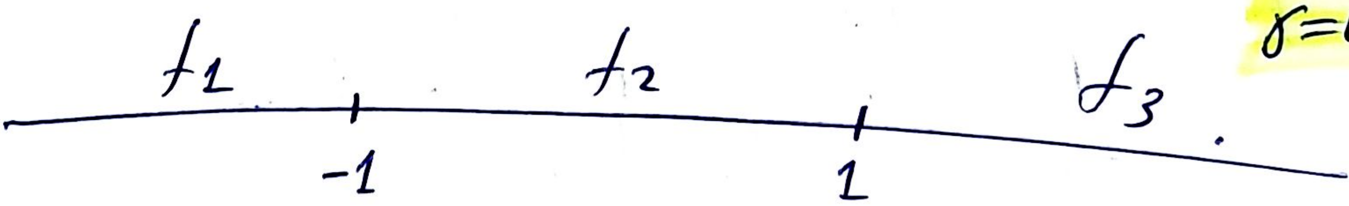
$\Leftrightarrow -3+\cancel{a} = 1+\cancel{a}+\beta \Leftrightarrow \beta = -4$

$\lim_{x \to 1^-} f(x) = 1 \Leftrightarrow \lim_{x \to 1^-}(x^2-ax+\beta) = 1 \Leftrightarrow 1-a-4=1$

$a = -4$

$\lim_{x \to 1^+} f(x) = 1 \Rightarrow \lim_{x \to 1^+}(x^3-ax^2+\gamma) = \cancel{1}-4+\gamma = \cancel{1}$

$\gamma = 4$



Επόμενο Μάθημα

Τετάρτη 8:30-10

Σελ 148 - 149

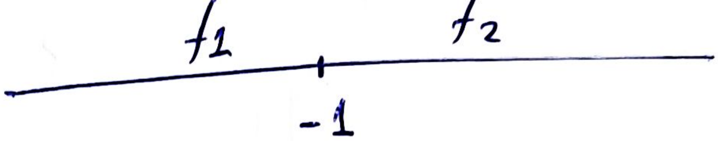
5 15 16 17.

α
β
γ

Επόμενη Θεωρία

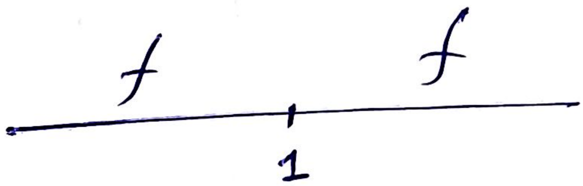
1 5 9 10 11.

(15) (a) $f(x)=\begin{cases}2x+3\,, & x\le -1\\ x^3+2\,, & x>-1\end{cases}$



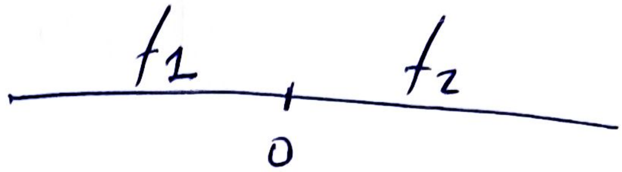
$$\left.\begin{aligned}\lim_{x\to -1^-} f(x) &= \lim_{x\to -1^-}(2x+3) = 1\\ \lim_{x\to -1^+} f(x) &= \lim_{x\to -1^+}(x^3+2) = 1\end{aligned}\right\}\ \lim_{x\to -1} f(x) = 1.$$

(B) $f(x)=\begin{cases}\dfrac{x^2-1}{x-1}\,, & x\neq 1\\ 3\,, & x=1.\end{cases}$



$$\lim_{x\to 1} f(x) = \lim_{x\to 1}\frac{x^2-1}{x-1} = \lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 2.$$

① $f(x)=\begin{cases} 2-\sigma\upsilon\nu x, & x<0 \\ 1, & x=0 \\ \dfrac{\sqrt{x+1}-1}{x}, & x>0 \end{cases}$



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(2-\sigma\upsilon\nu x) = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+1}-1}{x} =$$

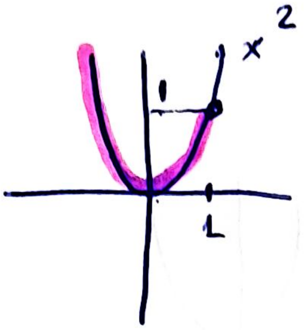
$$= \lim_{x\to 0^+} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)} = \frac{1}{2}$$

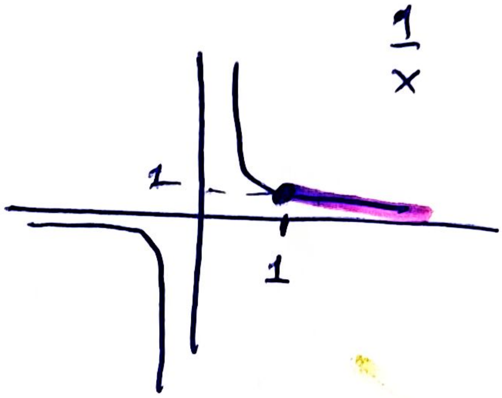
Το $\lim_{x\to 0} f(x)$ δεν

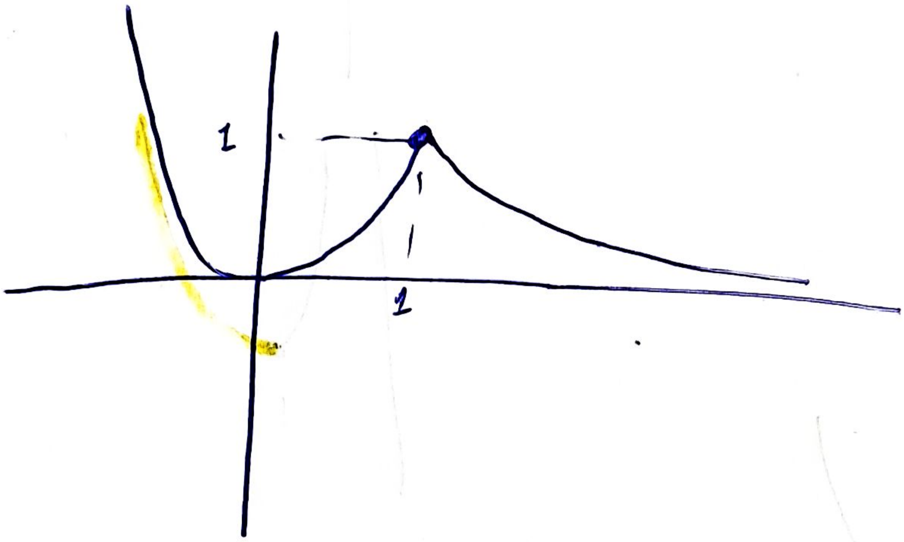
υπάρχει.

5

a) $f(x) = \begin{cases} x^2, & x \leq 1 \\ \frac{1}{x}, & x > 1 \end{cases}$

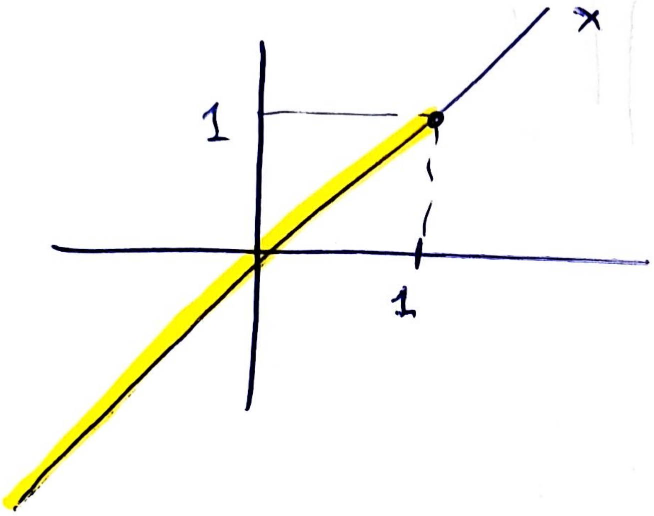


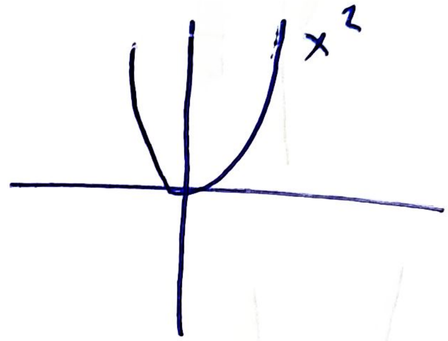


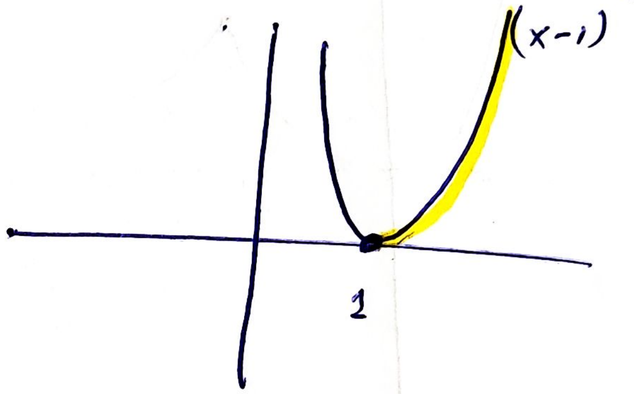


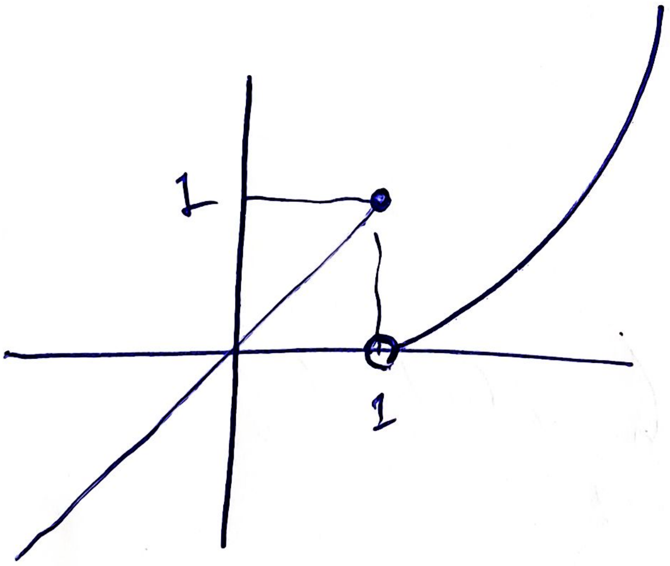
$$\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = 1 \\ \\ \lim_{x \to 1^+} f(x) = 1 \end{array}\right\} \lim_{x \to 1} f(x) = 1$$

(B). $f(x) = \begin{cases} x, & x \leq 1 \\ (x-1)^2, & x > 1 \end{cases}$









$$\left.\begin{array}{l} \lim\limits_{x\to1^-} f(x) = 1 \\ \\ \lim\limits_{x\to1^+} f(x) = 0 \end{array}\right\} \lim_{x\to1} f(x) \ \underline{\underline{\text{δεν}}} \ \text{υπαρχει.}$$

16

$$f(x)=\begin{cases} \dfrac{1-\sqrt{3x-2}}{x-1}, & x>1 \\ 2a+1, & x=1. \end{cases}$$

$$\boxed{\lim_{x\to 1} f(x)=f(1)}$$

$$\rightarrow \lim_{x\to 1} f(x)=\lim_{x\to 1}\frac{1-\sqrt{3x-2}}{x-1}=$$

$$=\lim_{x\to 1}\frac{1-(3x-2)}{(x-1)\left(1+\sqrt{3x-2}\right)}=\lim_{x\to 1}\frac{3-3x}{(x-1)(1+\sqrt{3x-2})}$$

$$=\lim_{x\to 1}\frac{-3\cancel{(x-1)}}{\cancel{(x-1)}\left(1+\sqrt{3x-2}\right)}=\lim_{x\to 1}\frac{-3}{1+\sqrt{3x-2}}$$

$$=\frac{-3}{2}.$$

$f(1)=2a+1$ άρα $2a+1=-\dfrac{3}{2}$

$4a+2=-3 \Rightarrow \cancel{4a=-5}$

$a=-\dfrac{5}{4}$

(17)

$$f(x) = \begin{cases} ax+2\beta, & x \le 1 \\ x^2+\beta x+2a, & x > 1 \end{cases}$$

$A(2,2)$

Το $\lim_{x\to 1} f(x)$ υπάρχει στο $\mathbb{R}$.

$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$$\lim_{x\to 1^-} (ax+2\beta) = \lim_{x\to 1^+} (x^2+\beta x+2a)$$

$$a+2\beta = 1+\beta+2a$$

$$\boxed{-a+\beta=1}$$

Επίσης $f(2)=2 \Leftrightarrow 4+2\beta+2a=2$

$2+\beta+a=1$

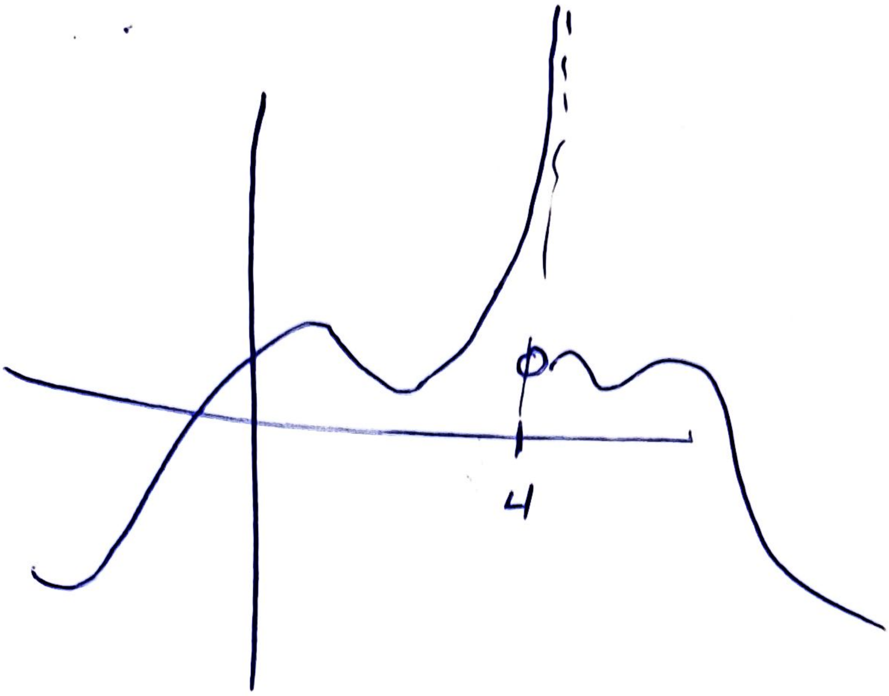
$\boxed{\beta+a=-1}$

$$\begin{cases} -a+\beta=1 \\ \beta+a=-1 \end{cases} \oplus \quad 2\beta=0$$

$\beta=0$

$0+a=-1$

$a=-1$



$$\lim_{x \to 4^-} f(x) = +\infty$$

Βασικο οριο

$$\frac{1}{0} = \infty$$

Αν ο παρονομαστης μηδενιζεται για

αλλα ειναι

i) θετικη ποσοτητα τοτε $+\infty$

ii) αρνητικη ποσοτητα τοτε $-\infty$.

Π.χ

$$\lim_{x\to 0} \frac{1}{x^2} \overset{\left(\frac{1}{0}\right)}{=\!=} +\infty$$

$$\lim_{x\to 0} \frac{1}{\sigma\upsilon\nu x - 1} \overset{\left(\frac{1}{0}\right)}{=\!=} -\infty.$$

- $-1 \le \sigma\upsilon\nu x \le 1$

$\sigma\upsilon\nu x \le 1$

$\boxed{\sigma\upsilon\nu x - 1 \le 0}$

γιατί το $\frac{1}{0}$ κάνει ∞ ;

$$\frac{1}{x}$$

$$\frac{1}{0{,}00001} = \frac{1}{\frac{1}{100000}} = 100000$$

$$\frac{1}{0{,}0000000000000001} = 1.000000000000$$

Σελ 169

(5) (γ) $\lim\limits_{x\to 0} \dfrac{1}{x^3} = \infty$;

x	0
x^3	$-$ 0 $+$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{1}{x^3} = -\infty \\ \\ \lim\limits_{x\to 0^+} \dfrac{1}{x^3} = +\infty \end{array}\right\}$$ Το $\lim\limits_{x\to 0} f(x)$ δεν υπάρχει.

(ε). $\lim\limits_{x\to 1} \dfrac{1}{\underbrace{|x-1|}_{+}} = +\infty$

Το $|x-1| \geq 0$

⑥ (B) $\lim_{x \to 2} \left(\dfrac{2x-5}{x^2-4x+4} \right) \overset{\left(\frac{a}{0}\right)}{=}$

Χωρίζω "καλούς" από "κακούς"

$= \lim_{x \to 2} (2x-5) \cdot \dfrac{1}{(x-2)^2} = (-1) \cdot (+\infty)$

$= \underline{\underline{-\infty}}$

(στ) $\lim_{x \to 1} \left(\dfrac{5x-3}{\sqrt{x-1}} \right) = \lim_{x \to 1} (5x-3) \dfrac{1}{\sqrt{x-1}} =$

$= 2 \cdot (+\infty) = +\infty$

7 δ

$$\lim_{x\to -1}\left(\frac{x^2-x}{x^3+3x^2+3x+1}\right)=$$

$$=\lim_{x\to -1}\frac{x^2-x}{(x+1)(x^2+2x+1)}=\lim_{x\to -1}\frac{x^2-x}{(x+1)^3}=$$

$$=\lim_{x\to -1}(x^2-x)\frac{1}{(x+1)^3}=$$

1	3	3	1	−1
↓	−1	−2	−1	
1	2	1	0	

$$=2\cdot\infty$$

x	-1
$(x+1)^3$	$-$ 0 $+$

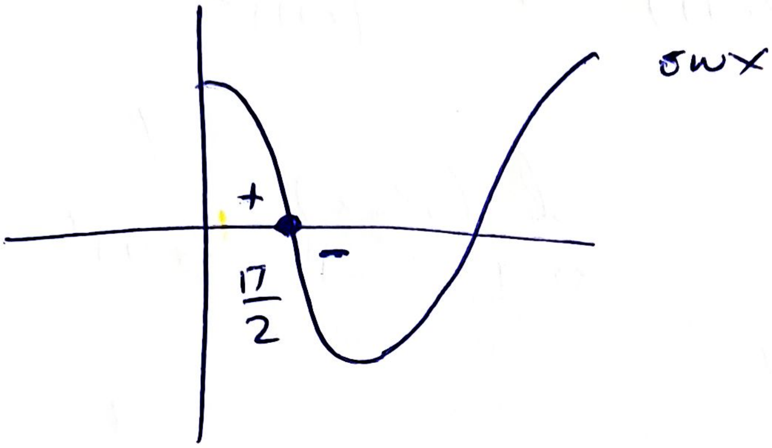
$$\rightarrow \lim_{x\to -1^-}(x^2-x)\frac{1}{(x+1)^3}=2(-\infty)=-\infty$$

$$\rightarrow \lim_{x\to -1^+}(x^2-x)\frac{1}{(x+1)^3}=2(+\infty)=+\infty$$

Το όριο δεν υπάρχει.

(52) $\lim\limits_{x \to \frac{\pi}{2}} \varepsilon\varphi x = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\eta\mu x}{\sigma\upsilon\nu x}$

$= \lim\limits_{x \to \frac{\pi}{2}} \eta\mu x \cdot \dfrac{1}{\sigma\upsilon\nu x} = 1 \cdot \infty$



$\to \lim\limits_{x \to \frac{\pi}{2}^-} \eta\mu x \, \dfrac{1}{\sigma\upsilon\nu x} = 1 \cdot (+\infty) = +\infty$

$\to \lim\limits_{x \to \frac{\pi}{2}^+} \eta\mu x \, \dfrac{1}{\sigma\upsilon\nu x} = 1(-\infty) = -\infty.$

Το όριο δεν υπάρχει.

(9) (γ) $\lim_{x\to 0}\left(\frac{2}{x}-\frac{2}{\sqrt{x}}\right) =$ Προϋπ. $x \geq 0$

$= \lim_{x\to 0^+} \frac{2\sqrt{x}-2x}{x\sqrt{x}} =$

$= \lim_{x\to 0^+} 2\frac{(\sqrt{x}-x)(\sqrt{x}+x)}{x\sqrt{x}(\sqrt{x}+x)} =$

$= \lim_{x\to 0^+} 2\frac{\cancel{\sqrt{x}^2}-x^2}{x\sqrt{x}(\sqrt{x}+x)} = \lim_{x\to 0^+} 2\frac{x-x^2}{x\sqrt{x}(\sqrt{x}+x)}$

$= \lim_{x\to 0^+} 2\frac{\cancel{x}(1-x)}{\cancel{x}\sqrt{x}(\sqrt{x}+x)} = \lim_{x\to 0^+} 2\frac{1-x}{\sqrt{x}(\sqrt{x}+x)}$

$= \lim_{x\to 0^+} 2(1-x)\frac{1}{\sqrt{x}(\sqrt{x}+x)} = 2\cdot(+\infty) = +\infty$

(+) στο $(0,+\infty)$

10 (B) $\lim\limits_{x\to 3} \dfrac{2x-5}{x^2-9} =$

$= \lim\limits_{x\to 3} \dfrac{2x-5}{(x-3)(x+3)} = \lim\limits_{x\to 3} \dfrac{2x-5}{x+3} \cdot \dfrac{1}{x-3}$

$= \dfrac{1}{6} \cdot \infty$

x	3
$x-3$	$-$ 0 $+$

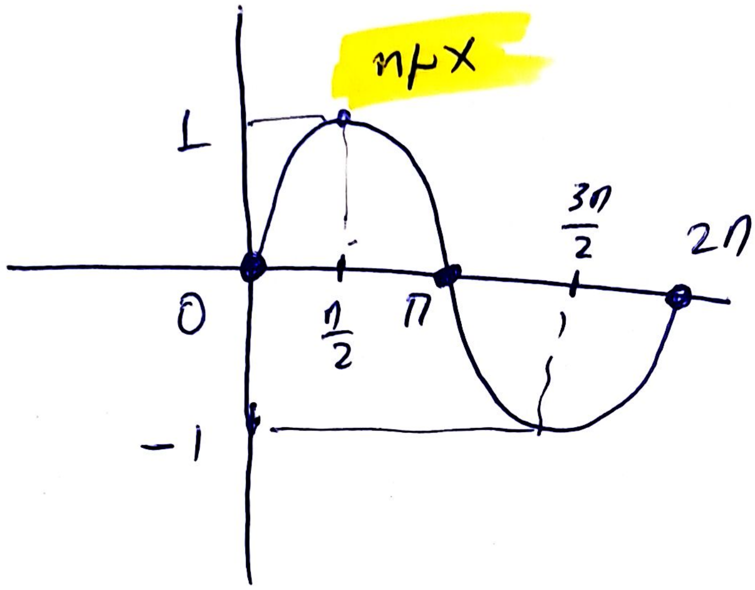
$\rightarrow \lim\limits_{x\to 3^-} \dfrac{2x-5}{x+3} \dfrac{1}{x-3} = \dfrac{1}{6}(-\infty) = -\infty$

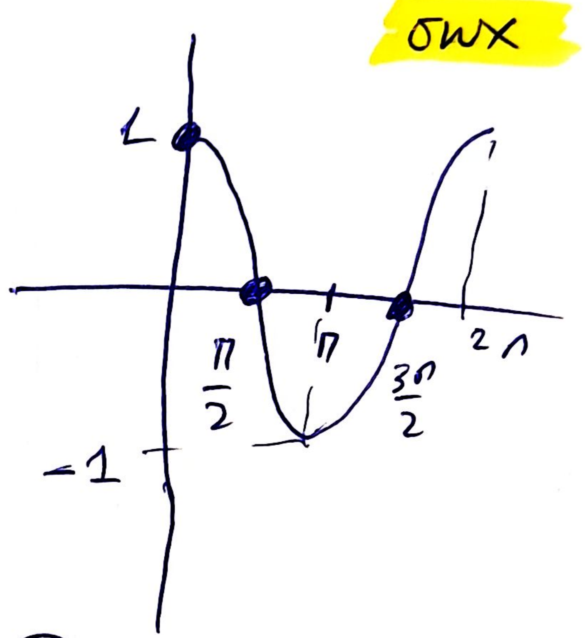
$\rightarrow \lim\limits_{x\to 3^+} \dfrac{2x-5}{x+3} \dfrac{1}{x-3} = \dfrac{1}{6}(+\infty) = +\infty$

Το όριο δεν υπάρχει.

Επομενο Μαθημα

Παρασκευη 12/7 (11-1)





Αυτα τα μαθαινεται απ' εξω.

Ασκησει (8) (9) αβ (10) α

Σελ 169

(5) αβδστ (6) αγδε (7) αβγε

Σελ 169

8 (α) $f(x) = \begin{cases} \dfrac{x}{x-1}, & x<1 \\ \dfrac{2-x}{x^3-1}, & x>1. \end{cases}$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left(\frac{x}{x-1}\right) = \lim_{x\to 1^-} x \cdot \frac{1}{x-1} =$$

$$= 1\cdot(-\infty) = -\infty$$

x		1	
$x-1$	$-$	0	$+$

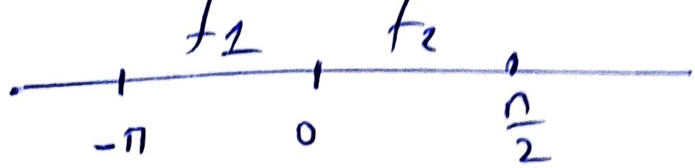
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{2-x}{x^3-1} = \lim_{x\to 1^+} (2-x)\frac{1}{x^3-1} =$$

$$= 1\cdot(+\infty) = +\infty$$

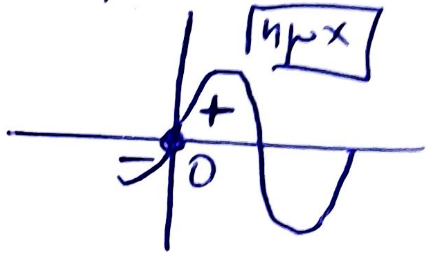
x		1	
x^3-1	$-$	0	$+$

Το όριο δεν υπάρχει!

(B) $f(x) = \begin{cases} \dfrac{x-1}{\eta\mu x}, & x\in(-\pi,0) \\[2ex] \dfrac{x+1}{\varepsilon\varphi x}, & x\in\left(0,\dfrac{\pi}{2}\right). \end{cases}$



$$\lim_{x\to0^-} f(x) = \lim_{x\to0^-}\frac{x-1}{\eta\mu x} = \lim_{x\to0^-}(x-1)\frac{1}{\eta\mu x} = -1\cdot(-\infty) = +\infty$$



$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}\frac{x+1}{\varepsilon\varphi x} = \lim_{x\to0^+}\frac{x+1}{\frac{\eta\mu x}{\sigma\upsilon\nu x}} =$$

$$= \lim_{x\to0^+}\frac{(x+1)\sigma\upsilon\nu x}{\eta\mu x} = \lim_{x\to0^+}(x+1)\sigma\upsilon\nu x\,\frac{1}{\eta\mu x} =$$

$$= 1\cdot(+\infty) = +\infty$$

Άρα

$$\lim_{x\to0} f(x) = +\infty .$$

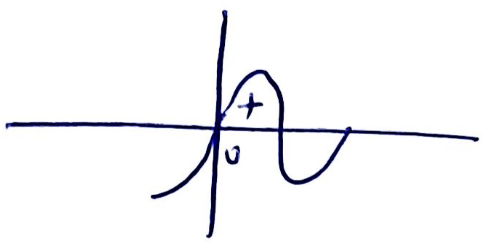
9 (α) $\lim\limits_{x\to 0} \dfrac{x+1+\sqrt{x}}{x} = \lim\limits_{x\to 0^+} (x+1+\sqrt{x}) \dfrac{1}{x} = 1\cdot(+\infty)$

$= +\infty$

λόγω του $\sqrt{x}$

(β) $\lim\limits_{x\to 0} \dfrac{1+\sqrt{x}}{\eta\mu x} = \lim\limits_{x\to 0^+} (1+\sqrt{x}) \dfrac{1}{\eta\mu x} = 1\cdot(+\infty)$

$= +\infty$



10 (α) $\lim\limits_{x\to 2} \dfrac{x-1}{x^2-4} = \lim\limits_{x\to 2} \dfrac{x-1}{(x-2)(x+2)} =$

$= \lim\limits_{x\to 2} \dfrac{x-1}{x+2} \cdot \dfrac{1}{x-2} = \dfrac{1}{4} \cdot \infty$

x		2	
$x-2$	$-$	0	$+$

$\rightarrow \lim\limits_{x\to 2^-} \dfrac{x-1}{x+2} \cdot \dfrac{1}{x-2} = \dfrac{1}{4} \cdot (-\infty) = -\infty$

$\rightarrow \lim\limits_{x\to 2^+} \dfrac{x-1}{x+2} \cdot \dfrac{1}{x-2} = \dfrac{1}{4} \cdot (+\infty) = +\infty$

Το όριο δεν υπάρχει.

Σελ 190

5 δ) $\lim_{x\to+\infty}(-5x^4+x-1)=\lim_{x\to+\infty}(-5x^4)=$

$= -5\cdot(+\infty)^4 = -\infty$

στ) $\lim_{x\to-\infty}(-2x^5+3x-1)=$

$=\lim_{x\to-\infty}(-2x^5) = -2\cdot(-\infty) = +\infty$

6 β) $\lim_{x\to-\infty}\dfrac{5x^2-x+1}{x^3+x+1}=\lim_{x\to-\infty}\dfrac{5x^2}{x^3}=$

$=\lim_{x\to-\infty}\dfrac{5}{x}=0$

Προσοχή

$\dfrac{1}{0}=\infty$ $\quad\quad$ $\dfrac{1}{\infty}=0$

(J) $\lim_{x\to-\infty} \frac{x^3}{4+x^2} =$

$= \lim_{x\to-\infty} \frac{x^3}{x^2} = \lim_{x\to-\infty} x = -\infty$

(ν) $\lim_{x\to-\infty} \frac{2x^2+1}{x^2-x+1} = \lim_{x\to-\infty} \frac{2\cancel{x^2}}{\cancel{x^2}} = 2$

7 (δ) $\lim_{x\to-\infty} \left(\frac{x^3}{1+x^2} - x \right) =$

$= \lim_{x\to-\infty} \frac{x^3}{1+x^2} - \frac{x(1+x^2)}{1+x^2} =$

$= \lim_{x\to-\infty} \frac{x^3 - x - x^3}{1+x^2} = \lim_{x\to-\infty} \frac{-x}{1+x^2}$

$= \lim_{x\to-\infty} \frac{-x}{x^2} = \lim_{x\to-\infty} -\frac{1}{x} = 0$

8 B $\lim_{x \to -\infty} \frac{|x^3 - 2x + 3| + x - 1}{x^4 + 2x + 2} \overset{(*)}{=}$

$\longrightarrow \lim_{x \to -\infty} (x^3 - 2x + 3) = \lim_{x \to -\infty} x^3 = -\infty$

$\overset{(*)}{=} \lim_{x \to -\infty} \frac{-x^3 + 2x - 3 + x - 1}{x^4 + 2x + 1} =$

$= \lim_{x \to -\infty} \frac{-x^3 + 3x - 4}{x^4 + 2x + 1} =$

$= \lim_{x \to -\infty} \frac{-x^3}{x^4} = \lim_{x \to -\infty} -\frac{1}{x} = 0.$

9 (β) $\lim\limits_{x \to -\infty} \sqrt{x^2 - 5x + 2} = \sqrt{+\infty} = +\infty$

$\longrightarrow \lim\limits_{x \to -\infty} x^2 - 5x + 2 = \lim\limits_{x \to -\infty} x^2 = +\infty$

(δ) $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 + 1} - x \right) =$

$= \lim\limits_{x \to -\infty} \left(\sqrt{x^2\left(1 + \frac{1}{x^2}\right)} - x \right) =$

$= \lim\limits_{x \to -\infty} \left(\overset{-}{|x|} \sqrt{1 + \frac{1}{x^2}} - x \right) =$

$= \lim\limits_{x \to -\infty} -x \sqrt{1 + \frac{1}{x^2}} - x =$

$= \lim\limits_{x \to -\infty} -x \left(\sqrt{1 + \frac{1}{x^2}} + 1 \right) =$

$= +\infty \left(2 \right) = +\infty .$

02. $$\lim_{x\to-\infty} \frac{x+1}{\sqrt{x^2+1}} = \lim_{x\to-\infty} \frac{x\left(1+\frac{1}{x}\right)}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}$$

$$= \lim_{x\to-\infty} \frac{x\left(1+\frac{1}{x}\right)}{\overset{\ominus}{|x|}\sqrt{1+\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{\cancel{x}\left(1+\frac{1}{x}\right)}{-\cancel{x}\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{1+\frac{1}{x}}{-\sqrt{1+\frac{1}{x^2}}} = \frac{1}{-1} = -1.$$

(10) (B) $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+1}-2x\right)=$

$$=\lim_{x\to+\infty}\left(\sqrt{x^2\left(4+\frac{1}{x^2}\right)}-2x\right)$$

$$=\lim_{x\to+\infty}\left(\overset{+}{|x|}\sqrt{4+\frac{1}{x^2}}-2x\right)$$

$$=\lim_{x\to+\infty}\left(x\sqrt{4+\frac{1}{x^2}}-2x\right)$$

$$=\lim_{x\to+\infty}x\left(\sqrt{4+\frac{1}{x^2}}-2\right)=$$

$$=+\infty(2-2)=+\infty\cdot 0\ ;$$

Άλλη στρατηγική

$$\lim_{x\to+\infty}\left(\sqrt{4x^2+1}-2x\right)=\lim_{x\to+\infty}\frac{\left(\sqrt{4x^2+1}-2x\right)\left(\sqrt{4x^2+1}+2x\right)}{\sqrt{4x^2+1}+2x}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{4x^2+1}^{\,2}-(2x)^2}{\sqrt{4x^2+1}+2x}=\lim_{x\to+\infty}\frac{\cancel{4x^2}+1-\cancel{4x^2}}{\sqrt{4x^2+1}+2x}=0.$$

(8) $\lim\limits_{x\to-\infty}\left(\sqrt{9x^2+x+1}+3x\right)$

$$= \lim_{x\to-\infty} \frac{\left(\sqrt{9x^2+x+1}+3x\right)\left(\sqrt{9x^2+x+1}-3x\right)}{\left(\sqrt{9x^2+x+1}-3x\right)}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{9x^2+x+1}^{\,2}-(3x)^2}{\sqrt{9x^2+x+1}-3x}$$

$$= \lim_{x\to-\infty} \frac{\cancel{9x^2}+x+1-\cancel{9x^2}}{\sqrt{9x^2+x+1}-3x} = \lim_{x\to-\infty} \frac{x+1}{\sqrt{9x^2+x+1}-3x}$$

$$= \lim_{x\to-\infty} \frac{x\left(1+\frac{1}{x}\right)}{\sqrt{x^2\left(9+\frac{1}{x}+\frac{1}{x^2}\right)}-3x} = \lim_{x\to-\infty} \frac{x\left(1+\frac{1}{x}\right)}{-x\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}-3x}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(1+\frac{1}{x}\right)}{-\cancel{x}\left(\sqrt{9+\frac{1}{x}+\frac{1}{x^2}}+3\right)} = \frac{1}{-6}.$$

12 ⓐ $\lim_{x \to +\infty} \left(\sqrt{4x^2+x+1} - 2x + 1 \right)$.

$$= \lim_{x \to +\infty} \left[\sqrt{4x^2+x+1} - (2x-1) \right]$$

$$= \lim_{x \to +\infty} \frac{\left[\sqrt{4x^2+x+1} - (2x-1)\right]\left[\sqrt{4x^2+x+1} + (2x-1)\right]}{\left(\sqrt{4x^2+x+1} + 2x-1\right)}$$

$$= \lim_{x \to +\infty} \frac{\cancel{\sqrt{4x^2+x+1}}^{\,2} - (2x-1)^2}{\sqrt{4x^2+x+1} + 2x - 1}$$

$$= \lim_{x \to +\infty} \frac{4x^2+x+1 - (4x^2-4x+1)}{\sqrt{4x^2+x+1} + 2x - 1}$$

$$= \lim_{x \to +\infty} \frac{5x}{\sqrt{4x^2+x+1} + 2x - 1} =$$

$$= \lim_{x \to +\infty} \frac{5x}{x\sqrt{4+\frac{1}{x}+\frac{1}{x^2}} + 2x - 1}$$

$$= \lim_{x \to +\infty} \frac{5\cancel{x}}{\cancel{x}\left(\sqrt{4+\cancel{\frac{1}{x}}+\cancel{\frac{1}{x^2}}} + 2 - \cancel{\frac{1}{x}}\right)}$$

$$= \frac{5}{4}.$$

(δ) $$\lim_{x \to -\infty} \frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2+1}} =$$

$$= \lim_{x \to -\infty} \frac{\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right)}{\left(x-\sqrt{x^2+1}\right)\left(x-\sqrt{x^2-1}\right)}$$

$$= \lim_{x \to -\infty} \frac{\left(x^2-\sqrt{x^2-1}^{\,2}\right)}{\left(x-\sqrt{x^2+1}\right)\left(x-\sqrt{x^2-1}\right)} = \frac{1}{\left(x-\sqrt{x^2+1}\right)\left(x-\sqrt{x^2-1}\right)}$$

$$= \frac{1}{-\infty \cdot (-\infty)} = \frac{1}{+\infty} = 0$$

Σcג 94

(12) $f : \mathbb{R} \to \mathbb{R}$ $\searrow$

(a) vdo $g(x) = f(x) - x$ $\searrow$

- $x_1 < x_2 \overset{f\searrow}{\Rightarrow} f(x_1) > f(x_2)$
- $x_1 < x_2 \Rightarrow -x_1 > -x_2$

$$\left.\begin{matrix} x_1 < x_2 \overset{f\searrow}{\Rightarrow} f(x_1) > f(x_2) \\ x_1 < x_2 \Rightarrow -x_1 > -x_2 \end{matrix}\right\} (+)$$

$$\underbrace{f(x_1) - x_1}_{g(x_1)} > \underbrace{f(x_2) - x_2}_{g(x_2)}$$

$$g \searrow$$

(B). Bpl λ $f(\lambda^2 - 3\lambda) - f(2\lambda - 6) = \lambda^2 - 5\lambda + 6$.

$$\underbrace{f(\lambda^2 - 3\lambda) - (\lambda^2 - 3\lambda)}_{g(\lambda^2 - 3\lambda)} = \underbrace{f(2\lambda - 6) - (2\lambda - 6)}_{g(2\lambda - 6)}.$$

g 1-1

$\lambda^2 - 3\lambda = 2\lambda - 6 \Rightarrow \lambda^2 - 5\lambda + 6 = 0$ $\lambda = 2$, $\lambda = 3$.

Επομενο Μαθημα

Σαββατο 10 - 11:30

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5. α β γ ε
6. α γ δ ε στ
7. α β γ
8. α
9. α γ ε
10. α γ
11. ΟΛΗ
12. β γ
13. ΟΛΗ .

Εφ 1.5.3

Μου δίνουν δεδομένο ένα όριο.

39 Αν $\lim_{x\to 1} \frac{f(x)-x^3}{x^2-1} = 2$

Βρείτε το $\lim_{x\to 1} f(x)$

και $\lim_{x\to 1} \frac{f(x)-x}{\sqrt{x}-1}$

Θεωρώ βοηθητική συνάρτηση.

$g(x) = \frac{f(x)-x^3}{x^2-1}$ ενώ $\boxed{\lim_{x\to 1} g(x) = 2}$

Λύνω ως προς $f(x)$.

$$\boxed{f(x) = g(x)(x^2-1) + x^3}$$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \left(g(x)(x^2-1) + x^3\right)$

$$\lim_{x\to1} f(x) = 2\cdot(1-1)+1$$

$$\lim_{x\to1} f(x) = 1$$

$$\lim_{x\to1}\left(\frac{h(x)-x}{\sqrt{x}-1}\right) \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to1}\frac{g(x)(x^2-1)+x^3-x}{\sqrt{x}-1}$$

$$= \lim_{x\to1}\frac{g(x)(x^2-1)}{\sqrt{x}-1} + \frac{x^3-x}{\sqrt{x}-1} = 8+4 = 12.$$

$$\to \lim_{x\to1}\frac{g(x)(x^2-1)}{\sqrt{x}-1} = \lim_{x\to1}\frac{g(x)\cancel{(x-1)}(x+1)(\sqrt{x}+1)}{\cancel{x-1}}$$

$$= 2\cdot2\cdot2 = 8.$$

$$\to \lim_{x\to1}\frac{x^3-x}{\sqrt{x}-1} = \lim_{x\to1}\frac{x\cancel{(x-1)}(x+1)(\sqrt{x}+1)}{\cancel{x-1}} = 4$$

(22) (B) $|x f(x) - x^3| \le x^2$

Ψάχνω $\lim_{x \to 0} f(x)$.

Ιδιότητα απολύτων

$|x| \le \theta \iff -\theta \le x \le \theta$

$-x^2 \le x f(x) - x^3 \le x^2$

$x^3 - x^2 \le x f(x) \le x^2 + x^3$

Αν $x < 0$

$\frac{x^3 - x^2}{x} \geq f(x) \geq \frac{x^2 + x^3}{x}$

$\boxed{x^2 - x \geq f(x) \geq x + x^2}$

$\left.\begin{array}{l} \lim_{x \to 0^-} x^2 - x = 0 \\ \lim_{x \to 0^-} x + x^2 = 0 \end{array}\right\}$ Από Κ.Π. $\lim_{x \to 0^-} f(x) = 0$

Αν $x>0$

$$\boxed{x^2 - x \le f(x) \le x + x^2}$$

$$\left.\begin{aligned} &\lim_{x\to 0^+} x^2 - x = 0 \\ &\lim_{x\to 0^+} x + x^2 = 0 \end{aligned}\right\} \text{ Από Κ.Π } \lim_{x\to 0^+} f(x) = 0.$$

Αφού $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 0$

Τότε $\lim_{x\to 0} f(x) = 0.$

21 Σελ 151.

(B) $3x - x^2 \le f(x) + 2 \le 3x + x^2$.

Βρείτε $\lim\limits_{x \to 0} f(x)$.

$$\boxed{3x - x^2 - 2 \le f(x) \le 3x + x^2 - 2}$$

$$\left.\begin{array}{l} \bullet \lim\limits_{x \to 0} (3x - x^2 - 2) = -2 \\ \bullet \lim\limits_{x \to 0} (3x + x^2 - 2) = -2 \end{array}\right\}$$

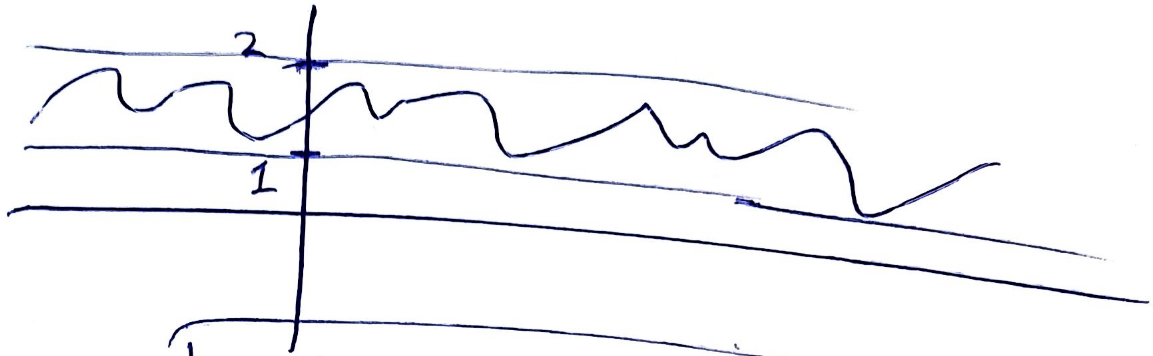
Από κριτήριο παρεμβολής

$$\lim_{x \to 0} f(x) = -2,$$

⑧ $f(\mathbb{R}) = (1,2)$

$\Rightarrow \quad f: \mathbb{R} \longrightarrow (1,2)$

ή



$\Rightarrow \quad \boxed{1 < f(x) < 2}$

Ψάχνω $\lim_{x\to 0}\left(x^4 f(x)\right)$

Παίρνω $\quad 1 < f(x) < 2$

$$\boxed{x^4 < x^4 f(x) < 2x^4}$$

$\left.\begin{array}{l} \lim_{x\to 0} x^4 = 0 \\ \lim_{x\to 0} 2x^4 = 0 \end{array}\right\}$ Από Κ.Π. $\lim_{x\to 0} x^4 f(x) = 0$

10 (a) $\lim\limits_{x\to+\infty} \sqrt{x^2+1} - x =$

$$= \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)}{\sqrt{x^2+1} + x}$$

$$= \lim_{x\to+\infty} \frac{\cancel{\sqrt{x^2+1}^{\,2}} - x^2}{\sqrt{x^2+1} + x}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{x^2+1} + x} = 0$$

(γ) $\lim\limits_{x\to-\infty} \left(\sqrt{x^2+1} + x\right) = \lim\limits_{x\to-\infty} \dfrac{\left(\sqrt{x^2+1} + x\right)\left(\sqrt{x^2+1} - x\right)}{\sqrt{x^2+1} - x}$

$$= \lim_{x\to-\infty} \frac{\cancel{\sqrt{x^2+1}^{\,2}} - x^2}{\sqrt{x^2+1} - x} = \lim_{x\to-\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1} - x}$$

$$= \lim_{x\to-\infty} \frac{1}{\sqrt{x^2+1} - x} = 0.$$

9 E $$\lim_{x \to +\infty} \frac{\sqrt{x^2+1}+x}{x} =$$

$$= \lim_{x \to +\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+x}{x} =$$

$$= \lim_{x \to +\infty} \frac{\overset{+}{|x|}\sqrt{1+\frac{1}{x^2}}+x}{x}$$

$$= \lim_{x \to +\infty} \frac{x\sqrt{1+\frac{1}{x^2}}+x}{x} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}\left(\sqrt{1+\frac{1}{x^2}}+1\right)}{\cancel{x}}$$

$$= 2.$$

Σελ 191

(7)

(B) $\lim_{x \to +\infty} \left(\frac{x^2}{x+1} - \frac{2x^2+3}{x} \right) =$

$= \lim_{x \to +\infty} \frac{x^2 x - (2x^2+3)(x+1)}{x(x+1)} =$

$= \lim_{x \to +\infty} \frac{x^3 - (2x^3+2x^2+3x+3)}{x^2+x} =$

$= \lim_{x \to +\infty} \frac{-x^3-2x^2-3x-3}{x^2+x}$

$= \lim_{x \to +\infty} \frac{-x^3}{x^2} = \lim_{x \to +\infty} -x = -\infty.$

Σελ 191

5

7

B $\lim_{x \to +\infty} \left(\frac{x^2}{x+1} - \frac{2x^2+3}{x} \right) =$

$$= \lim_{x \to +\infty} \frac{x^2 x - (2x^2+3)(x+1)}{x(x+1)} =$$

$$= \lim_{x \to +\infty} \frac{x^3 - (2x^3+2x^2+3x+3)}{x^2+x} =$$

$$= \lim_{x \to +\infty} \frac{-x^3-2x^2-3x-3}{x^2+x}$$

$$= \lim_{x \to +\infty} \frac{-x^3}{x^2} = \lim_{x \to +\infty} -x = -\infty.$$

(8)

(01) $\lim_{x \to +\infty} \left(|x^3 - x^2 - 1| - |-x^2 + 3| - 2x^3 \right)$

$= \lim_{x \to +\infty} x^3 - x^2 - 1 - (x^2 - 3) - 2x^3$

$= \lim_{x \to +\infty} \left(-x^3 - 2x^2 + 2 \right)$

$= \lim_{x \to +\infty} -x^3 = \underline{\underline{-\infty}}$

Σελ 168

③ (α) $\lim_{x \to 1} \frac{1}{f(x)} = -\infty$
$\ominus$

$\left[\lim_{x \to 1} f(x) = 0 \right.$ και η $f(x) < 0$ κοντά
στο 1 $\left. \right]$

(β). $\lim_{x \to 3} \frac{1}{f(x)} = \infty$;

$\to \lim_{x \to 3} f(x) = 0$

$\left. \begin{array}{l} \lim_{x \to 3^-} \frac{1}{f(x)} = -\infty \\ \lim_{x \to 3^+} \frac{1}{f(x)} = +\infty \end{array} \right\}$ $\lim_{x \to 3} \frac{1}{f(x)}$ <u>δεν</u> υπάρχει

(γ). $\lim_{x \to 5} \frac{1}{f(x)} = +\infty$
$\oplus$

$\to \lim_{x \to 5} f(x) = 0$.

κοντά στο 5 η $f(x) > 0$

(δ) $\lim_{x\to 5} \dfrac{1}{f(x)+(x-5)^2} = +\infty$

$\quad\quad\quad\quad\quad\quad (+) \quad (+)$

(ε) $\lim_{x\to 6} \dfrac{1}{f(x)}$.

$\longrightarrow \lim_{x\to 6^-} f(x) = +\infty$

$\longrightarrow \lim_{x\to 6^+} f(x) = 0$

Το όριο $\lim_{x\to 6} f(x)$ δεν υπάρχει.

$\lim_{x\to 6^-} \dfrac{1}{f(x)} = 0$

$\lim_{x\to 6^+} \dfrac{1}{f(x)} = +\infty$

Το $\lim_{x\to 6} \dfrac{1}{f(x)}$ δεν υπάρχει, (στ) $\lim\limits_{x\to 4} \dfrac{1}{f(f(x))+1} \underset{\substack{x\to 4\\ t\to 2}}{\overset{\substack{\text{Θέτω}\\ f(x)=t}}{=\!=}} \lim\limits_{t\to 2} \dfrac{1}{f(t)+1}$

$\rightarrow \lim\limits_{x\to 4} f(x) = 2$

$= \lim\limits_{x\to 2} \dfrac{1}{\underset{0}{f(x)+1}}\,(+) = +\infty$

Η $f(x)$ κοντά στο 2

είναι $f(x) > -1$

$f(x)+1 > 0$

(ζ) $\lim\limits_{x\to 4} \dfrac{1}{\underset{(-)}{f(x)-2}} = -\infty$

κοντά στο 4 η $f(x) < 2$

$f(x)-2 < 0$

(η) $\lim\limits_{x\to 3} \dfrac{1}{\underset{(+)}{(x-3)f(x)}} = +\infty.$

x	3
$x-3$	$-$ 0 $+$
$f(x)$	$-$ 0 $+$
$(x-3)f(x)$	$+$ | $+$

Επομενο Μαθημα

Δευτερα 11:30 - 1

11

12 β γ

13

\+ Ενημερωμενα

Τετραδια ή σημειωσεις,

Σελ 192

11 (α) $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \sqrt{x^2+1} = +\infty$

(β) $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \sqrt{x^2+1} = +\infty$

(γ) $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x}$

$= \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{1+\frac{1}{x^2}}}{\cancel{x}} = \lim_{x\to+\infty} \sqrt{1+\frac{1}{x^2}} = 1.$

(δ) $\lim_{x\to+\infty} (f(x)-x) = \lim_{x\to+\infty} \left(\sqrt{x^2+1} - x\right)$

$= \lim_{x\to+\infty} \frac{\left(\sqrt{x^2+1}-x\right)\left(\sqrt{x^2+1}+x\right)}{\sqrt{x^2+1}+x} = \lim_{x\to+\infty} \frac{\cancel{\sqrt{x^2+1}}^{2} - x^2}{\sqrt{x^2+1}+x}$

$= \lim_{x\to+\infty} \frac{1}{\sqrt{x^2+1}+x} = 0$

(ε) $\lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{\sqrt{x^2+1}}{x}$

$$= \lim_{x \to -\infty} \frac{-\cancel{x}\sqrt{1+\frac{1}{x^2}}}{\cancel{x}} = -1,$$

(5c) $\lim\limits_{x \to -\infty} \left(f(x)+x\right) = \lim\limits_{x \to -\infty} \left(\sqrt{x^2+1}+x\right)$

$$= \lim_{x \to -\infty} \frac{\left(\sqrt{x^2+1}+x\right)\left(\sqrt{x^2+1}-x\right)}{\sqrt{x^2+1}-x}$$

$$= \lim_{x \to -\infty} \frac{1}{\sqrt{x^2+1}-x} = 0$$

12

B $\lim_{x \to +\infty} \left(2x^2 - x\sqrt{4x^2+1}\right)$

$$= \lim_{x \to +\infty} \frac{\left(2x^2 - x\sqrt{4x^2+1}\right)\left(2x^2 + x\sqrt{4x^2+1}\right)}{2x^2 + x\sqrt{4x^2+1}}$$

$$= \lim_{x \to +\infty} \frac{4x^4 - x^2(4x^2+1)}{2x^2 + x\sqrt{4x^2+1}} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{4x^4} - \cancel{4x^4} - x^2}{2x^2 + x\sqrt{4x^2+1}} = \lim_{x \to +\infty} \frac{-x^2}{2x^2 + x\sqrt{4x^2+1}}$$

$$= \lim_{x \to +\infty} \frac{-x}{2x + \sqrt{4x^2+1}} = \lim_{x \to +\infty} \frac{-x}{2x + x\sqrt{4 + \frac{1}{x^2}}}$$

$$= \lim_{x \to +\infty} \frac{-1}{2 + \sqrt{4 + \frac{1}{x^2}}} = -\frac{1}{4}$$

(γ) $\lim\limits_{x\to-\infty}\left(x^2-1+x\sqrt{x^2+3}\right)$

$$=\lim_{x\to-\infty}\frac{\left(x^2-1+x\sqrt{x^2+3}\right)\left(x^2-1-x\sqrt{x^2+3}\right)}{x^2-1-x\sqrt{x^2+3}}$$

$$=\lim_{x\to-\infty}\frac{(x^2-1)^2-x^2(x^2+3)}{x^2-1-x\sqrt{x^2+3}}=$$

$$=\lim_{x\to-\infty}\frac{\cancel{x^4}-2x^2+1-\cancel{x^4}-3x^2}{x^2-1-x\sqrt{x^2+3}}$$

$$=\lim_{x\to-\infty}\frac{-5x^2+1}{x^2-1-x\sqrt{x^2+3}}=\lim_{x\to-\infty}\frac{x^2\left(-5+\frac{1}{x^2}\right)}{x^2-1+x^2\sqrt{1+\frac{3}{x^2}}}$$

$$=\lim_{x\to-\infty}\frac{\cancel{x^2}\left(-5+\frac{1}{x^2}\right)}{\cancel{x^2}\left(1-\frac{1}{x^2}+\sqrt{1+\frac{3}{x^2}}\right)}=\frac{-5}{2}.$$

24 (Β). $\lim\limits_{x\to 1}\left[(x-1)\,\eta\mu\frac{3x}{x-1}\right]$

Παρατηρώ ότι όταν το $x\to 1$ το $\frac{3x}{x-1}$ απειρίζεται.

Α' τρόπος

Γνωρίζω ότι $-1\le \eta\mu\frac{3x}{x-1}\le 1$

x		1	
$x-1$	$-$	0	$+$

Για $x<1$

$$\boxed{-(x-1)\ge (x-1)\,\eta\mu\frac{3x}{x-1}\ge x-1.}$$

$$\left.\begin{aligned}&\lim_{x\to 1^-} -(x-1)=0\\ &\lim_{x\to 1^-} x-1=0\end{aligned}\right\}\ \text{Από Κ.Π.}$$

$$\lim_{x\to 1^-}(x-1)\,\eta\mu\frac{3x}{x-1}=0.$$

Αν $x>1$

- $\lim_{x\to 1^+} -(x-1)=0$
- $\lim_{x\to 1^+} x-1=0$

$\Bigg\}$ Από Κ.Π $\lim_{x\to 1^+} (x-1)\,ημ\dfrac{3x}{x-1}=0$

Άρα $\lim_{x\to 1} (x-1)\,ημ\dfrac{3x}{x-1}=0$.

Β' τρόπος

Μηδενική × φραγμένη

$$\lim_{x\to 1} (x-1)\,ημ\frac{3x}{x-1}=0 \;;$$

Απόδειξη

$$-1\le ημ\frac{3x}{x-1}\le 1$$

$$\left|ημ\frac{3x}{x-1}\right|\le 1.$$

$$|x-1|\cdot\left|ημ\frac{3x}{x-1}\right|\le |x-1|$$

$$\left|(x-1)\,ημ\frac{3x}{x-1}\right|\le |x-1|$$

$$\boxed{-|x-1|\le (x-1)\,ημ\frac{3x}{x-1}\le |x-1|}$$

- $\lim_{x\to 1} -|x-1|=0$
- $\lim_{x\to 1} |x-1|=0$

$\Bigg\}$ Από Κ.Π $\lim_{x\to 1} (x-1)\,ημ\dfrac{3x}{x-1}=0.$

Пробохи

$$\lim_{x\to 0} \frac{\text{чрx}}{x} = 1.$$

$$\lim_{x\to 0} \frac{\text{owx} - 1}{x} = 0.$$

26 (γ) $\lim\limits_{x\to 0} \dfrac{\text{ημ}x \cdot \text{εφ}x}{x^2} =$

$$= \lim_{x\to 0} \frac{\text{ημ}x \cdot \frac{\text{ημ}x}{\text{συν}x}}{x^2} = \lim_{x\to 0} \frac{\text{ημ}^2 x}{x^2\,\text{συν}x} =$$

$$= \lim_{x\to 0} \frac{\text{ημ}^2 x}{x^2} \cdot \frac{1}{\text{συν}x} = 1 \cdot \frac{1}{1} = 1.$$

(δ) $\lim\limits_{x\to 0} \dfrac{1-\text{συν}^2 x}{x^2} = \lim\limits_{x\to 0} \dfrac{\text{ημ}^2 x}{x^2} = \underline{1}$

$\text{ημ}^2 x + \text{συν}^2 x = 1.$

$\text{ημ}^2 x = 1 - \text{συν}^2 x$

(ε) $\lim\limits_{x\to 0} \dfrac{\text{ημ}x\,\text{συν}x - \text{ημ}x}{x^2} = \lim\limits_{x\to 0} \dfrac{\text{ημ}x(\text{συν}x - 1)}{x \cdot x} = 1 \cdot 0 = 0$

25 (β) $\lim_{x\to 0}\left(\frac{\sigma\upsilon\nu x-1}{x}-\eta\mu x\right)=0-0=0$

(δ) $\lim_{x\to 0}\left(\frac{\eta\mu^2 x}{x^2}+x^2\eta\mu\frac{1}{x}\right)=1+0=1$

$\rightarrow \lim_{x\to 0}\frac{\eta\mu^2 x}{x^2}=\lim_{x\to 0}\left(\frac{\eta\mu x}{x}\right)^2=1^2=1$

$\rightarrow \lim_{x\to 0}\left(x^2\eta\mu\frac{1}{x}\right)$

$-1\le \eta\mu\frac{1}{x}\le 1$

$$\boxed{-x^2\le x^2\eta\mu\frac{1}{x}\le x^2}$$

$\left.\begin{array}{l}\lim_{x\to 0}-x^2=0\\ \lim_{x\to 0}x^2=0\end{array}\right\}$ Άρα Κ.Π. $\lim_{x\to 0}x^2\eta\mu\frac{1}{x}=0$

26

(γ) $$\lim_{x\to 0} \frac{ημx \cdot εφx}{x^2} =$$

$$= \lim_{x\to 0} \frac{ημx \cdot \frac{ημx}{συνx}}{x^2} = \lim_{x\to 0} \frac{ημ^2x}{x^2 συνx} =$$

$$= \lim_{x\to 0} \frac{ημ^2x}{x^2} \cdot \frac{1}{συνx} = 1 \cdot \frac{1}{1} = \underline{1}.$$

(δ) $$\lim_{x\to 0} \frac{1 - συν^2x}{x^2} = \lim_{x\to 0} \frac{ημ^2x}{x^2} = \underline{1}$$

$ημ^2x + συν^2x = 1$.

$ημ^2x = 1 - συν^2x$

(ε) $$\lim_{x\to 0} \frac{ημx συνx - ημx}{x^2} = \lim_{x\to 0} \frac{ημx(συνx - 1)}{x \cdot x} = 1 \cdot 0 = 0$$

27

(β) $$\lim_{x\to 0}\frac{\text{ημ}(\pi-x)}{x^2-x}=\lim_{x\to 0}\frac{\text{ημ}\,x}{x(x-1)}$$

$$=\lim_{x\to 0}\frac{\text{ημ}\,x}{x}\cdot\frac{1}{x-1}=1\cdot(-1)=-1.$$

(γ) $$\lim_{x\to 0}\frac{2\,\text{ημ}\,x-x}{3\,\text{ημ}\,x+x}=\lim_{x\to 0}\frac{2\frac{\text{ημ}\,x}{x}-\frac{x}{x}}{3\frac{\text{ημ}\,x}{x}+\frac{x}{x}}$$

$$=\frac{2\cdot 1-1}{3\cdot 1+1}=\frac{1}{4}=\frac{1}{4}.$$

(δ) $$\lim_{x\to 0}\frac{2x^2+\text{συν}\left(\frac{\pi}{2}-x\right)}{x+\text{συν}\,x-1}=\lim_{x\to 0}\frac{2x^2+\text{ημ}\,x}{x+\text{συν}\,x-1}$$

$$=\lim_{x\to 0}\frac{2x+\frac{\text{ημ}\,x}{x}}{1+\frac{\text{συν}\,x-1}{x}}=\frac{0+1}{1+0}=1.$$

(62) $$\lim_{x\to 0}\frac{\sqrt{x+1}-\cos x}{x}=$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{x+1}-\cos x\right)\left(\sqrt{x+1}+\cos x\right)}{x\left(\sqrt{x+1}+\cos x\right)}$$

$$=\lim_{x\to 0}\frac{\sqrt{x+1}^{2}-\cos^{2}x}{x\left(\sqrt{x+1}+\cos x\right)}=\lim_{x\to 0}\frac{x+1-\cos^{2}x}{x\left(\sqrt{x+1}+\cos x\right)}$$

$$=\lim_{x\to 0}\frac{x+\sin^{2}x}{x\left(\sqrt{x+1}+\cos x\right)}=$$

$$=\lim_{x\to 0}\frac{1+\frac{\sin x}{x}\cdot\sin x}{\sqrt{x+1}+\cos x}=\frac{1+1\cdot 0}{1+1}=\frac{1}{2}$$

29 (B) $\lim\limits_{x\to 0} \cancel{\frac{x^2}{\varepsilon\varphi x}}^{\to 0} \cdot \sigma\upsilon\nu\frac{2}{x} =$

Μ × Φ

$\rightarrow \lim\limits_{x\to 0} \frac{x^2}{\varepsilon\varphi x} = \lim\limits_{x\to 0} \frac{x^2}{\frac{\eta\mu x}{\sigma\upsilon\nu x}} =$

$= \lim\limits_{x\to 0} \frac{x^2\,\sigma\upsilon\nu x}{\eta\mu x} = \lim\limits_{x\to 0} \frac{x\,\sigma\upsilon\nu x}{\frac{\eta\mu x}{x}} = \frac{0\cdot 1}{1} = 0$

Απόδειξη

$-1 \le \sigma\upsilon\nu\frac{2}{x} \le 1$

$\left|\sigma\upsilon\nu\frac{2}{x}\right| \le 1$

$\left|\frac{x^2}{\varepsilon\varphi x}\right| \left|\sigma\upsilon\nu\frac{2}{x}\right| \le \left|\frac{x^2}{\varepsilon\varphi x}\right|$

$\left|\frac{x^2}{\varepsilon\varphi x}\sigma\upsilon\nu\frac{2}{x}\right| \le \left|\frac{x^2}{\varepsilon\varphi x}\right|$

$-\left|\frac{x^2}{\varepsilon\varphi x}\right| \le \frac{x^2}{\varepsilon\varphi x}\sigma\upsilon\nu\frac{2}{x} \le \left|\frac{x^2}{\varepsilon\varphi x}\right|$

$\lim\limits_{x\to 0} -\left|\frac{x^2}{\varepsilon\varphi x}\right| = 0$

$\lim\limits_{x\to 0} \left|\frac{x^2}{\varepsilon\varphi x}\right| = 0$

Από Κ.Π

$\lim\limits_{x\to 0} \frac{x^2}{\varepsilon\varphi x}\sigma\upsilon\nu\frac{2}{x}$

$\parallel$

0

(28) (B) $$\lim_{x\to 0}\left(\frac{2}{\eta\mu^2 x} - \frac{1}{1-\sigma\upsilon\nu x}\right)$$

$$= \lim_{x\to 0} \frac{2}{\eta\mu^2 x} - \frac{1+\sigma\upsilon\nu x}{(1-\sigma\upsilon\nu x)(1+\sigma\upsilon\nu x)}$$

$$= \lim_{x\to 0} \frac{2}{\eta\mu^2 x} - \frac{1+\sigma\upsilon\nu x}{1-\sigma\upsilon\nu^2 x}$$

$$= \lim_{x\to 0} \frac{2}{\eta\mu^2 x} - \frac{1+\sigma\upsilon\nu x}{\eta\mu^2 x}$$

$$= \lim_{x\to 0} \frac{2-1-\sigma\upsilon\nu x}{\eta\mu^2 x} = \lim_{x\to 0} \frac{1-\sigma\upsilon\nu x}{\eta\mu^2 x}$$

$$= \lim_{x\to 0} \frac{1-\sigma\upsilon\nu x}{1-\sigma\upsilon\nu^2 x} = \lim_{x\to 0} \frac{\cancel{1-\sigma\upsilon\nu x}}{(\cancel{1-\sigma\upsilon\nu x})(1+\sigma\upsilon\nu x)}$$

$$= \frac{1}{2}.$$

13

ⓐ $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \frac{x^2+1}{x^3+1} =$

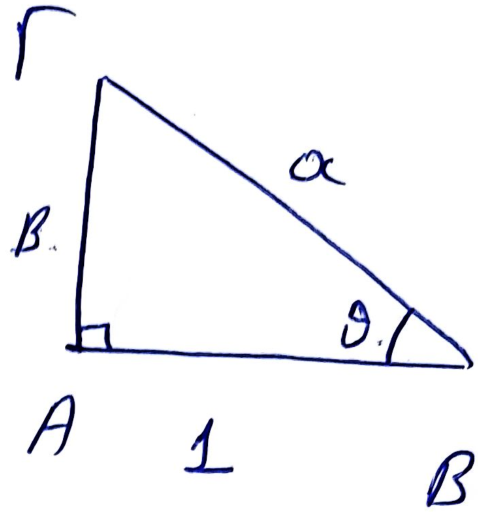
$= \lim\limits_{x \to +\infty} \frac{x^2}{x^3} = \lim\limits_{x \to +\infty} \frac{1}{x} = 0$

ⓑ $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \left(x + \sqrt{x^2+1}\right)$

$= \lim\limits_{x \to -\infty} \frac{\left(x+\sqrt{x^2+1}\right)\left(x-\sqrt{x^2+1}\right)}{x-\sqrt{x^2+1}} = \lim\limits_{x \to -\infty} \frac{x^2-(x^2+1)}{x-\sqrt{x^2+1}}$

$= \lim\limits_{x \to -\infty} \frac{-1}{x-\sqrt{x^2+1}} = 0$

30 (γ)



$$\lim_{\theta \to \frac{\pi}{2}} \frac{\beta}{\alpha} = \lim_{\theta \to \frac{\pi}{2}} \eta\mu\theta = \underline{1}$$

Επομενο Μαθημα
Τεταρτη 8:30 –10

1. Θέλω Δημητρη + Ραφαελα καλυτερη ταξινομηση στα τετραδια - σημειωσεις

2. Ασκησεις (Σελ 151 –).

21 α

22 α

23

24 α

25 α γ

26 α β δ.

27 α ε

28 α

29 α

30 α β.

22

(α) $|f(x) - x| \le x^2$

Ψάχνω το $\lim\limits_{x\to 0} f(x)$

$$|x| \le \theta \iff -\theta \le x \le \theta$$

$$-x^2 \le f(x) - x \le x^2$$

$$\boxed{x - x^2 \le f(x) \le x^2 + x}$$

Από Κ.Π.

$$\left.\begin{aligned} &\lim_{x\to 0} (x - x^2) = 0 \\ &\lim_{x\to 0} (x^2 + x) = 0 \end{aligned}\right\} \lim_{x\to 0} f(x) = 0.$$

24 (α) $\lim\limits_{x\to 0} x^2\,\text{συν}\dfrac{2}{x} =$

$$-1 \le \text{συν}\frac{2}{x} \le 1$$

$$\boxed{-x^2 \le x^2\,\text{συν}\frac{2}{x} \le x^2}$$

$$\left.\begin{aligned} &\lim_{x\to 0} (-x^2) = 0 \\ &\lim_{x\to 0} x^2 = 0 \end{aligned}\right\} \text{Από Κ.Π. } \lim_{x\to 0} x^2\,\text{συν}\frac{2}{x} = 0.$$

B' τρόπος

$$\lim_{x\to 0} x^2 \sigma \upsilon \nu \frac{2}{x} = 0 \qquad M \times \phi$$

$$-1 \le \sigma \upsilon \nu \frac{2}{x} \le 1$$

$$\left|\sigma \upsilon \nu \frac{2}{x}\right| \le 1$$

$$|x^2| \left|\sigma \upsilon \nu \frac{2}{x}\right| \le 1 \cdot |x^2|$$

$$\left|x^2 \sigma \upsilon \nu \frac{2}{x}\right| \le |x^2|$$

$$\boxed{-x^2 \le x^2 \sigma \upsilon \nu \frac{2}{x} \le x^2}$$

$$\left.\begin{aligned} &\lim_{x\to 0} -x^2 = 0 \\ &\lim_{x\to 0} x^2 = 0 \end{aligned}\right\} \text{Από K.П}$$

$$\lim_{x\to 0} x^2 \sigma \upsilon \nu \frac{2}{x} = 0.$$

25 (γ) $\lim\limits_{x\to 0}\left(x\,\etaμ\frac{1}{x}+\sigmaυν x\right)=0+1=1.$

$\rightarrow \lim\limits_{x\to 0} x\,\etaμ\frac{1}{x}=0$ (Κλx Φ)

$$-1\le \etaμ\frac{1}{x}\le 1$$

$$\left|\etaμ\frac{1}{x}\right|\le 1$$

$$|x|\left|\etaμ\frac{1}{x}\right|\le |x|$$

$$\left|x\,\etaμ\frac{1}{x}\right|\le |x|$$

$$\boxed{-|x|\le x\,\etaμ\frac{1}{x}\le |x|}$$

$$\left.\begin{aligned}&\lim_{x\to 0} -|x|=0\\ &\lim_{x\to 0}|x|=0\end{aligned}\right\}\ \text{Απο Κ.Π}\ \lim_{x\to 0} x\,\etaμ\frac{1}{x}=0.$$

26 σε $\lim\limits_{x\to 0} (\sigma\upsilon\nu x - 1)\,\eta\mu\frac{1}{x} = 0$

$(\mathrm{M} \times \Phi)$

$$-1 \le \eta\mu\frac{1}{x} \le 1$$

$$\left|\eta\mu\frac{1}{x}\right| \le 1$$

$$|\sigma\upsilon\nu x - 1|\left|\eta\mu\frac{1}{x}\right| \le |\sigma\upsilon\nu x - 1|$$

$$\left|(\sigma\upsilon\nu x - 1)\,\eta\mu\frac{1}{x}\right| \le |\sigma\upsilon\nu x - 1|$$

$$\boxed{-|\sigma\upsilon\nu x - 1| \le (\sigma\upsilon\nu x - 1)\,\eta\mu\frac{1}{x} \le |\sigma\upsilon\nu x - 1|}$$

$$\left.\begin{aligned}&\lim_{x\to 0} -|\sigma\upsilon\nu x - 1| = 0\\ &\lim_{x\to 0} |\sigma\upsilon\nu x - 1| = 0\end{aligned}\right\} \lim_{x\to 0} (\sigma\upsilon\nu x - 1)\,\eta\mu\frac{1}{x} = 0$$

θέλει απόδειξη,

Β' τρόπος

$$\lim_{x\to 0} (\sigma\upsilon\nu x - 1)\,\eta\mu\frac{1}{x} = \lim_{x\to 0} \frac{\sigma\upsilon\nu x - 1}{x}\, x\,\eta\mu\frac{1}{x} = 0$$

27 ε) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+1}-1}{\eta\mu^2 x} =$

$= \lim\limits_{x\to 0} \dfrac{x^2+1-1}{\eta\mu^2 x\left(\sqrt{x^2+1}+1\right)} = \lim\limits_{x\to 0} \dfrac{x^2}{\eta\mu^2 x\left(\sqrt{x^2+1}+1\right)}$

$= \lim\limits_{x\to 0} \dfrac{1}{\dfrac{\eta\mu^2 x}{x^2}\left(\sqrt{x^2+1}+1\right)} = \dfrac{1}{2}.$

29 α) $\lim\limits_{x\to 0} \dfrac{x^2}{\eta\mu x}\,\eta\mu\left(1-\dfrac{1}{x}\right) = \lim\limits_{x\to 0} \dfrac{x^2}{\eta\mu x}\,\eta\mu\left(\dfrac{x-1}{x}\right)$

$= 0$

$\to \lim\limits_{x\to 0} \dfrac{x^2}{\eta\mu x} = \lim\limits_{x\to 0} \dfrac{x}{\dfrac{\eta\mu x}{x}} = 0$ <u>ΜΧΦ</u>

<u>Απόδειξη</u>

$-1 \le \eta\mu\left(\dfrac{x-1}{x}\right) \le 1$ $\qquad \Leftrightarrow \left|\dfrac{x^2}{\eta\mu x}\right| \left|\eta\mu \dfrac{x-1}{x}\right| \le \left|\dfrac{x^2}{\eta\mu x}\right|$

$\left|\eta\mu \dfrac{x-1}{x}\right| \le 1$ $\qquad -\left|\dfrac{x^2}{\eta\mu x}\right| \le \dfrac{x^2}{\eta\mu x}\,\eta\mu \dfrac{x-1}{x} \le \left|\dfrac{x^2}{\eta\mu x}\right|$

κτλ.

28

ⓐ $\lim\limits_{x \to 0} \left(\frac{1}{x} - \frac{1}{x\,\sigma\upsilon\nu x} \right)$

$= \lim\limits_{x \to 0} \frac{\sigma\upsilon\nu x - 1}{x\,\sigma\upsilon\nu x} = \lim\limits_{x \to 0} \frac{\sigma\upsilon\nu x - 1}{x} \cdot \frac{1}{\sigma\upsilon\nu x}$

$= 0 \cdot 1 = 0.$

30

ⓐ $\lim\limits_{\theta \to \frac{\pi}{2}} (\alpha - \beta) = \lim\limits_{\theta \to \frac{\pi}{2}} \alpha \left(\frac{\alpha}{\alpha} - \frac{\beta}{\alpha} \right) =$

$= \lim\limits_{\theta \to \frac{\pi}{2}} \alpha (1 - \eta\mu\theta) = \alpha \cdot (1 - 1) = 0$

$= 0.$

ⓑ. $\lim\limits_{\theta \to \frac{\pi}{2}} (\alpha^2 - \beta^2) = \lim\limits_{\theta \to \frac{\pi}{2}} 1 = 1.$

$1 + \beta^2 = \alpha^2$

$1 = \alpha^2 - \beta^2$

Σελ 152

31 β) $\lim\limits_{x\to\pi} \dfrac{ημx}{\pi-x} = \lim\limits_{x\to\pi} \dfrac{ημ(\pi-x)}{\pi-x} = 1$

$\to \lim\limits_{x\to\pi} \dfrac{ημ(\pi-x)}{\pi-x} \underset{\substack{x\to\pi \\ t\to 0}}{\overset{\pi-x=t}{=\!=\!=}} \lim\limits_{t\to 0} \dfrac{ημt}{t} = 1$.

γ) $\lim\limits_{x\to 0} \dfrac{ημ3x}{εφ5x} = \lim\limits_{x\to 0} \dfrac{ημ3x}{\dfrac{ημ5x}{συν5x}} = \lim\limits_{x\to 0} \dfrac{ημ3x\,συν5x}{ημ5x}$

$= \lim\limits_{x\to 0} \dfrac{\dfrac{ημ3x}{x}\,συν5x}{\dfrac{ημ5x}{x}} = \lim\limits_{x\to 0} \dfrac{3\,\dfrac{ημ3x}{3x}\,συν5x}{5\,\dfrac{ημ5x}{5x}}$

$= \dfrac{3\cdot 1\cdot 1}{5\cdot 1} = \dfrac{3}{5}$

$\to \lim\limits_{x\to 0} \dfrac{ημ3x}{3x} \underset{\substack{x\to 0 \\ t\to 0}}{\overset{3x=t}{=\!=\!=}} \lim\limits_{t\to 0} \dfrac{ημt}{t} = 1$

$\to \lim\limits_{x\to 0} \dfrac{ημ5x}{5x} \underset{\substack{x\to 0 \\ u\to 0}}{\overset{5x=u}{=\!=\!=}} \lim\limits_{u\to 0} \dfrac{ημu}{u} = 1$.

Προσοχή

$$\lim_{x \to 0} \frac{\eta\mu x}{x} = 1 \quad \text{και} \quad \lim_{x \to x_0} \frac{\eta\mu f(x)}{f(x)} = 1$$

αν $\lim_{x \to x_0} f(x) = 0$.

$$\lim_{x \to 0} \frac{\sigma\upsilon\nu x - 1}{x} = 0 \quad \text{και} \quad \lim_{x \to x_0} \frac{\sigma\upsilon\nu f(x) - 1}{f(x)} = 0$$

αν $\lim_{x \to x_0} f(x) = 0$.

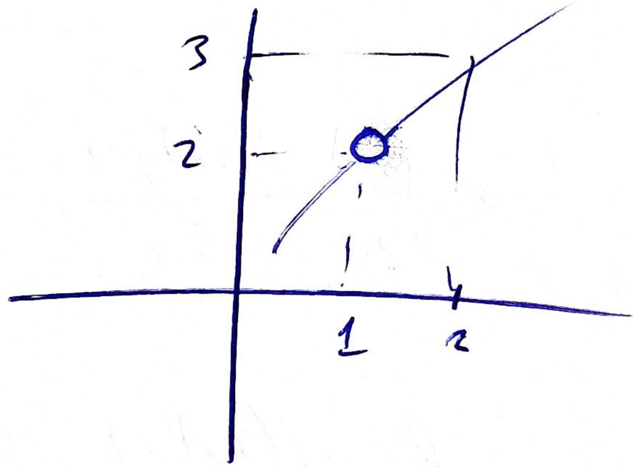
34

(B) $\lim_{x\to 0} \frac{f(x)}{x} = 5$

ψαχνω το $\lim_{x\to 0} \frac{f(2x)}{x} = \lim_{x\to 0} \frac{2f(2x)}{2x}$

$$= \lim_{x\to 0} 2\frac{f(2x)}{2x} \xlongequal[\substack{x\to 0\\ w\to 0}]{2x=w} \lim_{w\to 0} 2\frac{f(w)}{w} =$$

$$= 2\cdot 5 = 10.$$



35

$$\lim_{x\to 1} f(f(x)) \xlongequal[\substack{x\to 1\\ t\to}]{f(x)=t}$$

$$= \lim_{t\to 2} f(t) = 3.$$

(36) $\boxed{\lim_{x\to 0} \frac{f(x)-2}{x} = 3}$

(B) $\lim_{x\to 0} \frac{f(5x)-f(3x)}{x} = \lim_{x\to 0} \frac{f(5x)-2+2-f(3x)}{x}$

$= \lim_{x\to 0} \left(\frac{f(5x)-2}{x} - \frac{f(3x)-2}{x} \right)$

$= \lim_{x\to 0} 5\frac{f(5x)-2}{5x} - 3\frac{f(3x)-2}{3x} = 5\cdot 3 - 3\cdot 3$

$= 15-9=6.$

$\to \lim_{x\to 0} \frac{f(5x)-2}{5x} \underset{\substack{x\to 0\\ u\to 0}}{\overset{5x=u}{=\!=}} \lim_{u\to 0} \frac{f(u)-2}{u} = 3$

$\to \lim_{x\to 0} \frac{f(3x)-2}{3x} \underset{\substack{x\to 0\\ t\to 0}}{\overset{3x=t}{=\!=}} \lim_{t\to 0} \frac{f(t)-2}{t} = 3$

(37)

$$\lim_{h\to 0} \frac{f(2+h)-3}{h} = 5$$

Θέτω $2+h = x$

$h = x-2$

$h \to 0$

$x \to 2$

$$\boxed{\lim_{x\to 2} \frac{f(x)-3}{x-2} = 5}$$

(β) $$\lim_{h\to 0} \frac{f(2+3h)-f(2-h)}{h} =$$

$$= \lim_{h\to 0}\left(\frac{f(2+3h)-3}{h} - \frac{f(2-h)-3}{h}\right) \overset{(*)}{=}$$

$$\to \lim_{h\to 0} \frac{f(2+3h)-3}{h} \ \underset{\substack{3h=t-2\\ h=\frac{t-2}{3}}}{\overset{2+3h=t}{=\!=}} \ \lim_{t\to 2} \frac{f(t)-3}{\frac{t-2}{3}} =$$

$$= 3 \lim_{t \to 2} \frac{f(t)-3}{t-2} = 3 \cdot 5 = 15$$

$$\to \lim_{h \to 0} \frac{f(2-h)-3}{h} \quad \frac{2-h=t}{2-t=h} \quad \lim_{t \to 2} \frac{f(t)-3}{-(t-2)}$$

$h \to 0$

$t \to 2$

$$= -5 .$$

Apr

$$\textcircled{*} \quad 15-(-5)=20 .$$

38

$$\boxed{\lim_{x\to 2} \frac{f(x)-1}{x-2} = 3}$$

$$\lim_{x\to 2} \frac{f(x^2-x)-1}{x-2} = \lim_{x\to 2} \frac{f(x^2-x)-1}{(x^2-x)-2} \cdot \frac{x^2-x-2}{x-2}$$

$\underline{\underline{(*)}}$

$$\to \lim_{x\to 2} \frac{f(x^2-x)-1}{x^2-x-2} \xrightarrow[\substack{x\to 2\\ t\to 2}]{x^2-x=t} \lim_{t\to 2} \frac{f(t)-1}{t-2}$$

$$= 3.$$

$$\to \lim_{x\to 2} \frac{x^2-x-2}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}} = 3.$$

$\underline{\underline{(*)}}$ $3 \cdot 3 = 9.$

(41) $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1$.

(α) Θετω Βοηθητικη συναρτηση $g(x)=\dfrac{f(x)}{x}$

αρα $\lim\limits_{x\to 0} g(x)=1$

$$\boxed{f(x)=x\,g(x)}$$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0} x\,g(x)$$

$$\lim_{x\to 0} f(x)=0\cdot 1=0$$

$$\lim_{x\to 0} f(x)=0$$

(β) $$\lim_{x\to 0}\frac{f^2(x)+x\,ημx}{f^2(x)+x f(x)}=\lim_{x\to 0}\frac{\dfrac{f^2(x)}{x^2}+\dfrac{x}{x}\,\dfrac{ημx}{x}}{\dfrac{f^2(x)}{x^2}+\dfrac{x}{x}\,\dfrac{f(x)}{x}}$$

$$=\frac{1+1}{1+1}=1.$$

(γ) $$\lim_{x\to 0}\frac{f(x^3+x)}{x}=\lim_{x\to 0}\frac{f(x^3+x)}{x^3+x}\,\frac{x^3+x}{x}=1\cdot 1=1.$$

$\rightarrow \lim\limits_{x\to 0}\dfrac{f(x^3+x)}{x^3+x}\ \underline{\underline{x^3+x=t}}\ \lim\limits_{t\to 0}\dfrac{f(t)}{t}=1.$

44 $\quad x - x^2 \le f(x) \le x^2 + x$

a) $\lim\limits_{x\to 0} \dfrac{f(x)}{x}$

$\boxed{x<0}$

$$1 - x \geq \frac{f(x)}{x} \geq x + 1$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} 1 - x = 1 \\ \lim\limits_{x\to 0^-} x + 1 = 1 \end{array}\right\} \lim\limits_{x\to 0^-} \frac{f(x)}{x} = 1$$

$\boxed{x>0}$

$$1 - x \le \frac{f(x)}{x} \le x + 1$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^+} 1 - x = 1 \\ \lim\limits_{x\to 0^+} 1 + x = 1 \end{array}\right\} \lim\limits_{x\to 0^+} \frac{f(x)}{x} = 1$$

$$\lim\limits_{x\to 0} \frac{f(x)}{x} = 1$$

β) $$\lim_{x\to 0} \frac{f(x)}{\eta\mu x} = \lim_{x\to 0} \frac{\frac{f(x)}{x}}{\frac{\eta\mu x}{x}} = \frac{1}{1} = 1.$$

γ) $$\lim_{x\to 0} \frac{x+f(x)}{2x-f(x)} = \lim_{x\to 0} \frac{1+\frac{f(x)}{x}}{2-\frac{f(x)}{x}} =$$

$$= \frac{1+1}{2-1} = 2.$$

δ) $$\lim_{x\to 0} \frac{xf(x)+\sqrt{x^2+1}-1}{\eta\mu^2 x} = \lim_{x\to 0} \frac{\frac{x}{x}\cdot\frac{f(x)}{x}+\frac{\sqrt{x^2+1}-1}{x^2}}{\frac{\eta\mu^2 x}{x^2}}$$

$$\to \lim_{x\to 0} \frac{\sqrt{x^2+1}-1}{x^2} = \lim_{x\to 0} \frac{x^2+\cancel{1}-\cancel{1}}{x^2\left(\sqrt{x^2+1}+1\right)} =$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+1}+1} = \frac{1}{2}.$$

$$\frac{1+\frac{1}{2}}{1} = \frac{3}{2}.$$

Σελ 170

11 (B) $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \dfrac{x^2 - 2\lambda x + \lambda^2}{x^2 - 4x + 4}$

$= \lim\limits_{x \to 2} \left(x^2 - 2\lambda x + \lambda^2\right) \dfrac{1}{(x-2)^2}$

$= \left(4 - 4\lambda + \lambda^2\right)(+\infty)$

$= (2-\lambda)^2 (+\infty)$

Αν $\lambda \neq 2$ τότε $+\infty$

Αν $\lambda = 2$ τότε

$\lim\limits_{x \to 2} \dfrac{\cancel{x^2 - 4x + 4}}{\cancel{x^2 - 4x + 4}} = \underline{1}.$

13

$$\lim_{x\to 2} f(x) = \ell \in \mathbb{R}$$

$$\lim_{x\to 2} \frac{ax^2+(\beta-1)x+4}{x^2-4x+4} = \ell$$

$$\lim_{x\to 2} \left(ax^2+(\beta-1)x+4\right)\cdot \frac{1}{(x-2)^2} = \ell$$

$$(4a+2\beta-2+4)\cdot(+\infty) = \ell.$$

Αν $4a+2\beta+2 \neq 0$ τότε Άτοπο!

άρα $2a+\beta+1=0$

$$\boxed{\beta = -2a-1}$$

$$\lim_{x\to 2} \frac{ax^2+(-2a-1-1)x+4}{(x-2)^2} = \ell$$

$$\lim_{x\to 2} \frac{ax^2-(2a+2)x+4}{(x-2)^2} = \ell$$

$$\lim_{x\to 2} \frac{ax^2 - 2ax - 2x + 4}{(x-2)^2} = \ell$$

$$\lim_{x\to 2} \frac{ax\cancel{(x-2)} - 2\cancel{(x-2)}}{(x-2)^{\cancel{2}}} = \ell$$

$$\lim_{x\to 2} \frac{ax-2}{x-2} = \ell \qquad \lim_{x\to 2} g(x) = \ell$$

Θεωρώ $g(x) = \dfrac{ax-2}{x-2}$

$$g(x)(x-2) = ax-2$$

$$\lim_{x\to 2} g(x)(x-2) = \lim_{x\to 2} ax-2$$

$$\ell \cdot 0 = 2a-2$$

$$0 = 2a-2$$

$$a = 1$$

$$B = -3$$

B' τρόπος

$$\lim_{x\to 2} \frac{ax^2+(\beta-1)x+4}{x^2-4x+4} = \ell.$$

Θέτω $\frac{ax^2+(\beta-1)x+4}{x^2-4x+4} = g(x)$

$$ax^2+(\beta-1)x+4 = g(x)(x^2-4x+4)$$

$$\lim_{x\to 2} ax^2+(\beta-1)x+4 = \lim_{x\to 2} g(x)(x^2-4x+4)$$

$$4a+2(\beta-1)+4 = \ell \cdot 0$$

$$4a+2\beta-2+4=0$$

$$4a+2\beta+2=0 \longrightarrow \boxed{2a+\beta+1=0}$$

κτλ.

17 (δ) $\lim\limits_{x\to 0} f(x)\,\eta\mu\dfrac{1}{f(x)} \xrightarrow[\substack{x\to 0\\ t\to+\infty}]{f(x)=t} \lim\limits_{t\to+\infty} t\,\eta\mu\dfrac{1}{t}$

Γνωρίζω ότι $\lim\limits_{x\to 0} \underbrace{x^4 f(x)}_{g(x)} = 2$

$$x^4 f(x) = g(x)$$

$$f(x) = \frac{g(x)}{x^4}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{g(x)}{x^4} = +\infty$$

$$\lim_{x\to 0} f(x) = +\infty$$

$$\lim_{t\to+\infty} t\,\eta\mu\frac{1}{t} = \lim_{t\to+\infty} \frac{\eta\mu\frac{1}{t}}{\frac{1}{t}}\cdot\frac{1}{\cancel{t}}\cdot\cancel{t} = 1$$

16 (r) $\lim_{x \to 1} \frac{x^2-3}{f(x)} = +\infty$.

Ψαχνω $\lim_{x \to 1} f(x)$.

$\lim_{x \to 1} g(x) = +\infty$

$\frac{x^2-3}{f(x)} = g(x)$

$\frac{x^2-3}{g(x)} = f(x)$

$\lim_{x \to 1} \frac{x^2-3}{g(x)} = \lim_{x \to 1} f(x)$

$\frac{-2}{+\infty} = \lim_{x \to 1} f(x)$

$\lim_{x \to 1} f(x) = 0$

Επομενο Μαθημα

Παρασκευη 11-1

Σελ 152

31 α

34 α

36 α

37 α

39 α

40 ολη

42

43

Σελ 168

2

4

11 α

12

16 α β

17 α β γ.

Σελ 152

31 (α)

$$\lim_{x\to 0}\frac{ημ3x}{x} = \lim_{x\to 0} 3\,\frac{ημ3x}{3x} \;\underset{\substack{x\to 0\\ t\to 0}}{\overset{3x=t}{=\!=\!=}}$$

$$= \lim_{t\to 0} 3\,\frac{ημt}{t} = 3\cdot 1 = 3$$

34 (α)

$$\lim_{x\to 2} f(3x-1) \;\underset{\substack{x\to 2\\ y\to 5}}{\overset{3x-1=y}{=\!=\!=}}\; \lim_{y\to 5} f(y) = 1.$$

36

$$\lim_{x\to 0}\frac{f(x)-2}{x} = 3$$

(α)

$$\lim_{x\to 0}\frac{f(3x)-2}{x} = \lim_{x\to 0} 3\,\frac{f(3x)-2}{3x} \;\underset{\substack{x\to 0\\ w\to 0}}{\overset{3x=w}{=\!=\!=}}$$

$$\lim_{w\to 0} 3\,\frac{f(w)-2}{w} = 3\cdot 3 = 9.$$

37 $\lim_{h\to 0} \frac{f(2+h)-3}{h} = 5$

(α) $\lim_{h\to 0} \frac{f(2-3h)-3}{h} = \lim_{h\to 0} -3 \frac{f(2-3h)-3}{-3h}$ $\quad \frac{-3h = x}{\substack{h\to 0 \\ x\to 0}}$

$\lim_{x\to 0} -3 \frac{f(2-x)-3}{x} = -3\cdot 5 = -15.$

39 i) $\lim_{x\to -2} (2f(x)+1-x) = 3$

Θεωρώ $g(x) = 2f(x)+1-x$ άρα $\boxed{\lim_{x\to -2} g(x) = 3}$

$$\boxed{\frac{g(x)+x-1}{2} = f(x)}$$

$$\lim_{x\to -2} \frac{g(x)+x-1}{2} = \lim_{x\to -2} f(x)$$

$$\frac{3-2-1}{2} = \lim_{x\to -2} f(x)$$

$$0 = \lim_{x\to -2} f(x)$$

11). $\lim\limits_{x\to -2} \dfrac{f(x)-1}{x+2} = 3$

Θέτω $\dfrac{f(x)-1}{x+2} = g(x)$ οπότε $\lim\limits_{x\to -2} g(x) = 3$

Λύνω ως προς $f(x)$

$$\boxed{f(x) = g(x)(x+2) + 1}$$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \big(g(x)(x+2)+1\big)$$

$$\lim_{x\to -2} f(x) = 3\cdot(-2+2)+1$$

$$\boxed{\lim_{x\to -2} f(x) = 1}$$

40 $\lim_{x\to 1} \frac{f(x)}{x-1} = 3$ και $|g(x)-1| \le |f(x)|$

ⓐ $h(x) = \frac{f(x)}{x-1}$ άρα $\lim_{x\to 1} h(x) = 3$.

$f(x) = h(x)(x-1)$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} h(x)(x-1)$

$\lim_{x\to 1} f(x) = 3 \cdot 0 = 0 \Rightarrow \lim_{x\to 1} f(x) = 0$

ⓑ

$|g(x)-1| \le |f(x)|$

$-|f(x)| \le g(x) - 1 \le |f(x)|$

$$\boxed{1 - |f(x)| \le g(x) \le |f(x)| + 1}$$

$\lim_{x\to 1} 1 - |f(x)| = 1 - 0 = 1$.

$\lim_{x\to 1} |f(x)| + 1 = 1$

Από Κ.Π $\lim_{x\to 1} g(x) = 1$

(8). $\lim_{x \to 1} \dfrac{|g(x)-3|-2}{g^2(x)-g(x)}$ $\dfrac{g(x)=t}{\overline{\quad}}$

$x \to 1$

$t \to 1$

$= \lim_{t \to 1} \dfrac{|t-3|-2}{t^2-t} = \lim_{t \to 1} \dfrac{3-t-2}{t^2-t} =$

$= \lim_{t \to 1} \dfrac{1-t}{t^2-t} = \lim_{t \to 1} \dfrac{-(t-1)}{t^2-t}$

$= \lim_{t \to 1} \dfrac{-\cancel{(t-1)}}{t\cancel{(t-1)}} = \lim_{t \to 1} \dfrac{-1}{t} = \underline{\underline{-1}}$

43

$$\boxed{\eta\mu x - x^2 \le f(x) \le \eta\mu x + x^2}$$

(α) $\lim\limits_{x\to 0} \dfrac{f(x)}{x}$

$x<0$

$$\frac{\eta\mu x}{x} - x \ge \frac{f(x)}{x} \ge \frac{\eta\mu x}{x} + x$$

- $\lim\limits_{x\to 0^-}\left(\dfrac{\eta\mu x}{x} - x\right) = 1$
- $\lim\limits_{x\to 0^-}\left(\dfrac{\eta\mu x}{x} + x\right) = 1$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \lim_{x\to 0^-} \frac{f(x)}{x} = 1$$

$x>0$

ομοίως.

$$\lim_{x\to 0^+} \frac{f(x)}{x} = 1$$

$$\lim_{x\to 0} \frac{f(x)}{x} = 1$$

(β) $\lim\limits_{x\to 0} \dfrac{f(x)+2x}{x+\eta\mu x} = \lim\limits_{x\to 0} \dfrac{\frac{f(x)}{x}+2}{\frac{x}{x}+\frac{\eta\mu x}{x}} = \dfrac{1+2}{1+1} = \dfrac{3}{2}.$

42

$f: \mathbb{R} \to \mathbb{R}$

$$\lim_{x \to 0} \frac{f(x)}{x} = 3$$

(a) $$\lim_{x \to 0} \frac{f(x^2+x)}{x} = \lim_{x \to 0} \frac{f(x^2+x)}{x^2+x} \cdot \frac{x^2+x}{x} = 3 \cdot 1 = \underline{\underline{3}}$$

$$\to \lim_{x \to 0} \frac{f(x^2+x)}{x^2+x} \overset{x^2+x=t}{\underset{\substack{x\to 0 \\ t \to 0}}{=\!=}} \lim_{t \to 0} \frac{f(t)}{t} = 3$$

$$\to \lim_{x \to 0} \frac{x^2+x}{x} = \lim_{x \to 0} \frac{\cancel{x}(x+1)}{\cancel{x}} = 1$$

(B) $$\lim_{x \to 0} \frac{f^2(x) + x f(x) + x \eta \mu x}{f^2(x) + x^2 + \eta \mu^2 x} =$$

$$= \lim_{x \to 0} \frac{\frac{f^2(x)}{x^2} + \frac{x}{x}\frac{f(x)}{x} + \frac{x}{x}\frac{\eta \mu x}{x}}{\frac{f^2(x)}{x^2} + \frac{x^2}{x^2} + \frac{\eta \mu^2 x}{x^2}} = \frac{3^2 + 1 \cdot 3 + 1 \cdot 1}{3^2 + 1 + 1}$$

$$= \frac{13}{11}.$$

Σελ 168

(2) (α)

$$\lim_{x\to 1} f(x) = +\infty$$

$$\lim_{x\to 2} f(x) = -\infty$$

$$\lim_{x\to 3} f(x) = \text{δεν υπάρχει}$$

$$\lim_{x\to 3^-} f(x) = +\infty$$

$$\lim_{x\to 3^+} f(x) = -\infty$$

$$\lim_{x\to 4} f(x) = +\infty$$

(β)

$$\lim_{x\to 1} \frac{1}{f(x)} = 0$$

$$\lim_{x\to 2} \frac{1}{f(x)} = 0$$

$$\lim_{x\to 3} \frac{1}{f(x)}$$

$$\lim_{x\to 3^-} \frac{1}{f(x)} = 0$$

$$\lim_{x\to 3^+} \frac{1}{f(x)} = 0$$

$$= 0$$

$$\lim_{x\to 4} \frac{1}{f(x)} = 0.$$

(8) $\lim_{x \to 1} |f(x)| = +\infty$

$\lim_{x \to 2} |f(x)| = +\infty$

$$\lim_{x \to 3} |f(x)| = \begin{cases} \lim_{x \to 3^-} |f(x)| = |+\infty| = +\infty \\ \lim_{x \to 3^+} |f(x)| = |-\infty| = +\infty \end{cases} = \underline{\underline{+\infty}}$$

4

(α) $\lim_{x \to 0} (f(x) + g(x)) = -\infty + 3 = -\infty$

(β) $\lim_{x \to 0} (f(x) \cdot g(x)) = -\infty \cdot 3 = -\infty$.

(γ) $\lim_{x \to 1} \frac{g(x)}{f(x)} = \frac{2}{0} = \infty$; Δεν υπάρχει!

$\lim_{x \to 1^-} \frac{g(x)}{f(x)} = \lim_{x \to 1^-} g(x) \frac{1}{\underset{\ominus}{f(x)}} = 2 \cdot (-\infty) = -\infty$

$\lim_{x \to 1^+} \frac{g(x)}{f(x)} = \lim_{x \to 1^+} g(x) \frac{1}{\underset{\oplus}{f(x)}} = 2 \cdot (+\infty) = +\infty$

(δ) $\lim_{x \to 4} \frac{g(x)}{f(x)} = \frac{-2}{0} = +\infty \cdot (-2) = -\infty$.

$\lim_{x \to 4} g(x) \frac{1}{\underset{\oplus}{f(x)}} = -2 \cdot (+\infty) = -\infty$

(ε). $\lim_{x \to 5} \frac{f(x)}{x-5}$

- $\lim_{x \to 5^-} f(x) \frac{1}{x-5} = 1 \cdot (-\infty) = -\infty$

$-\infty$

- $\lim_{x \to 5^+} f(x) \frac{1}{x-5} = -2 \cdot (+\infty) = -\infty$

Σελ 170

(11) (α) $\lim_{x\to 1} \frac{2x^2-\lambda x-\lambda}{(x-1)^2} = \lim_{x\to 1} (2x^2-\lambda x-\lambda)\frac{1}{(x-1)^2}$

$= (2-2\lambda)\cdot(+\infty)$.

1. Αν $2-2\lambda>0$ τοτε $+\infty$

$\Rightarrow 2>2\lambda$

$1>\lambda$

2. Αν $2-2\lambda<0$ τοτε $-\infty$.

$\lambda>1$

3. Αν $2-2\lambda=0 \Rightarrow 2=2\lambda \Rightarrow$ Αν $\lambda=1$

Τοτε $\lim_{x\to 1} \frac{2x^2-x-1}{(x-1)^2} = \lim_{x\to 1} \frac{2\cancel{(x-1)}(x+\frac{1}{2})}{(x-1)^{\cancel{2}}}$

$= \lim_{x\to 1} 2(x+\frac{1}{2})\cdot\frac{1}{x-1}$ Το οριο δεν υπαρχει γιατι →

$$\lim_{x \to 1^-} 2\left(x+\frac{1}{2}\right) \frac{1}{x-1} = 3 \cdot (-\infty) = -\infty$$

$$\lim_{x \to 1^+} 2\left(x+\frac{1}{2}\right) \frac{1}{x-1} = 3(+\infty) = +\infty$$

(16) (a) $\lim_{x\to 1} (x-1)^2 f(x) = -3$

$g(x) = (x-1)^2 f(x)$ άρα $\lim_{x\to 1} g(x) = -3$

$f(x) = \dfrac{g(x)}{(x-1)^2}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{g(x)}{(x-1)^2} = \lim_{x\to 1} g(x) \dfrac{1}{(x-1)^2}$

$\lim_{x\to 1} f(x) = -3 \cdot (+\infty) = -\infty$

$\lim_{x\to 1} f(x) = -\infty$

(β) $\lim_{x\to 1} \dfrac{f(x)}{x^2+1} = -\infty$

$g(x) = \dfrac{f(x)}{x^2+1}$ $\lim_{x\to 1} g(x) = -\infty$

$f(x) = g(x)(x^2+1)$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)(x^2+1)$

$\lim_{x\to 1} f(x) = -\infty \cdot 2 = -\infty$

12

$$\lim_{x\to 2} \frac{x^2-2\lambda x+\lambda^2}{x^2-4x+4} = \lim_{x\to 2}\left(x^2-2\lambda x+\lambda^2\right)\frac{1}{(x-2)^2}$$

$$= \left(4-4\lambda+\lambda^2\right)(+\infty) = (2-\lambda)^2(+\infty)$$

$\forall \lambda \neq 2$ τότε $+\infty$.

<u>Αν $\lambda = 2$</u>

$$\lim_{x\to 2} \frac{\cancel{x^2-4x+4}}{\cancel{x^2-4x+4}} = 1.$$

17 $\lim\limits_{x \to 0} x^4 f(x) = 2$

(α) $\lim\limits_{x \to 0} f(x)$

$g(x) = x^4 f(x)$ άρα $\lim\limits_{x \to 0} g(x) = 2$

$f(x) = \dfrac{g(x)}{x^4}$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{g(x)}{x^4} = \lim_{x \to 0} g(x) \frac{1}{x^4} = 2 \cdot +\infty$$

$\lim\limits_{x \to 0} f(x) = +\infty$

(β) $\lim\limits_{x \to 0} \dfrac{x^2 - 3}{f(x)} = 0.$

(γ) $$\lim_{x \to 0} \frac{x}{f(x)\, ημ^5 x} = \lim_{x \to 0} \frac{\frac{1}{x^4}}{f(x) \, \frac{ημ^5 x}{x^5}} =$$

$$= \lim_{x \to 0} \frac{1}{\frac{ημ^5 x}{x^5}} \cdot \frac{1}{f(x)} \cdot \frac{1}{x^4} = \lim_{x \to 0} \frac{1}{\frac{ημ^5 x}{x^5}} \cdot \frac{1}{x^4 f(x)} = 1 \cdot 2$$

$$\underline{\underline{= 2}}$$

15 (B) $(1+x^2)f(x) \leq x^3$

Ψαχνω το $\lim\limits_{x\to-\infty} f(x)$.

$$f(x) \leq \frac{x^3}{x^2+1}$$

$$\lim_{x\to-\infty} f(x) \leq \lim_{x\to-\infty} \frac{x^3}{x^2+1}$$

$$\lim_{x\to-\infty} f(x) \leq \lim_{x\to-\infty} \frac{x^3}{x^2}$$

$$\lim_{x\to-\infty} f(x) \leq \lim_{x\to-\infty} x$$

$$\lim_{x\to-\infty} f(x) \leq -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

Σελ 192

14 (B) $\left|(x^2+1)f(x)-3x^2\right| \le |x|$

Ψάχνω $\lim\limits_{x\to+\infty} f(x)$

$$-|x| \le (x^2+1)f(x)-3x \le |x|$$

$$3x-|x| \le (x^2+1)f(x) \le |x|+3x$$

$$\boxed{\frac{3x-|x|}{x^2+1} \le f(x) \le \frac{|x|+3x}{x^2+1}}$$

- $$\lim_{x\to+\infty}\frac{3x-\overset{+}{|x|}}{x^2+1}=\lim_{x\to+\infty}\frac{3x-x}{x^2+1}=\lim_{x\to+\infty}\frac{2x}{x^2+1}$$

$$=\lim_{x\to+\infty}\frac{2\cancel{x}}{x^{\cancel{2}}}=\lim_{x\to+\infty}\frac{2}{x}=0$$

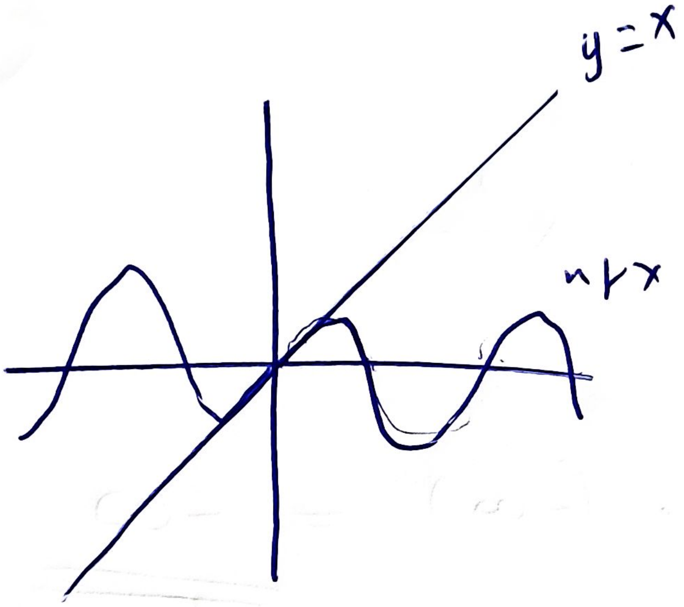
- $$\lim_{x\to+\infty}\frac{|x|+3x}{x^2+1}=\lim_{x\to+\infty}\frac{x+3x}{x^2+1}=\lim_{x\to+\infty}\frac{4x}{x^2}=0$$

Άρα Κ.Π. $\lim\limits_{x\to+\infty} f(x)=0$.

Προσοχη

Βασικη ανισοτητα της υλης.

$$|\eta\mu x| \le |x|$$



Παρατηρω οτι

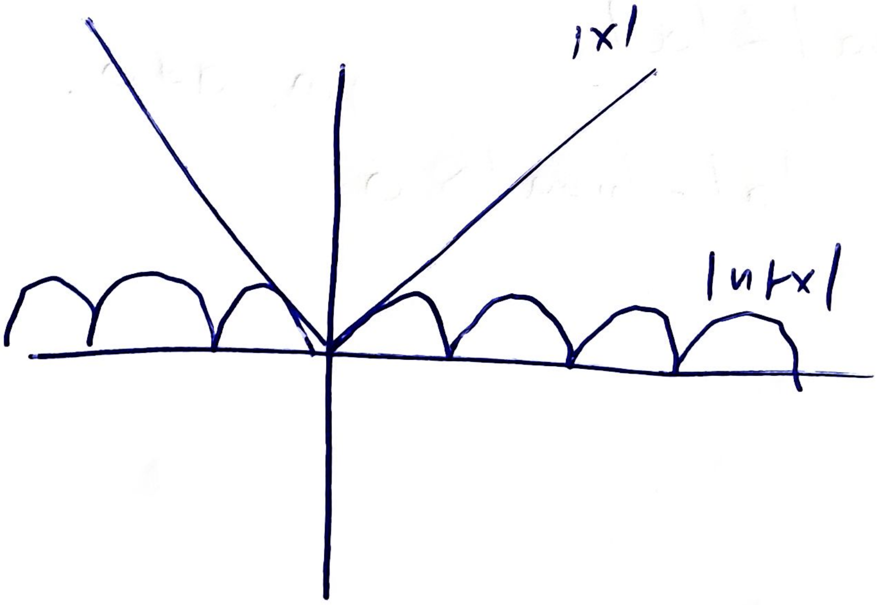
1. Οταν $x<0$

   $\eta\mu x > x$

2. Οταν $x>0$

   $\eta\mu x < x$

Αρα $\eta\mu x = x$ εχει μοναδικη λυση $x=0$.



(16) (β) $\lim_{x\to+\infty} \frac{\sqrt[3]{x^2}}{x} = \lim_{x\to+\infty} \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} =$

$= \lim_{x\to+\infty} \sqrt[3]{\frac{x^2}{x^3}} = \lim_{x\to+\infty} \sqrt[3]{\frac{1}{x}} = 0$

(17) (γ) $\lim_{x\to-\infty} \left(\left(|\alpha| - |\eta\mu\alpha| \right) x^3 - x - 1 \right)$ $\underline{\underline{\alpha \neq 0}}$

$= \lim_{x\to-\infty} \left(|\alpha| - |\eta\mu\alpha| \right) x^3 =$

$= \underset{+}{\left(|\alpha| - |\eta\mu\alpha| \right)} \cdot (-\infty) = \underline{\underline{-\infty}}$

Γνωρίζω ότι $|\eta\mu\alpha| < |\alpha|$ για $\alpha \neq 0$.

$|\alpha| - |\eta\mu\alpha| > 0$

(18) (B) $\lim_{x\to+\infty} (\lambda^2-2)x^4 - \lambda x + 1 =$

$= \lim_{x\to+\infty} (\lambda^2-2)x^4 = (\lambda^2-2)(+\infty)$

1. Αν $\lambda^2-2>0$ τοτε $+\infty$

λ	$-\sqrt{2}$ $\sqrt{2}$
λ^2-2	$+$ 0 $-$ 0 $+$

$\lambda\in(-\infty,-\sqrt{2})\cup(\sqrt{2},+\infty)$.

2. Αν $\lambda^2-2<0$ τοτε $-\infty$

$\lambda\in(-\sqrt{2},\sqrt{2})$.

3. Αν $\lambda=\sqrt{2}$ τοτε $\lim_{x\to+\infty} -\sqrt{2}x+1 =$

$= \lim_{x\to+\infty} -\sqrt{2}x = -\infty$

4. Αν $\lambda=-\sqrt{2}$ τοτε $\lim_{x\to+\infty} \sqrt{2}x = +\infty$

(8). $\lim\limits_{x\to+\infty}\left(x^3\ln\alpha+\alpha x^2-x\right)=$

$$=\lim_{x\to+\infty} x^3\ln\alpha = (+\infty)\ln\alpha$$

1. Αν $\ln\alpha>0$ τοτε $+\infty$.

   $\Rightarrow \underline{\alpha>1}$

2. Αν $\ln\alpha<0$ τοτε $-\infty$

   $\underline{0<\alpha<1}$

3. Αν $\ln\alpha=0 \Rightarrow \alpha=1$ τοτε

$$\lim_{x\to+\infty} x^3\cdot 0+x^2-x=$$

$$=\lim_{x\to+\infty} x^2-x=\underline{+\infty}$$

(19) (B) $\lim\limits_{x\to-\infty} \dfrac{(2\mu-1)x^3-3x+1}{(\mu+1)x^2+2} =$

$$= \lim_{x\to-\infty} \frac{(2\mu-1)x^3}{(\mu+1)x^2} = \lim_{x\to-\infty} \frac{2\mu-1}{\mu+1}\, x$$

$$= \frac{2\mu-1}{\mu+1} \cdot (-\infty),$$

1. Αν $\dfrac{2\mu-1}{\mu+1} > 0$ τοτε $-\infty$

μ		-1		$\frac{1}{2}$	
$2\mu-1$	$-$	‖	$-$	0	$+$
$\mu+1$	$-$	‖	$+$	|	$+$
$\frac{2\mu-1}{\mu+1}$	$+$	‖	$-$	|	$+$

$\mu \in (-\infty, -1) \cup (\frac{1}{2}, +\infty)$

2. Αν $\mu \in (-1, \frac{1}{2})$ τοτε $+\infty$

3. Αν $\mu = -1$ τότε

$$\lim_{x \to -\infty} \frac{-3x^3 - 3x + 1}{2} = \lim_{x \to -\infty} -\frac{3}{2}x^3$$

$$= -\frac{3}{2}(-\infty)$$

$$= +\infty$$

4. Αν $\mu = \frac{1}{2}$ τότε

$$\lim_{x \to -\infty} \frac{-3x + 1}{\frac{3}{2}x^2 + 1} = \lim_{x \to -\infty} \frac{-3\cancel{x}}{\frac{3}{2}x^{\cancel{2}}} = 0.$$

20 (B) $\lim_{x\to-\infty}\left(\mu^2 x+\sqrt{x^2+1}\right)$

$=\lim_{x\to-\infty}\left(\mu^2 x+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}\right)$

$=\lim_{x\to-\infty}\left(\mu^2 x-x\sqrt{1+\frac{1}{x^2}}\right)$

$=\lim_{x\to-\infty} x\left(\mu^2-\sqrt{1+\frac{1}{x^2}}\right)$

$=-\infty\left(\mu^2-1\right).$

1. Αν $\mu^2-1>0$ τότε $-\infty$

$\mu^2>1$

$\mu\in(-\infty,-1)\cup(1,+\infty)$

2. Αν $\mu^2-1<0$ τότε $+\infty$

$\mu\in(-1,1).$

3. Αν $\mu = 1$ τοτε

$$\lim_{x \to -\infty} \left(x + \sqrt{x^2+1}\right) = \lim_{x \to -\infty} \frac{x^2 - (x^2+1)}{x - \sqrt{x^2+1}}$$

$$= \lim_{x \to -\infty} \frac{-1}{x - \sqrt{x^2+1}} = 0$$

4. Αν $\mu = -1$ τοτε

$$\lim_{x \to -\infty} \left(x + \sqrt{x^2+1}\right) = 0.$$

Επόμενο Μάθημα

Δευτέρα 22/7/2024

11:30 – 1

Σελ 192

14 α

15 α

16 α

17 α β.

18 α γ

19 α

20 α .

Σελ 192

(15) α) $f(x) \geq x^3$

Ψάχνω το $\lim_{x \to +\infty} f(x)$

$f(x) \geq x^3$

$\lim_{x \to +\infty} f(x) \geq \lim_{x \to +\infty} x^3$

$\lim_{x \to +\infty} f(x) \geq +\infty$

$\lim_{x \to +\infty} f(x) = +\infty$

(16) α) $\lim_{x \to +\infty} \frac{\sqrt[3]{x}}{x} = \lim_{x \to +\infty} \frac{\sqrt[3]{x}}{\sqrt[3]{x^3}} =$

$= \lim_{x \to +\infty} \sqrt[3]{\frac{x}{x^3}} = \lim_{x \to +\infty} \sqrt[3]{\frac{1}{x^2}} = 0,$

18 (α) $\lim_{x \to -\infty} (\lambda-1)x^3 - x + 2 =$

$= \lim_{x \to -\infty} (\lambda-1)x^3 = (\lambda-1)(-\infty)$

1. Αν $\lambda - 1 > 0$ τότε $-\infty$

   $\underline{\underline{\lambda > 1}}$

2. Αν $\lambda - 1 < 0$ τότε $+\infty$

   $\underline{\underline{\lambda < 1}}$

3. Αν $\lambda = 1$ τότε $\lim_{x \to -\infty} (-x+2) = +\infty$.

(γ) $\lim_{x \to -\infty} \left((e^{a}-1)x^5 - (a-1)x^3 - 1 \right) = \lim_{x \to -\infty} (e^{a}-1)x^5$

$= (e^{a}-1)(-\infty)$

1. Αν $e^{a} - 1 < 0$ τότε $+\infty$.

   $e^{a} < 1$

   $\underline{\underline{a < 0}}$

2. Αν $e^{a} - 1 > 0$ τότε $-\infty$

   $\underline{\underline{a > 0}}$

3. Αν $a = 0$ τότε $\lim_{x \to -\infty} x^3 - 1 = -\infty$.

19 (α)
$$\lim_{x\to+\infty}\frac{(\mu-2)x^3+x^2+3}{\mu x^2-3x+5}=$$

$$=\lim_{x\to+\infty}\frac{(\mu-2)x^3}{\mu x^2}=\lim_{x\to+\infty}\frac{(\mu-2)x}{\mu}$$

$$=\lim_{x\to+\infty}\frac{\mu-2}{\mu}\cdot x=\frac{\mu-2}{\mu}\cdot(+\infty)$$

1. Αν $\frac{\mu-2}{\mu}>0$ τοτε $+\infty$

μ		0		2	
$\mu-2$	$-$		$-$		$+$
μ	$-$		$+$		$+$
$\frac{\mu-2}{\mu}$	$+$		$-$		$+$

$\mu\in(-\infty,0)\cup(2,+\infty)$

2. Αν $\frac{\mu-2}{\mu}<0$ τοτε $-\infty$

$\mu\in(0,2)$

3. Αν $\mu=0$ τοτε $\lim_{x\to+\infty}\frac{-2x^3+x^2+3}{-3x+5}=$

$$=\lim_{x\to+\infty}\frac{-2x^3}{-3x}=\lim_{x\to+\infty}\frac{2}{3}x^2=+\infty,$$

4. Αν $\mu=2$ τοτε

$$\lim_{x\to+\infty} \frac{x^2+3}{2x^2-3x+5} = \lim_{x\to+\infty} \frac{\cancel{x^2}}{2\cancel{x^2}} = \frac{1}{2}.$$

20 (α)

$$\lim_{x\to+\infty}\left(\sqrt{4x^2+x+1}-\mu x\right)$$

$$=\lim_{x\to+\infty}\left(x\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}-\mu x\right)$$

$$=\lim_{x\to+\infty}x\left(\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}-\mu\right)$$

$$=+\infty(2-\mu)$$

1. Αν $2-\mu>0$ τοτε $+\infty$

   $\underline{\underline{\mu<2}}$

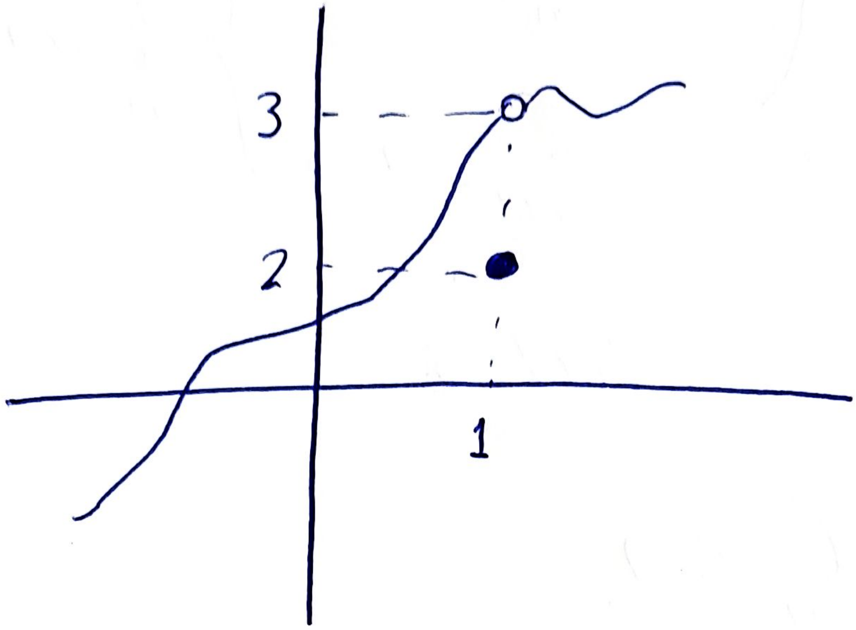
2. Αν $2-\mu<0$ τοτε $-\infty$

   $\underline{\underline{\mu>2}}$

3. Αν $\mu=2$ $\lim_{x\to+\infty}\left(\sqrt{4x^2+x+1}-2x\right)$

$$=\lim_{x\to+\infty}\frac{x+1}{\sqrt{4x^2+x+1}+2x}=\lim_{x\to+\infty}\frac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\left(\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}+2\right)}$$

$$=\frac{1}{4}.$$

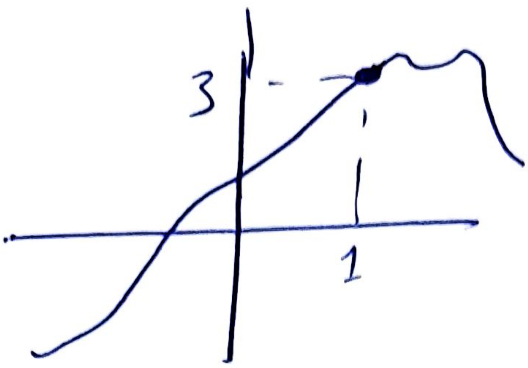


$$\left.\begin{aligned} &\lim_{x\to 1^-} f(x)=3 \\ &\lim_{x\to 1^+} f(x)=3 \end{aligned}\right\} \lim_{x\to 1} f(x)=3$$

$$f(1)=2$$

Αφού $f(1) \neq \lim_{x\to 1} f(x)$

η $f(x)$ δεν είναι συνεχής στο 1.

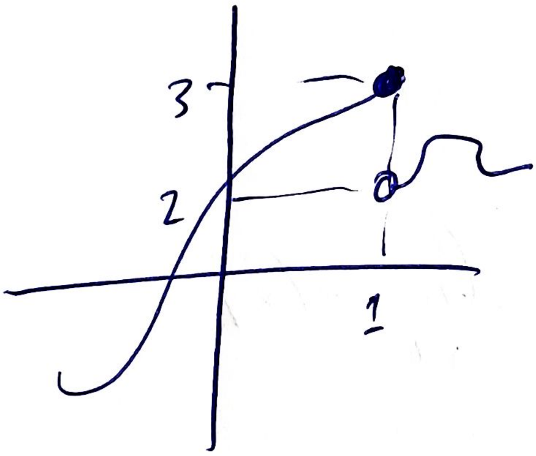


$$\lim_{x\to 1^-} f(x)=3, \quad \lim_{x\to 1^+} f(x)=3 \quad \Rightarrow \quad \lim_{x\to 1} f(x)=3$$

$$f(1)=3$$

Άρα $f(1)=\lim_{x\to 1} f(x)$.

Άρα η f συνεχής στο 1.



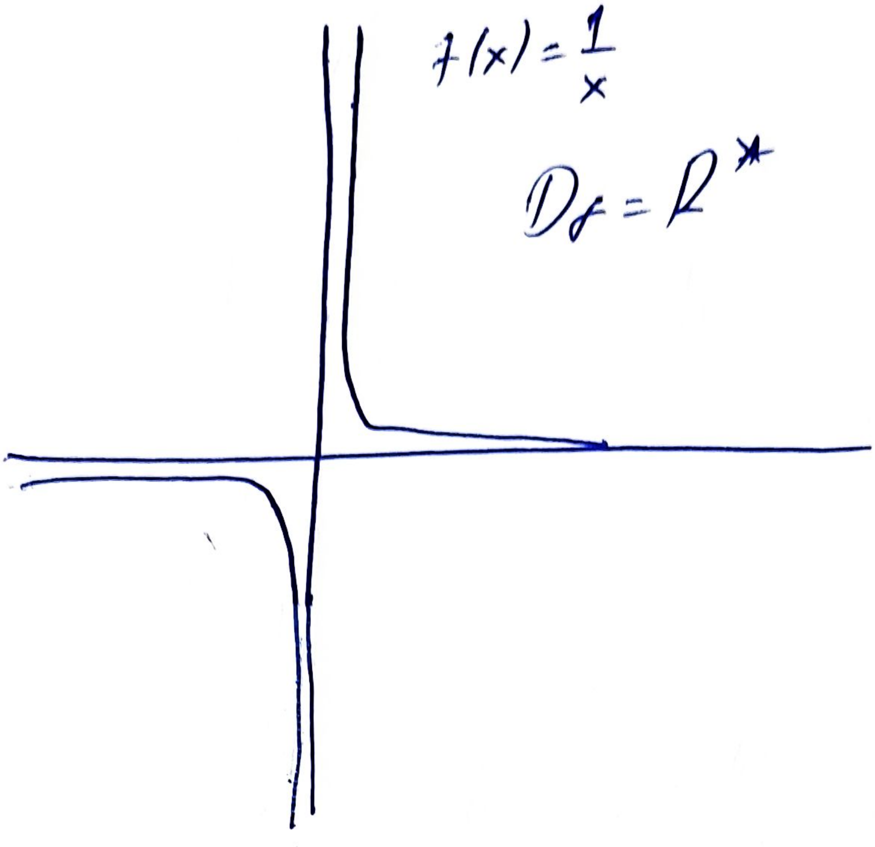
$$\lim_{x\to 1^-} f(x)=3, \quad \lim_{x\to 1^+} f(x)=2$$

Το $\lim_{x\to 1} f(x)$ δεν υπάρχει.

Προφανώς δεν είναι συνεχής στο 1.

$f(x) = \frac{1}{x}$

$D_f = \mathbb{R}^*$

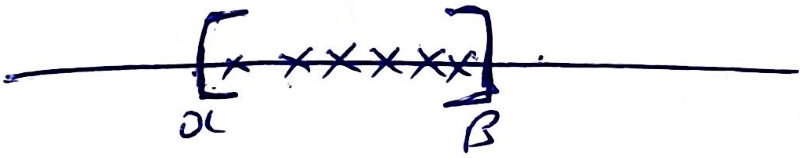


Η $f(x)$ ειναι συνεχης στο πεδιο ορισμου της παρολο που

<u>δεν</u> ειναι συνεχομενη γραμμη

Αν $\lim_{x \to x_0} f(x) = f(x_0)$ $\forall x_0 \in (\alpha, \beta)$

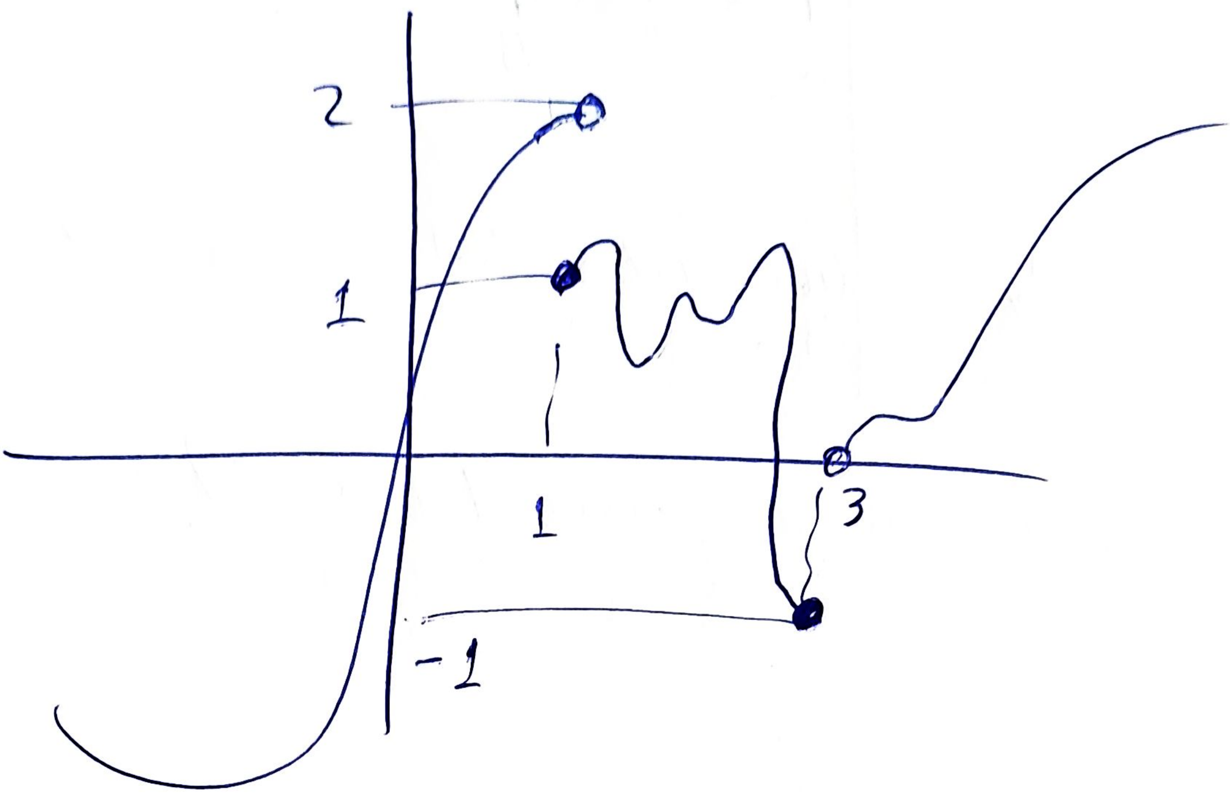
τότε η f συνεχής (α, β).



Αν η f συνεχής στο (α, β)

και επιπλέον

$\lim_{x \to \alpha^+} f(x) = f(\alpha)$ και $\lim_{x \to \beta^-} f(x) = f(\beta)$,



Είναι συνεχής στο 1;

$\lim_{x \to 1^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 1$

$\Big\}$ $\lim_{x \to 1} f(x)$ δεν υπάρχει άρα <u>ΟΧΙ</u> συνεχής στο 1.

Είναι συνεχής στο 3;

$\lim_{x \to 3^-} f(x) = -1$

$\lim_{x \to 3^+} f(x) = 0$

$\Big\}$ $\lim_{x \to 3} f(x)$ <u>δεν</u> υπάρχει άρα όχι συνεχής στο 3.

Είναι συνεχής στο $[1,3]$;

$\lim_{x \to 1^+} f(x) = 1 = f(1)$, $\lim_{x \to 3^-} f(x) = -1 = f(3)$ άρα συνεχής στο $[1,3]$

Σελ. 212

4 β) $f(x) = \begin{cases} e^x + \eta\mu x, & x \le 0 \\ \ln(x+1), & x > 0 \end{cases}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (e^x + \eta\mu x) = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \ln(x+1) = 0$$

Άρα $\boxed{\lim_{x\to 0} f(x) = \text{δεν υπάρχει}}$ άρα δεν είναι συνεχής στο 0.

$\boxed{f(0) = 1}$

δ) $f(x) = \begin{cases} \dfrac{\eta\mu x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

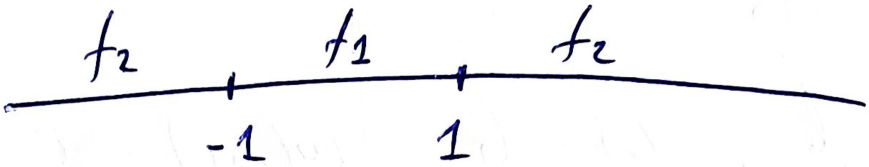
$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\eta\mu x}{x} = 1.$$

$f(0) = 0$

$f(0) \neq \lim_{x\to 0} f(x)$ όχι συνεχής στο 0.

7 (8) $f(x) = \begin{cases} x^2, & |x| \leq 1 \\ \frac{1}{x}, & |x| > 1 \end{cases}$

$f(x) = \begin{cases} x^2, & -1 \leq x \leq 1 \\ \frac{1}{x}, & x \in (-\infty, -1) \cup (1, +\infty). \end{cases}$



Η $f(x)$ είναι συνεχής στο $(-1, 1)$ ως πολυωνυμική.

Η $f(x)$ είναι συνεχής στο $(-\infty, -1)$ και στο $(1, +\infty)$

ως ρητή συνάρτηση

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{1}{x} = -1.$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^2 = 1$

Το $\lim_{x \to -1} f(x)$ δεν υπάρχει άρα η f <u>δεν</u> είναι συνεχής στο (-1).

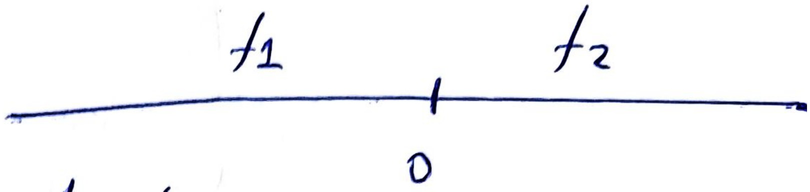
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} = 1$

$\lim_{x \to 1} f(x) = f(1) = 1$ άρα συνεχής στο 1.

8

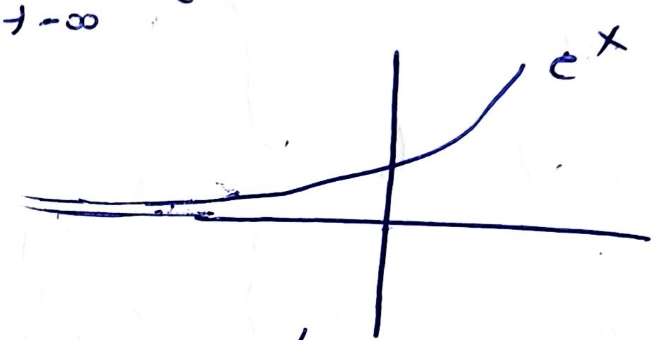
$$f(x) = \begin{cases} e^{1/x}, & x<0 \\ 1, & x=0 \\ x^2+1, & x>0 \end{cases}$$



(α)

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^{\frac{1}{x}} \xrightarrow[\substack{x\to 0^- \\ t\to -\infty}]{\frac{1}{x}=t} \lim_{t\to -\infty} e^{t} = 0$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2+1 = 1.$$



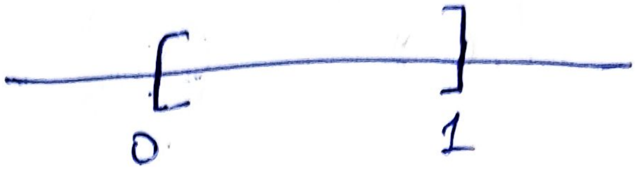
Το $\lim_{x\to 0^+} f(x)$ δεν υπάρχει!.

Άρα η f όχι συνεχής στο 0.

Η $f(x)$ συνεχής στο $(-\infty,0)$ και $(0,+\infty)$

ως πράξεις συνεχών συναρτήσεων.

B



$\lim\limits_{x \to 0^+} f(x) = 1$

$f(0) = 1$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1} (x^2+1) = 2$

$f(1) = 2$

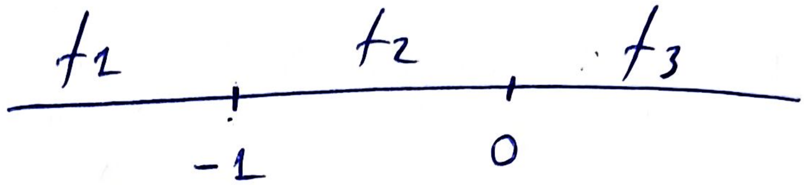
Η f συνεχής στο $(0,1)$.

Άρα συνεχής $[0,1]$.

(10)

Συνεχής!

$$f(x) = \begin{cases} e^{x+1} + \alpha x - 2, & x \leq -1 \\ 2x^2 - \beta, & -1 < x < 0 \\ \beta \eta\mu x - 7\sigma\upsilon\nu x + 2\ln(x+1), & x \geq 0 \end{cases}$$



$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$

$$\Leftrightarrow$$

$$\lim_{x\to -1^-} \left(e^{x+1} + \alpha x - 2\right) = \lim_{x\to -1^+} \left(2x^2 - \beta\right)$$

$$1 - \alpha - 2 = 2 - \beta \Leftrightarrow \boxed{-3 = \alpha - \beta}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x).$$

$$\lim_{x\to 0^-} \left(2x^2 - \beta\right) = \lim_{x\to 0^+} \left(\beta\eta\mu x - 7\sigma\upsilon\nu x + 2\ln(x+1)\right)$$

$$-\beta = -7$$

$$\beta = 7$$

$$\alpha = 4$$

Σελ 170

14 Β $x^2 f(x) + 1 \leq 0 \quad \forall x \neq 0$

i) Ψάχνω $\lim_{x\to 0} f(x)$

$$x^2 f(x) \leq -1$$

$$f(x) \leq -\frac{1}{x^2}$$

$$\lim_{x\to 0} f(x) \leq \lim_{x\to 0} -\frac{1}{x^2}$$

$$\lim_{x\to 0} f(x) \leq -\infty$$

$$\lim_{x\to 0} f(x) = -\infty$$

ii). $$\lim_{x\to 0} \frac{\eta\mu f(x)}{f(x)} \xrightarrow[\substack{x\to 0 \\ t\to -\infty}]{f(x)=t} \lim_{t\to -\infty} \frac{\eta\mu t}{t}$$

$$-1 \leq \eta\mu t \leq 1$$

$$\boxed{-\frac{1}{t} \geq \frac{\eta\mu t}{t} \geq \frac{1}{t}}$$

$t<0$ κοντα στο $-\infty$.

$$\lim_{t \to -\infty} -\frac{1}{t} = 0$$

$$\lim_{t \to -\infty} \frac{1}{t} = 0$$

Απο Κ.Π $\lim_{\cancel{x}\, t \to -\infty} \frac{\eta\mu t}{t} = 0$.

Σελ 193

23

$$\lim_{x\to+\infty} x\, f\left(\frac{x+1}{x}\right)=3.$$

Ψάχνω το $\lim_{x\to1} \frac{f(x)}{x-1}$,

Θέτω $\frac{x+1}{x}=t \iff x+1=tx$

$x-tx=-1$

$x(1-t)=-1$

$x=\frac{-1}{1-t}$

$x=\frac{1}{t-1}$

$x\to+\infty$

$t\to1$

$\lim_{x\to+\infty}\frac{x+1}{x}=1$

Γνωρίζω ότι

$$\lim_{t\to1}\frac{1}{t-1}\cdot f(t)=3$$

$$\lim_{t\to1}\frac{f(t)}{t-1}=3.$$

Ex 2 194

$\lim_{x \to 2} f(x) = +\infty$

(24) (B) $\lim_{x \to 2} \sqrt{f^2(x) + 1} - f(x)$ $\xrightarrow[\substack{x \to 2 \\ t \to +\infty}]{f(x) = t}$

$$\lim_{t \to +\infty} \left(\sqrt{t^2 + 1} - t \right) =$$

$$= \lim_{t \to +\infty} \frac{1}{\sqrt{t^2 + 1} + t} = 0 .$$

26

$$\boxed{\lim_{x\to+\infty} \frac{x f(x) - x + 1}{x+1} = 3}$$

ⓐ i) $\lim_{x\to+\infty} f(x)$

Θέτω $g(x) = \dfrac{x f(x) - x + 1}{x+1}$ τότε $\lim_{x\to+\infty} g(x) = 3$

Λύνω ως προς $f(x)$

$$\boxed{\frac{g(x)(x+1) + x - 1}{x} = f(x).}$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{g(x)(x+1) + x - 1}{x}$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \left(g(x) \frac{x+1}{x} + \frac{x-1}{x} \right) = 3 \cdot 1 + 1 = 4$$

$\to \lim_{x\to+\infty} \dfrac{x+1}{x} = \lim_{x\to+\infty} \dfrac{x}{x} = 1$

$\to \lim_{x\to+\infty} \dfrac{x-1}{x} = \lim_{x\to+\infty} \dfrac{x}{x} = 1$

$$\lim_{x\to+\infty} f(x) = 4$$

ii). $$\lim_{x \to +\infty} \frac{2x^2 f(x) + x^2 - x + 1}{2x + x f(x)}$$

$$= \lim_{x \to +\infty} \frac{2x f(x) + x - 1 + \frac{1}{x}}{2 + f(x)}$$

$$= \frac{2 \cdot (+\infty) \cdot 4 + (+\infty) - 1 + 0}{6}$$

$$= \frac{+\infty}{6} = +\infty.$$

$$\Sigma T_f = (4, +\infty).$$

(B) $f(0, +\infty) = (4, +\infty) \longrightarrow$ $4 < f(x) < +\infty$

$f(x) > 4$

$$\lim_{x \to +\infty} \frac{x}{f(x) - 4} = \lim_{x \to +\infty} x \cdot \frac{1}{f(x) - 4} \quad (+)$$

$\longrightarrow$ Αφού $f(x) > 4$

$f(x) - 4 > 0$

$$= +\infty \cdot (+\infty) = +\infty;$$

27

$$\lim_{x \to -\infty} \frac{f(x)}{x} = 2$$

$$\lim_{x \to -\infty} (f(x) - 2x) = 3.$$

$$\lim_{x \to -\infty} \frac{2f(x) + \lambda x - 1}{x f(x) - 2x^2 + 1} = 1,$$

$$\lim_{x \to -\infty} \frac{2\frac{f(x)}{x} + \lambda - \frac{1}{x}}{f(x) - 2x + \frac{1}{x}} = 1$$

$$\frac{2 \cdot 2 + \lambda}{3} = 1$$

$$\frac{4 + \lambda}{3} = 1$$

$$4 + \lambda = 3$$

$$\lambda = -1$$

Επομενο Μαθημα

Τεταρτη 8:30 - 10

Σελ 212

2

4 αγ

5

7 α β γ

9

Σελ 171

21

Σελ 170

14 α

15

Σελ 194

24 α

(21) Σελ 171

(α) $\lim\limits_{x\to 0} \dfrac{1}{\underset{\ominus}{ημ^2x - x^2}} = -\infty$

• $|ημx| \le |x| \Leftrightarrow |ημx|^2 \le |x|^2 \Leftrightarrow ημ^2x \le x^2$

$ημ^2x - x^2 \le 0.$

(β) $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{|x|}\right) = \lim\limits_{x\to 0} \dfrac{\overset{0}{|x|-x}}{x|\overset{0}{x}|}$

x	0
x	− 0 +

$\to \lim\limits_{x\to 0^-} \dfrac{\overset{\ominus}{|x|-x}}{x|\overset{\ominus}{x}|} = \lim\limits_{x\to 0^-} \dfrac{-x-x}{-x^2} = \lim\limits_{x\to 0^-} \dfrac{-2x}{-x^2}$

$= \lim\limits_{x\to 0^-} \dfrac{2}{x} = -\infty$

$\to \lim\limits_{x\to 0^+} \dfrac{\overset{\oplus}{|x|-x}}{x|\overset{\oplus}{x}|} = \lim\limits_{x\to 0^+} \dfrac{x-x}{x^2} = 0$

γιατί $f(x) = \begin{cases} \dfrac{2}{x}, & x<0 \\ 0, & x>0 \end{cases}$

(γ) $\lim_{x \to 0} \dfrac{1}{\underset{(+)}{|x| - x}} = +\infty$.

• $|x| \geq x \Rightarrow |x| - x \geq 0$.

Ιδιότητα απολύτων α' λυκείου,

• $|x| \geq x$

• $|x| \geq -x$.

24 Σελ 194

$$\boxed{\lim_{x\to 2} f(x) = +\infty}$$

ⓐ $$\lim_{x\to 2} \frac{f^3(x) - f(x) + 1}{2f^2(x) - f(x) + 3} \xlongequal[\substack{x\to 2 \\ t\to +\infty}]{f(x)=t}$$

$$= \lim_{t\to +\infty} \frac{t^3 - t + 1}{2t^2 - t + 3} = \lim_{t\to +\infty} \frac{t^3}{2t^2} =$$

$$= \lim_{t\to +\infty} \frac{t}{2} = +\infty .$$

5 Σελ 212

Ⓑ $$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\sqrt{x^2+3} - 2}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(\sqrt{x^2+3}+2)}$$

$$= \frac{2}{4} = \frac{1}{2} \neq 3 = f(1)$$

όχι συνεχής στο 1.

⑨ Σελ 213

ⓐ Αφού είναι συνεχής

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$1+\alpha = 0$$

$$\underline{\underline{\alpha = -1}}$$

ⓑ όριο $\lim_{x\to -1} f(x) = f(-1)$

$$\lim_{x\to -1} \frac{x^3+1}{x+1} = \alpha$$

$$\lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}} = \alpha$$

$$\underline{\underline{\alpha = 3}}$$

(28) Σελ 194

Β) $\lim_{x \to +\infty} \eta\mu \dfrac{x}{x^2+1} = \eta\mu 0 = 0$

$\to \lim_{x \to +\infty} \dfrac{x}{x^2+1} = \lim_{x \to +\infty} \dfrac{\cancel{x}}{x^{\cancel{2}}} = 0$

(29) Β) $\lim_{x \to -\infty} \left(\sqrt{x^2+1} + x\right) \eta\mu x = 0$

$M \times \Phi$

$\to \lim_{x \to -\infty} \left(\sqrt{x^2+1} + x\right) = \lim_{x \to -\infty} \dfrac{1}{\sqrt{x^2+1} - x} = 0$

Απόδειξη

$-1 \le \eta\mu x \le 1$

$|\eta\mu x| \le 1$

$\left|\sqrt{x^2+1} + x\right| |\eta\mu x| \le 1 \cdot \left|\sqrt{x^2+1} + x\right|$

$\left|\left(\sqrt{x^2+1} + x\right)\eta\mu x\right| \le \left|\sqrt{x^2+1} + x\right|$

$$\boxed{-\left|\sqrt{x^2+1} + x\right| \le \left(\sqrt{x^2+1} + x\right)\eta\mu x \le \left|\sqrt{x^2+1} + x\right|}$$

$\lim_{x\to-\infty} -|\sqrt{x^2+1}+x| = 0$

$\lim_{x\to-\infty} |\sqrt{x^2+1}+x| = 0$

$\Big\}$ Άρα Κ.Π

$$\lim_{x\to-\infty} \left(\sqrt{x^2+1}+x\right)\eta\mu x = 0$$

30 (i) $\lim_{x\to+\infty} x\,\eta\mu\frac{1}{x} + \frac{1}{x}\eta\mu x = 1+0 = \underline{1}$

$\rightarrow \lim_{x\to+\infty} x\,\eta\mu\frac{1}{x} = \lim_{x\to+\infty} \cancel{x}\,\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}}\,\cancel{\frac{1}{x}} = \underline{1}$

$\rightarrow \lim_{x\to+\infty} \frac{1}{x}\eta\mu x = 0$

$M \times \Phi$.

Απαιτείται απόδειξη.

(ε) $$\lim_{x\to+\infty}\frac{x\,\eta\mu\frac{1}{x}-1}{x+1}=\lim_{x\to+\infty}\frac{\cancel{x}\,\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}}\,\frac{1}{\cancel{x}}-1}{x+1}$$

$$=\lim_{x\to+\infty}\frac{\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}}-1}{x+1}=$$

$$=\lim_{x\to+\infty}\left(\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}}-1\right)\cdot\frac{1}{x+1}=0\cdot 0=0.$$

Το $\frac{0}{\infty}$ δεν είναι πρόβλημα

γιατί $\frac{0}{\infty}=0\cdot\frac{1}{\infty}=0\cdot 0$

$=0$

31 B $\lim\limits_{x\to+\infty} \dfrac{x^2+x\eta\mu x+1}{x^2+\eta\mu x+3} =$

$$= \lim_{x\to+\infty} \frac{1+\cancelto{0}{\dfrac{\eta\mu x}{x}}+\dfrac{1}{x^2}}{1+\cancelto{0}{\dfrac{\eta\mu x}{x^2}}+\dfrac{3}{x^2}} = \frac{1+0+0}{1+0+0} = 1.$$

$\to \lim\limits_{x\to+\infty} \dfrac{\eta\mu x}{x} = 0$

$-1 \leq \eta\mu x \leq 1$

Το $x>0$ στο $+\infty$

$$\boxed{-\frac{1}{x} \leq \frac{\eta\mu x}{x} \leq \frac{1}{x}}$$

$$\left.\begin{array}{l} \lim\limits_{x\to+\infty} -\dfrac{1}{x}=0 \\ \lim\limits_{x\to+\infty} \dfrac{1}{x}=0 \end{array}\right\} \text{Από Κ.Π.}$$

Όμοια $\lim\limits_{x\to+\infty} \dfrac{\eta\mu x}{x^2} = 0.$

β) $\lim\limits_{x\to+\infty} \dfrac{x^3 \eta\mu \frac{1}{x}}{\sqrt{x^2+1}-1} = \lim\limits_{x\to+\infty} \dfrac{x^3}{\sqrt{x^2+1}-1} \left(\dfrac{\eta\mu \frac{1}{x}}{\frac{1}{x}} \right) \dfrac{1}{x}$ ΙΞ

$\underline{\underline{\ast}}$

$\rightarrow \lim\limits_{x\to+\infty} \dfrac{x^2}{\sqrt{x^2+1}-1} = \lim\limits_{x\to+\infty} \dfrac{\cancel{x^2}\left(\sqrt{x^2+1}+1\right)}{\cancel{x^2}} = +\infty$

$\underline{\underline{\ast}} \quad +\infty \cdot 1 = +\infty$

$\rightarrow \lim\limits_{x\to+\infty} \dfrac{\eta\mu \frac{1}{x}}{\frac{1}{x}} \overset{\frac{1}{x}=u}{\underset{\substack{x\to+\infty \\ u\to 0}}{=}} \lim\limits_{u\to 0} \dfrac{\eta\mu u}{u} = 1.$

32 (B) $\lim\limits_{x\to-\infty}\left(\dfrac{x^2}{x+1}+\sigma\upsilon\nu x\right) = \underline{\underline{-\infty}}$

$-1 \le \sigma\upsilon\nu x \le 1$

$$\boxed{\frac{x^2}{x+1}-1 \le \frac{x^2}{x+1}+\sigma\upsilon\nu x \le \frac{x^2}{x+1}+1.}$$

$\cdot \lim\limits_{x\to-\infty} \dfrac{x^2}{x+1}-1 = -\infty$

$\cdot \lim\limits_{x\to-\infty} \dfrac{x^2}{x+1}-1 = -\infty$

Από Κ.Π

$\to \lim\limits_{x\to-\infty} \dfrac{x^2}{x+1} = \lim\limits_{x\to-\infty} \dfrac{x^{\cancel{2}}}{\cancel{x}} = -\infty$

(33) (B) $\lim\limits_{x \to +\infty} \dfrac{x^2}{2-ημx} = +\infty$.

$$-1 \leq ημx \leq 1$$

$$1 \geq -ημx \geq -1$$

$$3 \geq 2-ημx \geq 1$$

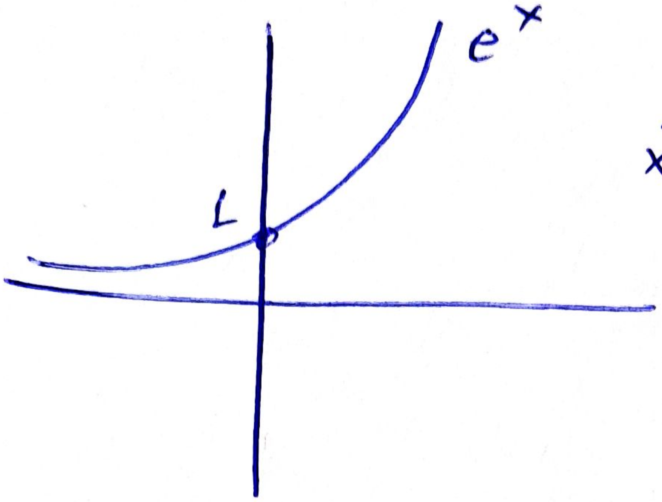
$$\frac{1}{3} \leq \frac{1}{2-ημx} \leq 1$$

$$\boxed{\frac{x^2}{3} \leq \frac{x^2}{2-ημx} \leq x^2}$$

- $\lim\limits_{x \to +\infty} \dfrac{x^2}{3} = +\infty$
- $\lim\limits_{x \to +\infty} x^2 = +\infty$

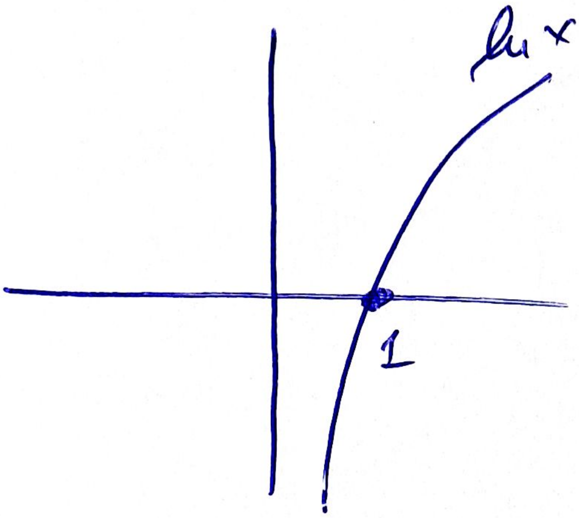
$\Big\}$ Από Κ.Π

Κωδικι Ονοματσια



$\lim_{x \to -\infty} e^x = 0$

$\lim_{x \to +\infty} e^x = +\infty$



$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to +\infty} \ln x = +\infty.$

$e^{-\infty} = 0 \qquad e^{+\infty} = +\infty$

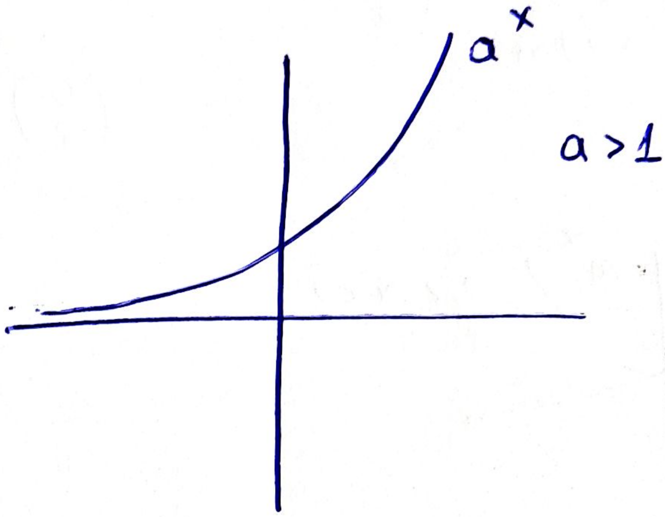
$\ln 0 = -\infty \qquad \ln +\infty = +\infty.$

(34) (γ) $\lim\limits_{x\to+\infty} \dfrac{2^{3x}}{3^{x+1}} = \lim\limits_{x\to+\infty} \dfrac{(2^3)^x}{3^x \cdot 3^1} =$

$= \lim\limits_{x\to+\infty} \dfrac{8^x}{3 \cdot 3^x} = \lim\limits_{x\to+\infty} \dfrac{1}{3} \cdot \dfrac{8^x}{3^x}$

$= \lim\limits_{x\to+\infty} \dfrac{1}{3} \cdot \left(\dfrac{8}{3}\right)^x$

$= \dfrac{1}{3} \cdot +\infty = +\infty$.



(35) (γ) $\lim\limits_{x\to-\infty} (e^x - \pi^x - 3^x) =$

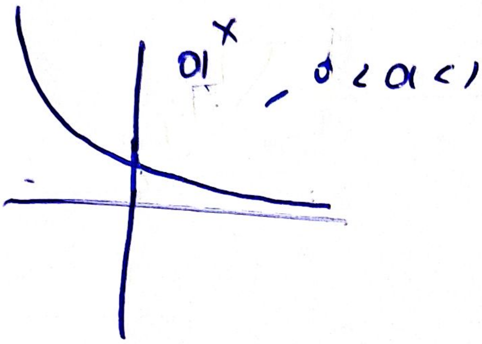
$= 0 - 0 - 0 = 0$

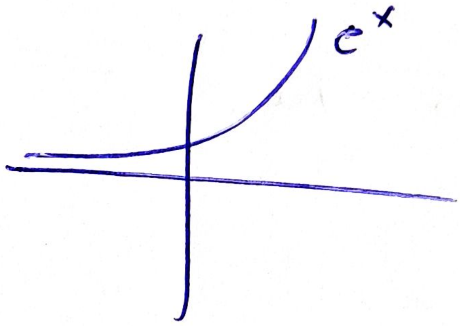
(36) (β) $\lim\limits_{x\to+\infty} \dfrac{e^x - 3 \cdot 2^x - 1}{e^x + 3^x - 1} =$,

Οταν το $x \to +\infty$ βγαζω κοινο παραγοντα το μεγαλυτερο εκθετικο πανω και κατω ξεχωριστα, ενω οταν $x \to -\infty$ βγαζω το μικροτερο εκθετικο κοινο παραγοντα.

$$= \lim_{x \to +\infty} \frac{e^x\left(1 - 3\,\frac{2^x}{e^x} - \frac{1}{e^x}\right)}{3^x\left(\frac{e^x}{3^x} + 1 - \frac{1}{3^x}\right)} =$$

$$= \lim_{x \to +\infty} \cancelto{0}{\left(\frac{e}{3}\right)^x} \cdot \frac{1 - 3\cancelto{0}{\left(\frac{2}{e}\right)^x} - \cancelto{0}{\frac{1}{e^x}}}{\cancelto{0}{\left(\frac{e}{3}\right)^x} + 1 - \cancelto{0}{\frac{1}{3^x}}} = 0 \cdot \frac{1}{1} = 0.$$

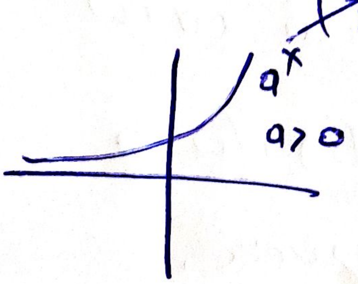




⑧ $$\lim_{x \to -\infty} \frac{2^{x+1} - 3^{x+2}}{e^x - 2^{x+1}} = \lim_{x \to -\infty} \frac{2^x \cdot 2^1 - 3^x \cdot 3^2}{e^x - 2^x \cdot 2^1} =$$

$$= \lim_{x \to -\infty} \frac{2 \cdot 2^x - 9 \cdot 3^x}{e^x - 2 \cdot 2^x} = \lim_{x \to -\infty} \frac{\cancel{2^x}\left(2 - 9 \cdot \frac{3^x}{2^x}\right)}{\cancel{2^x}\left(\frac{e^x}{2^x} - 2\right)}$$

$$= \lim_{x \to -\infty} \frac{2 - 9 \cdot \cancelto{0}{\left(\frac{3}{2}\right)^x}}{\cancelto{0}{\left(\frac{e}{2}\right)^x} - 2} = \frac{2 - 0}{0 - 2} = \underline{-1}$$



38 γ) $\lim\limits_{x\to -\infty} \dfrac{e^x}{1+x^2} = \lim\limits_{x\to -\infty} e^x \cdot \dfrac{1}{1+x^2}$

$= 0 \cdot 0 = 0,$

39 δ) $\lim\limits_{x\to -\infty} e^{-x^2} = e^{-\infty} = 0$

στ) $\lim\limits_{x\to 0^-} e^{\frac{1}{x}} = e^{-\infty} = 0$

ζ) $\lim\limits_{x\to 1^+} e^{\frac{x}{x-1}} = e^{+\infty} = +\infty.$

40 γ) $\lim\limits_{x\to +\infty} \left(e^{-x} + \dfrac{1}{x+1} - 1\right) = e^{-\infty} + 0 - 1 =$

$= 0 - 1 = -1$

δ) $\lim\limits_{x\to 0^+} \dfrac{e^{\frac{1}{x}}}{x} = \lim\limits_{x\to 0^+} e^{\frac{1}{x}} \cdot \dfrac{1}{x} = e^{+\infty} (+\infty)$

$= (+\infty) \cdot (+\infty) = \underline{\underline{+\infty}}$

Προσοχη

$$\frac{\infty}{0} = \infty \cdot \frac{1}{0} = \infty \cdot \infty = \infty$$

$$\frac{0}{\infty} = 0 \cdot \frac{1}{\infty} = 0 \cdot 0 = 0$$

$$\left.\begin{array}{l} \dfrac{\infty}{\infty} \\ \\ \dfrac{0}{0} \end{array}\right\} \text{Απροσδιοριστα}$$

40 σε $\lim\limits_{x \to 0} \dfrac{e^x}{e^{-x^2}-1} = -\infty$

$\ominus$

- $x^2 \geq 0$

$-x^2 \leq 0$

$e^{-x^2} \leq e^0$

$e^{-x^2} \leq 1$

$$\boxed{e^{-x^2} - 1 \leq 0}$$

41 B $\lim\limits_{x \to +\infty} \dfrac{\sigma\omega x}{e^x} = 0.$

$-1 \leq \sigma\omega x \leq 1$

$$\boxed{-\frac{1}{e^x} \leq \frac{\sigma\omega x}{e^x} \leq \frac{1}{e^x}}$$

$$\left.\begin{aligned} &\lim_{x \to +\infty} -\frac{1}{e^x} = 0 \\ &\lim_{x \to +\infty} \frac{1}{e^x} = 0 \end{aligned}\right\}$$

42

$$\lim_{x\to+\infty} \frac{e^x}{2+\eta\mu x} = +\infty$$

$$-1 \le \eta\mu x \le 1$$

$$1 \le 2+\eta\mu x \le 3$$

$$1 \ge \frac{1}{2+\eta\mu x} \ge \frac{1}{3}$$

$$\boxed{e^x \ge \frac{e^x}{2+\eta\mu x} \ge \frac{e^x}{3}.}$$

$$\left.\begin{array}{l} \cdot \lim\limits_{x\to+\infty} e^x = +\infty \\ \cdot \lim\limits_{x\to+\infty} \dfrac{e^x}{3} = +\infty \end{array}\right\} \text{Από Κ.Π}$$

Επομενο Μαθημα
Παρασκευη 11-1

Σελ 194 - 195 - 196

28 α

29 α

30 α β δ

31 α γ

32 α

33 α

34 α β

35 α β

36 α γ

38 α β

39 α β γ ε

40 α β ε

41 α

42 α β

28 Σελ 194

(α) $\lim\limits_{x\to+\infty}\left(ημ\frac{1}{x} - συν\frac{2}{x}\right) = ημ0 - συν0 =$

$= 0 - 1 = -1$

α' τρόπος

29 (α) $\lim\limits_{x\to+\infty}\frac{xημx}{x^2+1} = \lim\limits_{x\to+\infty}\frac{x}{x^2+1}\cdot ημx = 0$

(Μ × Φ)

$\rightarrow \lim\limits_{x\to+\infty}\frac{x}{x^2+1} = \lim\limits_{x\to+\infty}\frac{x}{x^2} = \lim\limits_{x\to+\infty}\frac{1}{x} = 0$

Απόδειξη

$-1 \le ημx \le 1$

$|ημx| \le 1$

$\left|\frac{x}{x^2+1}\right||ημx| \le 1\cdot\left|\frac{x}{x^2+1}\right|$

$\left|\frac{x}{x^2+1}ημx\right| \le \left|\frac{x}{x^2+1}\right|$

$$\boxed{\left|\frac{x}{x^2+1}\right| \le \frac{xημx}{x^2+1} \le \left|\frac{x}{x^2+1}\right|}$$

$$\lim_{x\to+\infty} -\left|\frac{x}{x^2+1}\right| = 0$$

$$\lim_{x\to+\infty} \left|\frac{x}{x^2+1}\right| = 0$$

Από Κ.Π

$$\lim_{x\to+\infty} \frac{x}{x^2+1}\,ημx = 0.$$

B' τρόπος

$$-1 \le ημx \le 1$$

$$-x \le x\,ημx \le x$$

Το $x>0$ κοντά στο $+\infty$.

$$\boxed{-\frac{x}{x^2+1} \le \frac{x\,ημx}{x^2+1} \le \frac{x}{x^2+1}}$$

$$\lim_{x\to+\infty} -\frac{x}{x^2+1} = 0$$

$$\lim_{x\to+\infty} \frac{x}{x^2+1} = 0$$

Από Κ.Π

$$\lim_{x\to+\infty} \frac{x\,ημx}{x^2+1} = 0$$

30

(α) $\lim\limits_{x\to-\infty} x\,\eta\mu\frac{1}{x} = \lim\limits_{x\to-\infty} \cancel{x}\,\frac{\eta\mu\frac{1}{x}}{\frac{1}{x}}\,\cancel{\frac{1}{x}}$

$= \lim\limits_{x\to-\infty} \frac{\eta\mu\frac{1}{x}}{\frac{1}{x}} = 1$

(β) $\lim\limits_{x\to-\infty} \frac{\eta\mu x}{x}$

$-1 \leq \eta\mu x \leq 1$

$\boxed{-\frac{1}{x} \geq \frac{\eta\mu x}{x} \geq \frac{1}{x}}$ γιατί το $x<0$ κοντά στο $-\infty$.

$$\left.\begin{aligned} &\lim_{x\to-\infty} -\frac{1}{x} = 0 \\ &\lim_{x\to-\infty} \frac{1}{x} = 0 \end{aligned}\right\} \lim_{x\to-\infty} \frac{\eta\mu x}{x} = 0$$

Από Κ.Π.

⑧ $\lim\limits_{x\to-\infty} \eta\mu x \cdot \eta\mu \frac{1}{x} = 0$

Λύση

$-1 \le \eta\mu x \le 1$

$|\eta\mu x| \le 1$

$|\eta\mu x|\,|\eta\mu \frac{1}{x}| \le 1 \cdot |\eta\mu \frac{1}{x}|$

$|\eta\mu x\, \eta\mu \frac{1}{x}| \le |\eta\mu \frac{1}{x}|$

$$\boxed{-|\eta\mu \tfrac{1}{x}| \le \eta\mu x\, \eta\mu \tfrac{1}{x} \le |\eta\mu \tfrac{1}{x}|}$$

$$\left.\begin{array}{l} \lim\limits_{x\to-\infty} -|\eta\mu \frac{1}{x}| = 0 \\ \lim\limits_{x\to-\infty} |\eta\mu \frac{1}{x}| = 0 \end{array}\right\} \text{Από Κ.Π}$$

$\lim\limits_{x\to 0} \eta\mu x\, \eta\mu \frac{1}{x} = 0$

31 (α) $\lim_{x \to +\infty} \dfrac{x + \eta\mu x}{x+1}$

$-1 \le \eta\mu x \le 1$

$x - 1 \le x + \eta\mu x \le 1 + x$

Το $x+1>0$ κοντά στο $+\infty$,

$$\boxed{\frac{x-1}{x+1} \le \frac{x+\eta\mu x}{x+1} \le 1.}$$

$\lim_{x \to +\infty} \dfrac{x-1}{x+1} = \lim_{x \to +\infty} \dfrac{x}{x} = 1$.

$\lim_{x \to +\infty} 1 = 1$

Από Κ.Π

$\lim_{x \to +\infty} \dfrac{x+\eta\mu x}{x+1} = 1,$

(γ) $\lim_{x\to+\infty} \dfrac{x^2 \eta\mu \frac{1}{x}}{2x+3} = \lim_{x\to+\infty} \dfrac{\cancel{x^2}\, \dfrac{\eta\mu \frac{1}{x}}{\frac{1}{x}}\, \cancel{\frac{1}{x}}}{2x+3}$

$= \lim_{x\to+\infty} \dfrac{x}{2x+3} \cdot \dfrac{\eta\mu \frac{1}{x}}{\frac{1}{x}} = \dfrac{1}{2} \cdot 1 = \dfrac{1}{2}$

$\rightarrow \lim_{x\to+\infty} \dfrac{x}{2x+3} = \lim_{x\to+\infty} \dfrac{\cancel{x}}{2\cancel{x}} = \dfrac{1}{2}$

32 (α) $\lim\limits_{x\to+\infty} x^2+\eta\mu x$

$$-1 \le \eta\mu x \le 1$$

$$\boxed{x^2-1 \le x^2+\eta\mu x \le x^2+1}$$

$$\left.\begin{array}{l} \cdot \lim\limits_{x\to+\infty} x^2-1=+\infty \\ \cdot \lim\limits_{x\to+\infty} x^2+1=+\infty \end{array}\right\}$$

Απο Κ.Π $\lim\limits_{x\to+\infty} x^2+\eta\mu x=+\infty$.

33 (α) $\lim\limits_{x\to+\infty} \dfrac{x}{2+\eta\mu x}$

$$-1 \le \eta\mu x \le 1$$

$$1 \le 2+\eta\mu x \le 3$$

$$1 \ge \frac{1}{2+\eta\mu x} \ge \frac{1}{3}$$

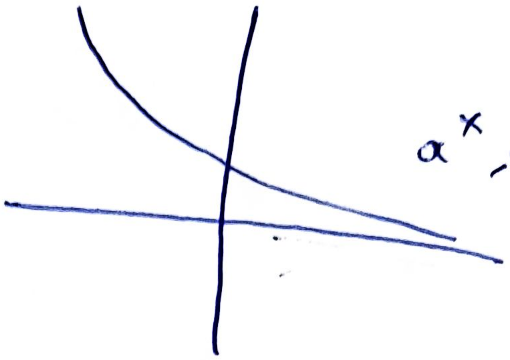
$$\boxed{x \ge \frac{x}{2+\eta\mu x} \ge \frac{x}{3}}$$

$$\left.\begin{array}{l} \cdot \lim\limits_{x\to+\infty} x=+\infty \\ \cdot \lim\limits_{x\to+\infty} \dfrac{x}{3}=+\infty \end{array}\right\}$$

Απο Κ.Π

$$\lim\limits_{x\to+\infty} \frac{x}{2+\eta\mu x}=+\infty$$

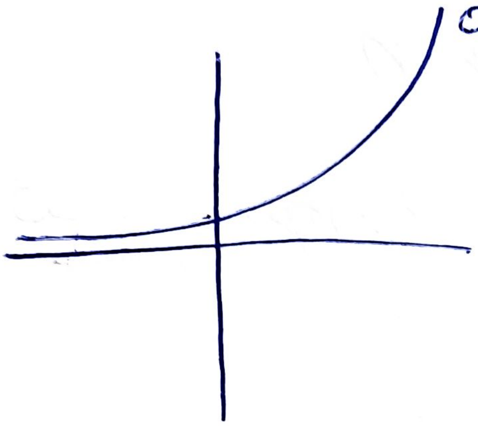
34 (a) $\lim\limits_{x \to +\infty} \left(\frac{e}{3}\right)^x = 0$



a^x, $0<a<1$

(B) $\lim\limits_{x \to -\infty} \left(\frac{\pi}{3}\right)^x = 0.$

a^x



, $a>1$.

35 (a) $\lim\limits_{x \to +\infty} \left(e^x - 2^x + 1\right) = \lim\limits_{x \to +\infty} e^x\left(1 - \frac{2^x}{e^x} + \frac{1}{e^x}\right)$

$= \lim\limits_{x \to +\infty} e^x\left(1 - \cancel{\left(\frac{2}{e}\right)^x}^{\,0} + \cancel{\frac{1}{e^x}}^{\,0}\right) = +\infty(1-0-0)$

$= +\infty.$

(B) $\lim\limits_{x \to +\infty} \left(e^x - 3^{x+1} - 2\right) = \lim\limits_{x \to +\infty} \left(e^x - 3^x \cdot 3 - 2\right)$

$= \lim\limits_{x \to +\infty} 3^x\left(\frac{e^x}{3^x} - 3 - \frac{2}{3^x}\right) = \lim\limits_{x \to +\infty} 3^x\left(\cancel{\left(\frac{e}{3}\right)^x}^{\,0} - 3 - \cancel{\frac{2}{3^x}}\right)$

$= +\infty\,(-3) = -\infty.$

36 (α) $\lim\limits_{x\to+\infty} \dfrac{3^x+3\cdot 2^x}{3^x-2^x} =$

$= \lim\limits_{x\to+\infty} \dfrac{\cancel{3^x}\left(1+3\cdot\frac{2^x}{3^x}\right)}{\cancel{3^x}\left(1-\frac{2^x}{3^x}\right)} =$

$= \lim\limits_{x\to+\infty} \dfrac{1+3\cdot\cancelto{0}{\left(\frac{2}{3}\right)^x}}{1-\cancelto{0}{\left(\frac{2}{3}\right)^x}} = 1$

(γ) $\lim\limits_{x\to-\infty} \dfrac{3^{x+1}+5e^x}{2\cdot 3^x-e^x} = \lim\limits_{x\to-\infty} \dfrac{3^x\cdot 3+5e^x}{2\cdot 3^x-e^x}$

$= \lim\limits_{x\to-\infty} \dfrac{\cancel{e^x}\left(\frac{3^x}{e^x}3+5\right)}{\cancel{e^x}\left(2\cdot\frac{3^x}{e^x}-1\right)} = \lim\limits_{x\to-\infty} \dfrac{\cancelto{0}{\left(\frac{3}{e}\right)^x}3+5}{2\cdot\cancelto{0}{\left(\frac{3}{e}\right)^x}-1}$

$= -5$

38

(α) $\lim\limits_{x\to-\infty} \dfrac{e^x}{x} = \lim\limits_{x\to-\infty} e^x \dfrac{1}{x} = 0\cdot 0 = 0$

(β) $\lim\limits_{x\to-\infty} \dfrac{x}{e^x} = \lim\limits_{x\to-\infty} x \cdot \dfrac{1}{e^x} = (-\infty)\cdot(+\infty)$

$= -\infty$.

39

(α) $\lim\limits_{x\to+\infty} e^{-x} = e^{-\infty} = 0$,

(β) $\lim\limits_{x\to-\infty} e^{-x} = e^{+\infty} = +\infty$

(γ) $\lim\limits_{x\to+\infty} e^{-x^2} = e^{-\infty} = 0$

(ε) $\lim\limits_{x\to0^+} e^{\frac{1}{x}} = e^{+\infty} = +\infty$,

40 α) $\lim\limits_{x\to+\infty} \left(e^{x} + \frac{1}{x} - 1\right) = e^{+\infty} + 0 - 1 = +\infty$

β) $\lim\limits_{x\to-\infty} e^{x}\, ημ\frac{1}{x} = e^{-\infty}\, ημ0 = 0 \cdot 0 = 0,$

ε) $\lim\limits_{x\to0} \dfrac{1}{\underset{(+)}{e^{|x|}-1}} = +\infty$

$\bullet\ |x| \geq 0 \Rightarrow e^{|x|} \geq e^{0} \Rightarrow e^{|x|} \geq 1$

$e^{|x|} - 1 \geq 0$

41 α) $\lim\limits_{x\to-\infty} e^{x}\, ημx = 0$

$-1 \leq ημx \leq 1$

$\boxed{-e^{x} \leq e^{x}\, ημx \leq e^{x}}$

$\bullet \lim\limits_{x\to+\infty} -e^{x} = 0$

$\bullet \lim\limits_{x\to+\infty} e^{x} = 0$

42 α) $\lim\limits_{x\to+\infty} \left(e^{x} + ημx\right) = +\infty,$

$-1 \leq ημx \leq 1$

$\boxed{e^{x} - 1 \leq e^{x} + ημx \leq e^{x} + 1}$

$\lim\limits_{x\to+\infty} e^{x} - 1 = +\infty$ $\qquad$ $\lim\limits_{x\to+\infty} e^{x} + 1 = +\infty$

(B) $\lim\limits_{x\to+\infty} e^{x+\eta\mu x} = +\infty,$

$-1 \le \eta\mu x \le 1$

$x-1 \le x+\eta\mu x \le x+1$

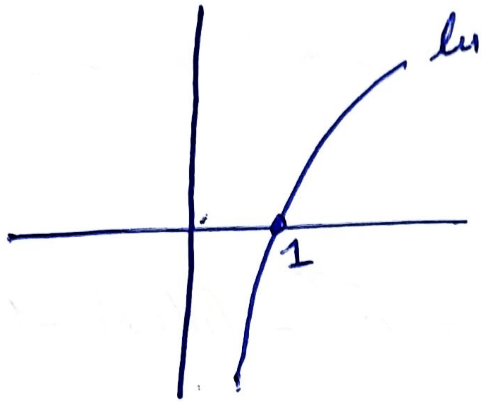
$$\boxed{e^{x-1} \le e^{x+\eta\mu x} \le e^{x+1}}$$

$$\left.\begin{aligned} &\lim_{x\to+\infty} e^{x-1} = +\infty \\ &\lim_{x\to+\infty} e^{x+1} = +\infty \end{aligned}\right\} \text{Από Κ.Π}$$

43 β) $\lim_{x\to 0}\left(\ln x - \frac{1}{x} + 1\right) = \lim_{x\to 0^+}\left(\ln x - \frac{1}{x} + 1\right)$

$= -\infty - \infty + 1$

$= -\infty$



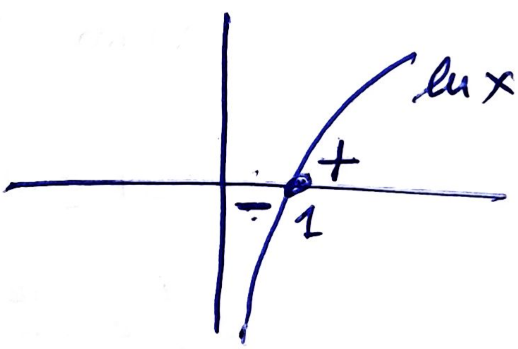
$$\boxed{\begin{array}{l} \ln 0 = -\infty \\ \ln +\infty = +\infty \end{array}}$$

δ) $\lim_{x\to 0}\left(e^{\frac{1}{x}} \ln x\right) = \lim_{x\to 0^+} e^{\frac{1}{x}} \ln x =$

$= e^{+\infty} \ln 0 = +\infty(-\infty) = -\infty$.

44 γ) $\lim_{x\to 1} \frac{x+\sqrt{1-x}}{\ln x} = \lim_{x\to 1^-} \left(x+\sqrt{1-x}\right) \frac{1}{\ln x}$

$= 1 \cdot (-\infty) = -\infty$.



45 ① $\lim_{x\to 0} e^{\frac{\ln x}{x}} = e^{-\infty} = 0$

$\rightarrow \lim_{x\to 0^+} \frac{\ln x}{x} = \lim_{x\to 0^+} \ln x \, \frac{1}{x} = (-\infty)(+\infty) = -\infty$

46 ② $\lim_{x\to +\infty} \left(e^{-x} - \ln x - 1\right) = e^{-\infty} - \ln(+\infty) - 1$

$= 0 - \infty - 1 = -\infty$

⑧ $\lim_{x\to 0} \left(e^{\frac{1}{x}} - \ln x\right) = \lim_{x\to 0^+} \left(e^{\frac{1}{x}} - \ln x\right)$

$= e^{+\infty} - \ln 0 = +\infty - (-\infty) = +\infty + \infty = +\infty$

47 ② $\lim_{x\to +\infty} \frac{\eta\mu x}{\ln x} = 0,$

$-1 \le \eta\mu x \le 1$

$\bullet \lim_{x\to +\infty} -\frac{1}{\ln x} = 0$

$\boxed{-\frac{1}{\ln x} \le \frac{\eta\mu x}{\ln x} \le \frac{1}{\ln x}}$

$\bullet \lim_{x\to +\infty} \frac{1}{\ln x} = 0.$

48 (B) $\lim\limits_{x\to-\infty} \ln\left(\sqrt{x^2+1}+x\right) = \ln 0 = -\infty$

$\to \lim\limits_{x\to-\infty} \sqrt{x^2+1}+x = \lim\limits_{x\to-\infty} \dfrac{1}{\sqrt{x^2+1}-x} = 0$

(δ) $\lim\limits_{x\to+\infty} \ln\left(e^{\frac{1}{x}}-1\right) = \ln 0 = -\infty.$

$\to \lim\limits_{x\to+\infty} e^{\frac{1}{x}}-1 = e^0-1 = 1-1 = 0$

(στ) $\lim\limits_{x\to+\infty} \left[\ln(1+e^x)-x\right] =$

$= \lim\limits_{x\to+\infty} \left(\ln(1+e^x)-\ln e^x\right) =$

$= \lim\limits_{x\to+\infty} \ln\left(\dfrac{1+e^x}{e^x}\right) = \ln 1 = 0.$

$\to \lim\limits_{x\to+\infty} \dfrac{1+e^x}{e^x} = \lim\limits_{x\to+\infty} \dfrac{\cancel{e^x}\left(\frac{1}{e^x}+1\right)}{\cancel{e^x}} = \underline{1}$

(50)

$$f(x)=\begin{cases} e^{\frac{1}{x}}+x, & x<0 \\ \frac{\ln x}{x}, & x>0 \end{cases}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\left(e^{\frac{1}{x}}+x\right)=e^{-\infty}+0=$$
$$=0+0=0$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{\ln x}{x}=\lim_{x\to 0^+}\ln x\cdot\frac{1}{x}=$$
$$=\ln 0\cdot(+\infty)=$$
$$=-\infty\cdot(+\infty)=-\infty$$

Το $\lim_{x\to 0} f(x)$ δεν υπάρχει!

(51) (B) $f(x) > ημx + \ln x \quad , x > 0,$

Ψάχνω το $\lim_{x \to +\infty} f(x)$.

$f(x) > ημx + \ln x$

$\lim_{x \to +\infty} f(x) > \lim_{x \to +\infty} (ημx + \ln x)$

$-1 \leq ημx \leq 1$

$\boxed{\ln x - 1 \leq \ln x + ημx \leq \ln x + 1}$

$$\left.\begin{array}{l} \lim_{x \to +\infty} \ln x - 1 = +\infty \\ \lim_{x \to +\infty} \ln x + 1 = +\infty \end{array}\right\} \lim_{x \to +\infty} \ln x + ημx = +\infty$$

$\lim_{x \to +\infty} f(x) > +\infty \Rightarrow \lim_{x \to +\infty} f(x) = +\infty$

(52) $f(x) = e^x + \ln(x-1)$

(α) • $x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2}$ (+)

• $x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1 \Rightarrow \ln(x_1-1) < \ln(x_2-1)$

$f(x_1) < f(x_2)$

$f \nearrow \Rightarrow f$ γν. μονότονη $\Rightarrow f$ ''1-1'' $\Rightarrow f$ αντιστρ.

(β) $\lim\limits_{x\to+\infty} x^2 f^{-1}(x)$. $f^{-1}(x) = t$ (∗).

$f(f^{-1}(x)) = f(t)$

$x = f(t)$

$x \to +\infty$

$t \to +\infty$,

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} e^x + \ln(x-1) = +\infty$.

Όταν $x \to +\infty$ το $f(x) \to +\infty$

Όταν $x \to +\infty$ το $t \to +\infty$

(∗) $\lim\limits_{t\to+\infty} f^2(t)\, t = +\infty \cdot (+\infty) = +\infty$

53 B $\lim_{x \to +\infty} (x^2+1)^x =$

$= \lim_{x \to +\infty} e^{\ln (x^2+1)^x} = \lim_{x \to +\infty} e^{x \ln(x^2+1)}$

$= e^{+\infty} = +\infty$

$\to \lim_{x \to +\infty} x \ln(x^2+1) = +\infty$

Σελ 214

(12) $f: \mathbb{R} \to \mathbb{R}$ συνεχής στο 0!

$x f(x) = 2x + 3\eta\mu x \quad \forall x \in \mathbb{R}$

Βρείτε τον τύπο της $f(x)$.

Για $x \neq 0$

$$\boxed{f(x) = 2 + 3\frac{\eta\mu x}{x}.}$$

Αφού f συνεχής στο 0 το $f(0) = \lim_{x \to 0} f(x)$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(2 + 3\frac{\eta\mu x}{x}\right) = 2 + 3 \cdot 1 = 5.$$

Άρα $f(0) = 5$.

$$f(x) = \begin{cases} 2 + 3\frac{\eta\mu x}{x}, & x \neq 0 \\ 5, & x = 0 \end{cases}$$

(14) $x f(x) + 2 = f(x) + \sqrt{x^2+3} \quad \forall x \in R.$

Αν f συνεχής στο 1 Τότε f(x)

$x f(x) - f(x) = \sqrt{x^2+3} - 2$

$f(x)(x-1) = \sqrt{x^2+3} - 2$

$$\boxed{f(x) = \frac{\sqrt{x^2+3} - 2}{x-1}, \quad x \neq 1}$$

Αφού είναι συνεχής στο 1.

Το $\lim_{x \to 1} f(x) = f(1)$.

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{\sqrt{x^2+3} - 2}{x-1} =$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(\sqrt{x^2+3}+2)} = \frac{2}{4} = \frac{1}{2}.$$

$$f(x) = \begin{cases} \dfrac{\sqrt{x^2+3} - 2}{x-1}, & x \neq 1 \\ \dfrac{1}{2}, & x = 1 \end{cases}$$

16

- $\lim\limits_{x\to 0} \dfrac{f(x)}{x} = 1 + f(0)$

- $f: \mathbb{R} \to \mathbb{R}$ συνεχής

(α) Βρείτε το $f(0)$.

$$\boxed{\text{Θέτω } g(x) = \frac{f(x)}{x} \Leftrightarrow f(x) = g(x)\,x}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x)\,x \qquad \lim_{x\to 0} g(x) = 1 + f(0)$$

f συνεχής

$$f(0) = \big(1 + f(0)\big) \cdot 0$$

$$f(0) = 0$$

$$\lim_{x\to 0} \frac{f(x)}{x} = 1$$

(β) $$\lim_{x\to 0} \frac{x + f^2(x)}{x + \eta\mu x} = \lim_{x\to 0} \frac{\frac{x}{x} + f(x)\frac{f(x)}{x}}{\frac{x}{x} + \frac{\eta\mu x}{x}} = \frac{1 + 0\cdot 1}{1 + 1} = \frac{1}{2}$$

(18)

$$\lim_{x \to 0} \frac{f(x)+1+\eta\mu x}{x^2+x} = \lambda$$

$f(0)=-1$

(α) Αρκεί νδο $f(0)=\lim_{x\to 0} f(x)$

Θέτω $g(x)=\dfrac{f(x)+1+\eta\mu x}{x^2+x}$ άρα $\lim_{x\to 0} g(x)=\lambda$

$$\boxed{f(x)=g(x)\left(x^2+x\right)-1-\eta\mu x}$$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0} g(x)\left(x^2+x\right)-1-\eta\mu x$$

$$\lim_{x\to 0} f(x)=\lambda\cdot\left(0^2+0\right)-1-0$$

$\lim_{x\to 0} f(x)=-1$ όμως $f(0)=-1$

Άρα $\lim_{x\to 0} f(x)=f(0)$ συνεπώς f συνεχής στο 0.

B) $\lim\limits_{x\to 0} \dfrac{f(x)+1}{ημx} = 3$

Είναι $\lim\limits_{x\to 0} \dfrac{f(x)+1+ημx}{x^2+x} = λ.$

$$\lim_{x\to 0} \frac{\dfrac{f(x)+1}{ημx} + \dfrac{ημx}{ημx}}{\dfrac{x^2+x}{ημx}} = λ.$$

$$\to \lim_{x\to 0} \frac{x^2+x}{ημx} = \lim_{x\to 0} \frac{x+1}{\dfrac{ημx}{x}} = \frac{1}{1} = 1$$

$$\frac{3+1}{1} = λ \Rightarrow λ = 4$$

(19)

$$x + ημx \le x f(x) \le 2x^2 + x + ημx$$

f συνεχής βρες το $f(0)$. $\forall x \in \mathbb{R}$.

Για $x \neq 0$

$$1 + \frac{ημx}{x} \le f(x) \le 2x + 1 + \frac{ημx}{x}$$

$$\left.\begin{aligned} &\lim_{x\to 0}\left(1 + \frac{ημx}{x}\right) = 1 + 1 = 2 \\ &\lim_{x\to 0}\left(2x + 1 + \frac{ημx}{x}\right) = 2 \end{aligned}\right\}$$

Από κ.π

$\lim_{x\to 0} f(x) = 2$ και αφού f

συνεχής $f(0) = 2$

25 $f(x)=\begin{cases} 2x^2-1+\ln a, & x\leq 1 \\ \sqrt{x}+1-\sqrt{a}, & x>1. \end{cases}$

f συνεχής στο 1.

Βρείτε το a.

Αφού είναι συνεχής στο 1

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$$

$$\lim_{x\to 1^-} 2x^2-1+\ln a=\lim_{x\to 1^+} \sqrt{x}+1-\sqrt{a}$$

$$1+\ln a=2-\sqrt{a}$$

$$\ln a+\sqrt{a}-1=0 \Rightarrow g(a)=0$$

$g(a)=g(1)$

g 1-1

$a=1$

$$\boxed{g(x)=\ln x+\sqrt{x}-1}$$

- $x_1<x_2 \Rightarrow \ln x_1<\ln x_2$
- $x_1<x_2 \Rightarrow \sqrt{x_1}-1<\sqrt{x_2}-1$

$\Big\}+ \; g(x_1)<g(x_2)$

$g \nearrow$

Επομενο Μαθημα

Αυριο 10-12

Σελ 196

43 α γ

44 α β

45 α β

46 α γ

47 α

48 α γ ε

49

51 α

53 α

54.

$\lim_{x \to x_0} f(x)$

- απλη αντικατασταση
- $\frac{0}{0}$
  - $\frac{\text{πολυωνυμο}}{\text{πολυωνυμο}}$
  - ριζα → ριτοποιηση
  - απολυτα → $|\overset{\oplus}{x}|$, $|\overset{\ominus}{x}|$, $|\overset{0}{x}|$ → περιπτωσεις
  - τριγωνομετρικα → $\lim_{x \to 0} \frac{ημx}{x} = 1$, $\lim_{x \to 0} \frac{συνx - 1}{x} = 0$ (χωριζω καλους - κακους).
- $\frac{\alpha}{0} = \alpha \cdot \frac{1}{0} = \alpha \cdot \infty = \infty$

Μεγαλη σημασια να βρουμε το προσημο του παρονομαστη κοντα στο x_0.

$\lim_{x \to \infty} f(x)$

- $\frac{0}{0}$, $\frac{\infty}{\infty}$ απροσδιοριστια.
- $\frac{0}{\infty} = 0 \cdot \frac{1}{\infty} = 0 \cdot 0 = 0$
- $\frac{\infty}{0} = \infty \cdot \frac{1}{0} = \infty \cdot \infty = \infty$
- Μεγαλη προσοχη στα τριγωνομετρικα ορια.

→ Οταν το ημιτονο απειριζεται χειζω η' αι × φ

→ Οταν το ημιτονο μηδενιζεται το δινω οτι θελω.

Εφαρμογή ορίων

1. Όταν $\lim_{x\to 0} \frac{f(x)-3x}{x^2} = 4$ έχω ένα δεδομένο γνωστό όριο που περιέχει την $f(x)$ τότε θέτω βοηθητική συνάρτηση

έστω $g(x) = \frac{f(x)-3x}{x^2}$ άρα $\lim_{x\to 0} g(x) = 4$

$$\Rightarrow \boxed{f(x) = g(x)\, x^2 + 3x}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x)\, x^2 + 3x$$

$$\lim_{x\to 0} f(x) = 0$$

Μετά μπορεί να ζητάν

$$\lim_{x\to 0} \frac{f(x)-5x}{x^3} = \lim_{x\to 0} \frac{g(x)x^2+3x-5x}{x^3} =$$

$$= \lim_{x \to 0} \frac{g(x)x^2 - 2x}{x^3} = \lim_{x \to 0} \frac{g(x)x - 2}{x^2}$$

$$= \lim_{x \to 0} (g(x)x - 2)\frac{1}{x^2} = (4 \cdot 0 - 2) \cdot (+\infty)$$

$$= -2(+\infty) = -\infty$$

2. $3x + 2\,\text{ημ}x \leq x f(x) \leq 5x \quad , \quad \underline{\underline{x > 0}}$

$$3 + 2\frac{\text{ημ}x}{x} \leq f(x) \leq 5$$

$$\left.\begin{aligned} &\lim_{x \to 0} 3 + 2\frac{\text{ημ}x}{x} = 5 \\ &\lim_{x \to 0} 5 = 5 \end{aligned}\right\} \underline{\underline{\lim_{x \to 0} f(x) = 5}}$$

3. $|x f(x) - \text{ημ}x| \leq x^2 \quad , \quad x > 0$

$$-x^2 \leq x f(x) - \text{ημ}x \leq x^2$$

$$\text{ημ}x - x^2 \leq x f(x) \leq x^2 + \text{ημ}x$$

$$\boxed{\frac{\text{ημ}x}{x} - x \leq f(x) \leq x + \frac{\text{ημ}x}{x}}$$

Σελ 196

43 (α) $\lim\limits_{x\to 0}\left(x^2+1+\ln x\right) = 0+1+(-\infty) = -\infty$

(γ) $\lim\limits_{x\to 0}\left(\ln x - \dfrac{1}{e^x-1}\right) = \lim\limits_{x\to 0^+}\left(\ln x - \dfrac{1}{\underset{(+)}{e^x-1}}\right) =$

$= -\infty - (+\infty) = -\infty - \infty = \underline{\underline{-\infty}}$

Αν $x>0 \Rightarrow e^x > e^0 \Rightarrow e^x > 1 \Rightarrow e^x - 1 > 0$

44 (α) $\lim\limits_{x\to 0}\dfrac{3x+1}{\ln x} = \lim\limits_{x\to 0^+}\dfrac{3x+1}{\ln x} =$

$= \lim\limits_{x\to 0^+}(3x+1)\dfrac{1}{\ln x} = 1\cdot 0 = 0.$

(β) $\lim\limits_{x\to 0}\dfrac{2x-5}{\ln x} = \lim\limits_{x\to 0^+}(2x-5)\dfrac{1}{\ln x} = -5\cdot 0 = 0,$

45 (α) $\lim\limits_{x\to 0} \dfrac{1+\ln x}{x} =$

$$= \lim_{x\to 0^+} (1+\ln x)\,\frac{1}{x} = (-\infty)\cdot(+\infty) = -\infty$$

(β) $\lim\limits_{x\to 0} \dfrac{\sigma\upsilon\nu x-1}{\ln x} = \lim\limits_{x\to 0} (\sigma\upsilon\nu x-1)\,\dfrac{1}{\ln x} = 0\cdot 0 = 0$

46 (α) $\lim\limits_{x\to +\infty} \left(\ln x - \dfrac{1}{x} - 1\right) = +\infty - 0 - 1 = +\infty$

(β) $\lim\limits_{x\to +\infty} x\ln x = (+\infty)\cdot(+\infty) = +\infty$.

47 (α) $\lim\limits_{x\to +\infty} \ln x + \eta\mu x$

$$-1 \le \eta\mu x \le 1$$

$$\boxed{\ln x - 1 \le \ln x + \eta\mu x \le \ln x + 1}$$

$$\left.\begin{aligned} &\cdot \lim_{x\to +\infty} \ln x - 1 = +\infty \\ &\cdot \lim_{x\to +\infty} \ln x + 1 = +\infty \end{aligned}\right\} \lim_{x\to +\infty} \ln x + \eta\mu x = +\infty.$$

(48) (a) $\lim\limits_{x \to 1^-} \ln \dfrac{x}{1-x} = \ln(+\infty) = +\infty$.

$\rightarrow \lim\limits_{x \to 1^-} \dfrac{x}{1-x} = +\infty$

x		1	
$1-x$	+	0	−

(b) $\lim\limits_{x \to -\infty} \ln(1-e^x) = \ln 1 = 0$

(c) $\lim\limits_{x \to -\infty} \ln(1+x^2) - \ln(2-x) =$

$= \lim\limits_{x \to -\infty} \ln\left(\dfrac{1+x^2}{2-x}\right) = \ln(+\infty) = +\infty$.

$\rightarrow \lim\limits_{x \to -\infty} \dfrac{1+x^2}{2-x} = \lim\limits_{x \to -\infty} \dfrac{x^2}{-x} = \lim\limits_{x \to -\infty} \dfrac{x}{-1} = +\infty$

49

$$f(x) = \begin{cases} \ln x, & x > 0 \\ \dfrac{2x+1}{e^x - 1}, & x < 0 \end{cases}$$

- $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \ln x = -\infty$

- $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{2x+1}{\underset{\ominus}{e^x - 1}} = -\infty$

$\left(x < 0 \Rightarrow e^x < e^0 \Rightarrow e^x < 1 \Rightarrow e^x - 1 < 0 \right)$

Άρα $\lim\limits_{x \to 0} f(x) = -\infty$

51 a) $f(x) + x^2 - e^x < 0$

$\lim\limits_{x\to-\infty} f(x)$

$$\boxed{f(x) < e^x - x^2}$$

$$\lim_{x\to-\infty} f(x) < \lim_{x\to-\infty} (e^x - x^2)$$

$$\lim_{x\to-\infty} f(x) < -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

---

53 a) $$\lim_{x\to+\infty} (x-1)^{x-1} = \lim_{x\to+\infty} e^{\ln(x-1)^{x-1}} =$$

$$= \lim_{x\to+\infty} e^{(x-1)\ln(x-1)} = e^{+\infty} = +\infty.$$

$$\rightarrow \lim_{x\to+\infty} (x-1)\ln(x-1) = +\infty$$

(54) $f(x) = x^{\ln x}$

(a) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^{\ln x} = \lim_{x \to +\infty} e^{\ln x^{\ln x}} =$

$= \lim_{x \to +\infty} e^{\ln x \ln x} = e^{+\infty} = +\infty .$

$\rightarrow \lim_{x \to +\infty} \ln x . \ln x = (+\infty)(+\infty) = +\infty$

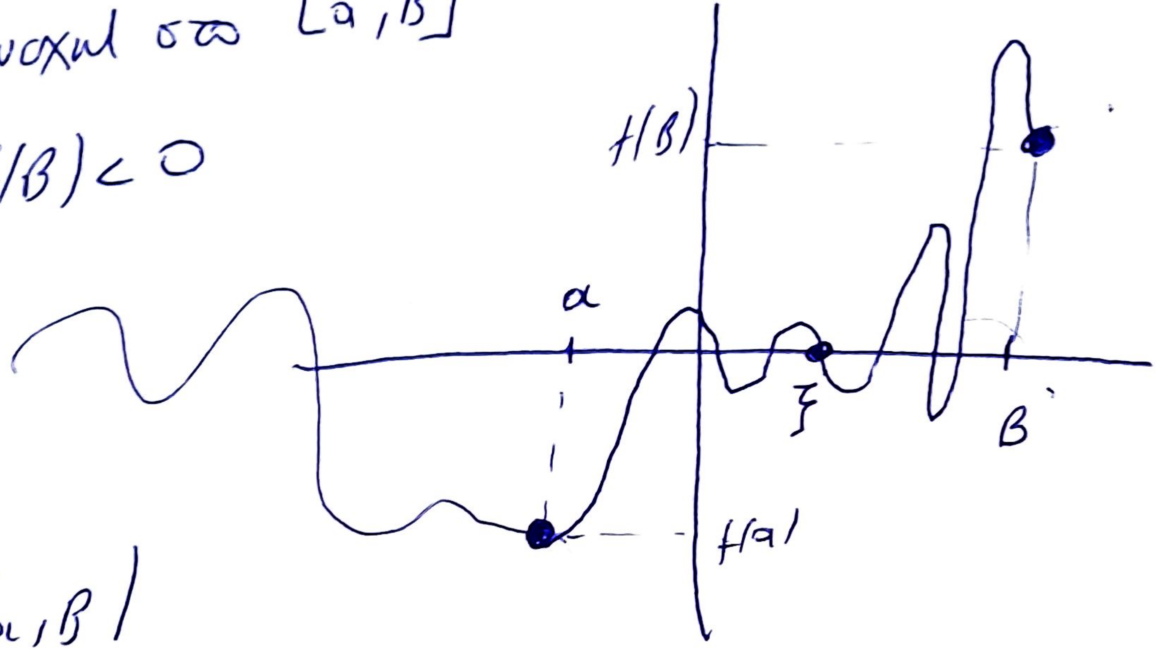
(B) $\lim_{x \to 0} f(x) = \lim_{x \to 0} x^{\ln x} = \lim_{x \to 0} e^{\ln x^{\ln x}} =$

$= \lim_{x \to 0} e^{\ln x \ln x} = e^{+\infty} = +\infty .$

$\rightarrow \lim_{x \to 0^+} \ln x \ln x = (-\infty)(-\infty) = +\infty$

Θεωρημα Bolzano

- f συνεχης στο $[\alpha, \beta]$
- $f(\alpha) f(\beta) < 0$



$\exists\, \xi \in (\alpha, \beta)$

τ.ω $f(\xi) = 0$

Το Θεωρημα του Bolzano μας εξασφαλιζει ριζα.

Τουλαχιστον μια, μπορει να εχει αλλα 10 δεν το ξερω.

Ουτε ξερω ποια ειναι η ριζα και δεν με ενδιαφερει.

6 (α) Νδο $\exists x_0 \in (0,1)$

τ.ω $\ln(x_0+1) = 1 - x_0$

Δηλαδη αρκει νδο η εξισωση $\ln(x+1) = 1 - x$

εχει τουλαχιστον μια λυση στο $(0,1)$.

$$\underbrace{\ln(x+1) - 1 + x}_{f(x)} = 0$$

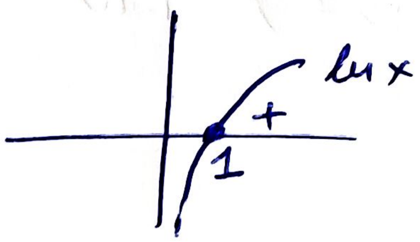
• Η $f(x)$ ειναι συνεχης στο $[0,1]$ ως

πρ. συνεχων συναρτησεων.

• $f(0) = -1$

• $f(1) = \ln 2 > 0$

$$\left.\begin{matrix} f(0) = -1 \\ f(1) = \ln 2 > 0 \end{matrix}\right\} f(0)f(1) < 0$$



Bolzano $\exists x_0 \in (0,1)$

τ.ω $f(x_0) = 0$.

$\ln(x_0+1) - 1 + x_0 = 0$

$\ln(x_0+1) = 1 - x_0$

Σελ 226

② (B) Νδο η εξισωση $x^3-3x+1=0$ εχει μια τουλ. ριζα στο $(0,1)$

$\boxed{\text{Θεωρω } f(x)=x^3-3x+1.}$

Η $f(x)$ ειναι συνεχης στο $[0,1]$ ως πρ. συνεχων συναρτησεων.

$$\left.\begin{aligned} f(0)&=1 \\ f(1)&=-1 \end{aligned}\right\} f(0)f(1)<0$$

Απο Bolzano $\exists\, \xi \in (0,1)$ τ.ω

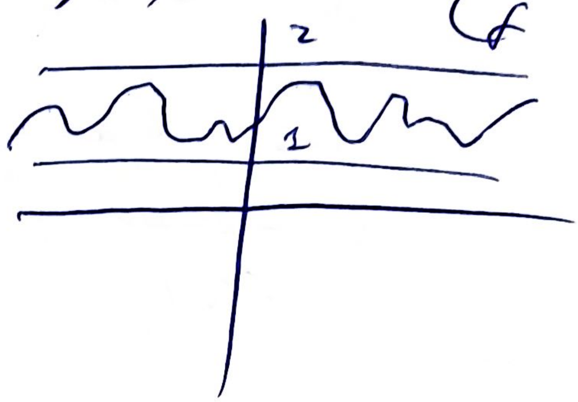
$f(\xi)=0$

$\xi^3-3\xi+1=0$

$\xi \in (0,1)$.

7 $f(\mathbb{R}) = (1,2)$.

$D_f = \mathbb{R}$ $\quad$ $\Sigma T_f = (1,2)$



$1 < f(x) < 2$

Νδο η εξίσωση

$f(x) = 2 - x$ έχει ρίζα στο $(0,1)$

$$\underbrace{f(x) - 2 + x}_{g(x)} = 0$$

• Η $g(x)$ συνεχής $[0,1]$ ως πράξη συνεχών συναρτήσεων.

$1 < f(0) < 2$

$f(0) - 2 < 0$

• $g(0) = f(0) - 2 < 0$

$g(1) = f(1) - 1 > 0$

$1 < f(1) < 2$

$0 < f(1) - 1$

Άρα $g(0)g(1) < 0$

Bolzano $\exists\, \xi \in (0,1)$ που $g(\xi) = 0$

$f(\xi) - 2 + \xi = 0 \Rightarrow f(\xi) = 2 - \xi$, $\xi \in (0,1)$.

6 B Νδο $\exists \alpha \in (0, \pi)$ τ.ω $\eta\mu\alpha = \alpha - 1$

$$\eta\mu x = x - 1$$

$$\underbrace{\eta\mu x - x + 1}_{f(x)} = 0$$

- $f(x)$ συνεχής στο $[0, \pi]$ ως πράξη συνεχών συναρτήσεων.

- $f(0) = 1$
- $f(\pi) = 1 - \pi < 0$

$\Big\}\ f(0)f(\pi) < 0$

Bolzano $\exists \alpha \in (0, \pi)$ τ.ω

$$f(\alpha) = 0.$$

$$\eta\mu\alpha - \alpha + 1 = 0$$

$$\eta\mu\alpha = \alpha - 1$$

Επανάληψη Θερινών Μαθημάτων

Θεωρία

1, 5, 7, 9, 10, 11, 20, 23

\+ αντιπαράδειγμα (Σελ 10)

[Αν f: 1-1 ⟹ f γν. μονότονη]

Για Σ-Λ SOS σελίδα.

3, 4, 8, 9, 11, 16, 17, 18, 19, 26

Ασκήσεις

Ενότητα 2 : 13, 18

Ενότητα 3 : 2, 6, 8

Ενότητα 4 : 9, 10, 12, 16,

Ενότητα 6 : 2, 4, 7, 8

Ενότητα 7 : 7, 18, 19, 20, 26, 29, 31

Ενότητα 8 : 4, 10, 13, 15, 19, 20
21, 22, 24, 26, 39

Ενότητα 9 : 5, 6, 7, 10,

Ενότητα 10 : 5, 6, 8, 9, 10, 28, 29, 30, 31
34, 35, 36, 41, 42, 43, 44, 45
48, 51, 53