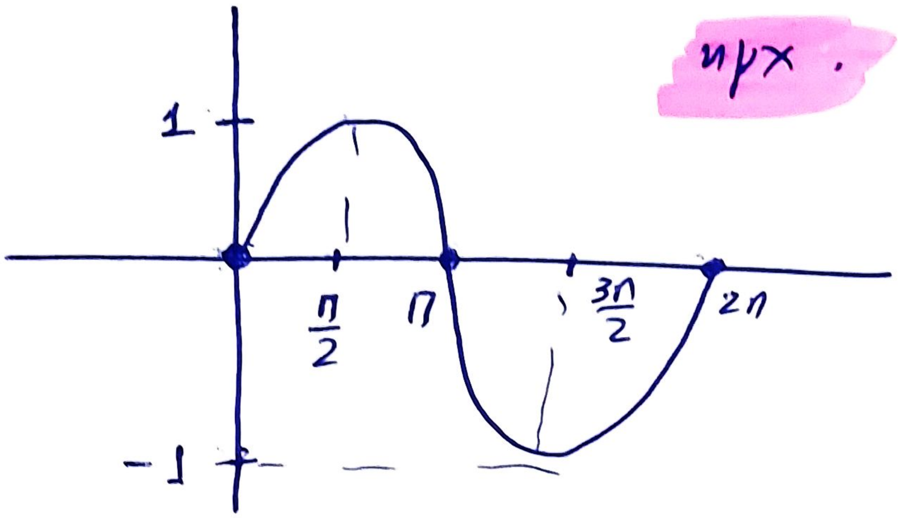
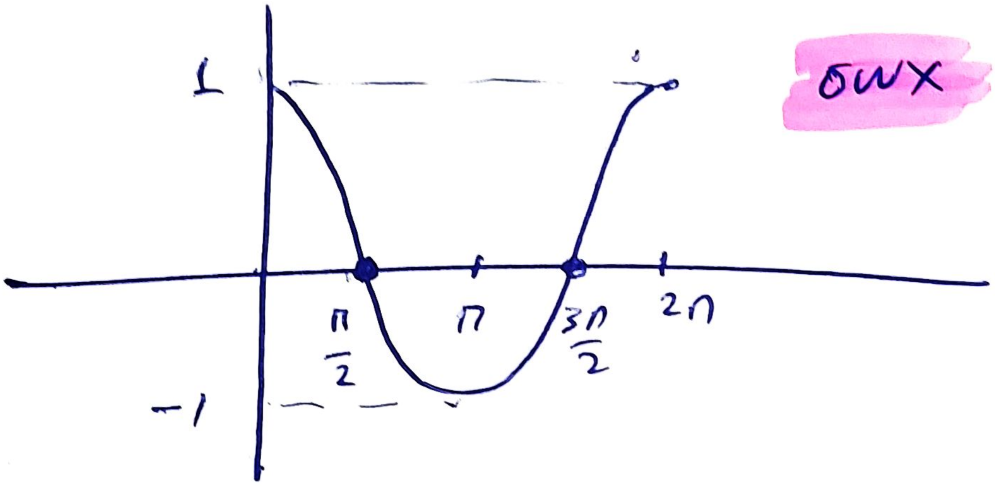
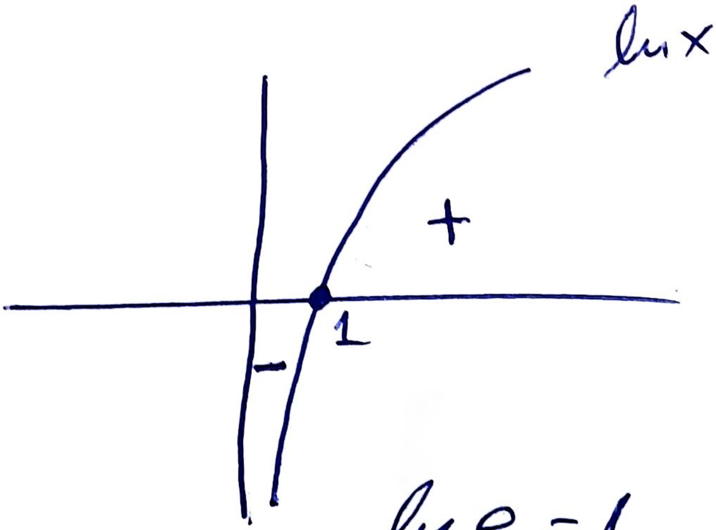
Βασικές συναρτήσεις





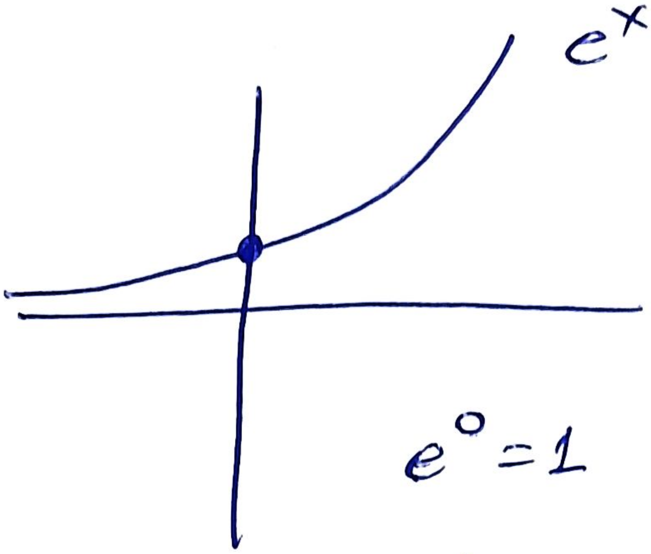


Το lnx δεν είναι θετικό.

Το x είναι θετικό δηλαδή η ποσότητα που βάζουμε μέσα στο lnx.

$\ln e = 1$

$\ln 1 = 0$.



$e^0 = 1$

$e^1 = e$.

Το e^x ειναι <u>παντα</u> θετικο οτι και να μπει στη θεση του x.

Πρεπει $x^2-4\neq 0$

$x^2 \neq 4$

$x\neq 2$ κ' $x\neq -2$

Ασκηση 1

$$f(x)=\frac{x^2+2|x|}{x^2-4}$$

$D_f = \mathbb{R}-\{2,-2\}$.

$$g(x)=\frac{|x|}{|x|-2}.$$

$D_g = \mathbb{R}-\{2,-2\}$.

πρεπει $|x|-2\neq 0$

$\longrightarrow$ $|x|-2=0$

$|x|=2$

$x=2$ ή $x=-2$.

Ειναι ισες;

$$f(x)=\frac{x^2+2|x|}{x^2-4}=\frac{|x|^2+2|x|}{|x|^2-4}=\frac{|x|\,\cancel{(|x|+2)}}{(|x|-2)\,\cancel{(|x|+2)}}$$

$|x|^2=x^2=|x^2|$

$$=\frac{|x|}{|x|-2}=g(x)$$

Εχουν ιδιο πεδιο ορισμου

και ιδιο τυπο αρα

Ισες!

Ασκηση 2

Εστω $g(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ οπου $g : [1,+\infty) \to \mathbb{R}$.

Επισης $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$ οπου $f : [1,+\infty) \to \mathbb{R}$.

Βρειτε τις $(f+g)$, $(f-g)$, $f \cdot g$, $\frac{f}{g}$.

$\longrightarrow (f+g)(x) = f(x) + g(x) = \sqrt{x} - \cancel{\frac{1}{\sqrt{x}}} + \sqrt{x} + \cancel{\frac{1}{\sqrt{x}}} = 2\sqrt{x}$

$D_{f+g} = D_f \cap D_g = [1,+\infty)$.

$\longrightarrow (f-g)(x) = f(x) - g(x) = \sqrt{x} - \frac{1}{\sqrt{x}} - \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

$= \cancel{\sqrt{x}} - \frac{1}{\sqrt{x}} - \cancel{\sqrt{x}} - \frac{1}{\sqrt{x}} = -\frac{2}{\sqrt{x}}$.

$D_{f-g} = D_f \cap D_g = [1,+\infty)$.

$\rightarrow (f\cdot g)(x) = f(x)\cdot g(x) = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

$$= \sqrt{x}^2 - \left(\frac{1}{\sqrt{x}}\right)^2 = x - \frac{1}{x}$$

$D_{f\cdot g} = D_f \cap D_g = [1,+\infty)$.

$$\rightarrow \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x} - \frac{1}{\sqrt{x}}}{\sqrt{x} + \frac{1}{\sqrt{x}}} = \frac{\sqrt{x}^2 - 1}{\sqrt{x}^2 + 1}$$

$$= \frac{x-1}{x+1},$$

<u>$D_{f/g}$</u>

$D_f \cap D_g$ και επιπλέον $g(x) \neq 0$

$\downarrow$

$[1,+\infty)$

$\rightarrow g(x) = 0$

$\sqrt{x} + \frac{1}{\sqrt{x}} = 0$

Αδύνατη

γιατί $g(x) > 0$

$$\underline{D_{f/g} = [1,+\infty)}$$

Άσκηση 3

Έστω $f(x) = \dfrac{x+1}{x-1}$, $x \geq 0$

και $g(x) = \ln x$, $x > 0$

① $(f \circ g)(x) = f(g(x)) = \dfrac{g(x)+1}{g(x)-1} = \dfrac{\ln x + 1}{\ln x - 1}$

Πρέπει $x \in D_g$ και $g(x) \in D_f$

$x > 0$ $\qquad \ln x \geq 0$

$e^{\ln x} \geq e^0$

$x \geq 1$

$D_{f \circ g} = [1, +\infty)$.

② $(g \circ f)(x) = g(f(x)) = \ln f(x) = \ln\left(\dfrac{x+1}{x-1}\right)$

πρέπει $x \in D_f$ και $f(x) \in D_g$

$x \geq 0$ $\qquad \dfrac{x+1}{x-1} > 0$.

$x \in (-\infty, -1) \cup (1, +\infty)$

x		-1		1	
$x+1$	$-$	0	$+$		$+$
$x-1$	$-$		$-$	0	$+$
$\frac{x+1}{x-1}$	$+$		$-$		$+$

$D_{g \circ f} = (1, +\infty)$.

(γ) $(f\circ f)(x) = f(f(x)) = \dfrac{f(x)+1}{f(x)-1} =$

$$= \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{x+1+x-1}{x+1-x+1} = \frac{2x}{2} = x$$

πρέπει $x \in D_f$ και $f(x) \in D_f$

$x \geq 0$ και $\dfrac{x+1}{x-1} \geq 0$

$x \in (-\infty, -1] \cup (1, +\infty)$

$D_{f\circ g} = (1, +\infty)$.

(δ). $(g\circ g)(x) = g(g(x)) = \ln(\ln x)$

$x \in D_g$ και $g(x) \in D_g$

$\underline{\underline{x > 0}}$

$\ln x > 0$

$e^{\ln x} > e^0$

$\underline{\underline{x > 1}}$.

$D_{g\circ g} = (1, +\infty)$.

Άσκηση 4

$f(x) = \frac{x+1}{x-1} \quad D_f = \mathbb{R} - \{1\}.$

$g(x) = \frac{1}{x} \quad D_g = \mathbb{R}^*$

Βρείτε $\quad g \circ \frac{1}{f}$

$$\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)} = \frac{1}{\frac{x+1}{x-1}} = \frac{x-1}{x+1}$$

πρέπει $f(x) \neq 0$

$\longrightarrow f(x) = 0 \iff \frac{x+1}{x-1} = 0 \iff x+1 = 0$

$\underline{\underline{x = -1}}$

$D_{1/f} = \mathbb{R} - \{-1\}.$

$$\left(g \circ \frac{1}{f}\right)(x) = g\left(\left(\frac{1}{f}\right)(x)\right) = \frac{1}{\frac{x-1}{x+1}} = \frac{x+1}{x-1}$$

$x \in D_{1/f}$ και $\left(\frac{1}{f}\right)(x) \in D_g \quad D_{g \circ \frac{1}{f}} = \mathbb{R} - \{-1, 1\}$

$x \neq -1 \qquad \frac{x-1}{x+1} \neq 0 \iff x \neq 1$

Επομενο Μαθημα
Τεταρτη 8:30-10

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1. Δινονται οι συναρτησεις f, g.
Να εξετασεις εαν οι f και g ειναι ισες.

(α) $f(x) = \dfrac{x^2+4x+3}{x^2-1}$ και $g(x) = \dfrac{x^2-9}{x^2-4x+3}$

(β) $f(x) = \sqrt{x-2}\sqrt{x+5}$ και $g(x) = \sqrt{x^2+3x-10}$

2. Δινονται οι συναρτησεις
$f(x) = x+2+\dfrac{6}{x-3}$ και $g(x) = x+1+\dfrac{3}{x-3}$

Να βρεθουν οι $f+g$, $f-g$, $f\cdot g$, $\dfrac{f}{g}$

3. Δινονται οι συναρτησεις f, g.
Να βρεθουν οι $f\circ g$, $g\circ g$, $f\circ f$, $g\circ g$.

(α) $f(x) = \dfrac{x}{x+1}$ και $g(x) = \dfrac{1}{x+3}$

(β) $f(x) = \sqrt{x+3}$ και $g(x) = x^2+1$

Ασκηση 1

Εστω $f(x) = e^{1-x} + \frac{1}{x} - 1 \quad , x>0$

ⓐ Μονοτονια.

ⓑ $x e^{1-x} - 2x + 1 = 0$ εξισωση.

ⓒ $2x e^{x-1} > x + e^{x-1}$ ανισωση.

Λυση

ⓐ. Οσα x τοσα χτισιματα.

• $x_1 < x_2 \Rightarrow -x_1 > -x_2 \Leftrightarrow 1 - x_1 > 1 - x_2$

$$\boxed{e^{1-x_1} > e^{1-x_2}}$$

(+)

• $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow \boxed{\frac{1}{x_1} - 1 > \frac{1}{x_2} - 1}.$

$$\underbrace{e^{1-x_1} + \frac{1}{x_1} - 1}_{f(x_1)} > \underbrace{e^{1-x_2} + \frac{1}{x_2} - 1}_{f(x_2)} \quad f \downarrow$$

(β) $x e^{1-x} - 2x + 1 = 0$

Διαιρώ με το x το οποίο $x \neq 0$.

$e^{1-x} - 2 + \frac{1}{x} = 0$

$e^{1-x} + \frac{1}{x} - 1 = 2 - 1$

$f(x) = 1$.

$f(x) = f(1)$

f 1-1

$\underline{\underline{x = 1}}$

(γ). $2x e^{x-1} > x + e^{x-1}$, $x > 0$.

$2e^{x-1} > 1 + \frac{e^{x-1}}{x}$

$\frac{2\cancel{e^{x-1}}}{\cancel{e^{x-1}}} > \frac{1}{e^{x-1}} + \cancel{\frac{e^{x-1}}{x e^{x-1}}}$

$2 > \frac{1}{e^{x-1}} + \frac{1}{x} \Leftrightarrow 2 > (e^{x-1})^{-1} + \frac{1}{x}$

$2 > e^{1-x} + \frac{1}{x} \Leftrightarrow 1 > e^{1-x} + \frac{1}{x} - 1 \rightarrow f(1) > f(x)$

$f \downarrow$

$\underline{1 < x}$

Σελ 93

2) (γ). $f(x) = \dfrac{x-3}{x+1}$. $D_f = \mathbb{R} - \{-1\}$.

$\boxed{\text{Έστω } f(x_1) = f(x_2)}$

$$\frac{x_1-3}{x_1+1} = \frac{x_2-3}{x_2+1}$$

$$(x_1-3)(x_2+1) = (x_2-3)(x_1+1)$$

$$\cancel{x_1x_2} + x_1 - 3x_2 - \cancel{3} = \cancel{x_2x_1} + x_2 - 3x_1 - \cancel{3}$$

$$4x_1 = 4x_2$$

$$\boxed{x_1 = x_2}.$$

f 1-1.

(β). $f(x) = \ln\left(\dfrac{x}{1-x}\right)$

πρέπει $\dfrac{x}{1-x} > 0$ και $1-x \neq 0$

$x \neq 1$

x		0		1	
x	$-$		$+$		$+$
$1-x$	$+$		$+$		$-$
	$-$		$+$		$-$

$D_f = (0, 1)$.

$$\boxed{\text{Έστω } f(x_1)=f(x_2)}$$

$$\ln\frac{x_1}{1-x_1}=\ln\frac{x_2}{1-x_2}$$

$$\frac{x_1}{1-x_1}=\frac{x_2}{1-x_2}$$

$$x_1(1-x_2)=x_2(1-x_1)$$

$$x_1-\cancel{x_1x_2}=x_2-\cancel{x_1x_2}$$

$$\boxed{x_1=x_2}$$

ΠΕΠ $1-e^x>0$

$1>e^x$

$\ln 1>\ln e^x$

$0>x$

(σ2) $f(x)=x-\ln(1-e^x)$.

• $x_1<x_2 \Rightarrow e^{x_1}<e^{x_2} \Rightarrow -e^{x_1}>-e^{x_2}$

$1-e^{x_1}>1-e^{x_2}$

$\ln(1-e^{x_1})>\ln(1-e^{x_2})$

• $\boxed{x_1<x_2}$

(+)

$$\boxed{-\ln(1-e^{x_1})<-\ln(1-e^{x_2})}$$

$f(x_1)<f(x_2)$

$f\nearrow \Rightarrow f$ 1-1

4 δ f 1-1

$f(2)=3$

εξίσωση $f(f(x)-1)=3$.

Ισότητα

$f(x_1)=f(x_2)$

f 1-1

$\Rightarrow x_1=x_2$

$f(f(x)-1)=f(2)$

f 1-1

$f(x)-1=2$

$f(x)=3$

$f(x)=f(2)$

f 1-1

$x=2$

7) $f(x) = e^{x-2} + x - 3$

α) • $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2 \Rightarrow \boxed{e^{x_1-2} < e^{x_2-2}}$

• $x_1 < x_2 \rightarrow \boxed{x_1 - 3 < x_2 - 3}$ (+)

$$\underbrace{e^{x_1-2} + x_1 - 3}_{f(x_1)} < \underbrace{e^{x_2-2} + x_2 - 3}_{f(x_2)}$$

$f \nearrow$ άρα f 1-1.

β) i) $x + e^{x-2} = 3$

$$\underbrace{x + e^{x-2} - 3}_{} = 0$$

$f(x) = 0$

$f(x) = f(2)$

f 1-1

$\underline{\underline{x = 2}}$

ii). $e^{x^2-2} + x^2 = 3$.

$e^{x^2-2} + x^2 - 3 = 0$

$f(x^2) = 0$

$f(x^2) = f(2)$

f 1-1

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$.

iii). $e^{3x-2} - e^{x^2} = x^2 - 3x + 2$.

$\boxed{f(x) = e^{x-2} + x - 3}$

$e^{3x-2} + 3x - 3 = e^{(x^2+2)-2} + (x^2+2) - 3$

$f(3x) = f(x^2+2)$.

iv). $f\left((9-3x)e^{2-x} - 1\right) = 0$.

$f\left((9-3x)e^{2-x} - 1\right) = f(2)$

f 1-1

$(9-3x)e^{2-x} - 1 = 2 \Rightarrow (9-3x)e^{2-x} = 3$

$(9-3x)\,e^{2-x} = 3$,

$(3-x)\,e^{2-x} = 1$.

$$\frac{(3-x)\cancel{e^{2-x}}}{\cancel{e^{2-x}}} = \frac{1}{e^{2-x}}$$

$3-x = e^{x-2} \Leftrightarrow 0 = e^{x-2} + x - 3$

$0 = f(x)$

$f(2) = f(x)$

f 1-1

$x = 2$.

γ). $e^{f(x)} + f(x) - 1 = 0$

Θετω $f(x) = t$.

$e^t + t - 1 = 0 \longrightarrow \varphi(t) = 0$

$\varphi(t) = \varphi(0)$

φ 1-1

$t = 0$

$f(x) = 0$

$f(x) = f(2)$

f 1-1

$\underline{x = 2}$

$\boxed{\text{Θεωρω } \varphi(x) = e^x + x - 1}$

$\left.\begin{array}{l} \cdot\ x_1 < x_2 \Rightarrow e^{x_1} < e^{x_2} \\ \cdot\ x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1 \end{array}\right\} \oplus$

$\varphi(x_1) < \varphi(x_2)$

$\varphi \nearrow \Rightarrow \varphi$ 1-1.

(8) (B) $e^{\frac{1}{x}} - \ln x = e$.

$$\underbrace{e^{\frac{1}{x}} - \ln x - e}_{f(x)} = 0 \Rightarrow f(x) = 0$$

$f(x) = f(1)$

f 1-1

$\underline{\underline{x = 1}}$

• $x_1 < x_2 \Rightarrow \frac{1}{x_1} > \frac{1}{x_2} \Rightarrow \boxed{e^{\frac{1}{x_1}} > e^{\frac{1}{x_2}}}$

• $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2 \Rightarrow -\ln x_1 > -\ln x_2$

(+)

$$\boxed{-e - \ln x_1 > -e - \ln x_2}$$

$f(x_1) > f(x_2)$

$f \searrow$

άρα

f 1-1.

9 (B). $e^{x^3-x}+x^3=x+1$.

$$e^{x^3-x}+x^3-x-1=0$$

Θέτω $x^3-x=t$

$e^t+t-1=0 \Rightarrow \varphi(t)=0$

$\boxed{\varphi(x)=e^x+x-1}$

- $x_1<x_2 \Rightarrow e^{x_1}<e^{x_2}$
- $x_1<x_2 \Rightarrow x_1-1<x_2-1$

$\Big\} \oplus$

$\varphi \nearrow \Rightarrow \varphi$ 1-1

$\varphi(t)=\varphi(0)$

φ 1-1

$t=0$

$x^3-x=0$

$x(x^2-1)=0$

$x(x-1)(x+1)=0$

$x=0$ $x=1$ $x=-1$.

(10) (B) $\ln \varepsilon\varphi x = \sigma\upsilon\nu x - \eta\mu x$

$x \in \left(0, \frac{\pi}{2}\right)$.

$$\ln \frac{\eta\mu x}{\sigma\upsilon\nu x} = \sigma\upsilon\nu x - \eta\mu x$$

$$\ln \eta\mu x - \ln \sigma\upsilon\nu x = \sigma\upsilon\nu x - \eta\mu x$$

$$\ln \eta\mu x + \eta\mu x = \ln \sigma\upsilon\nu x + \sigma\upsilon\nu x.$$

$$\boxed{f(x) = \ln x + x}$$

$$f(\eta\mu x) = f(\sigma\upsilon\nu x),$$

f 1-1

Μονοτονία $f(x)$

- $x_1 < x_2 \Rightarrow \ln x_1 < \ln x_2$
- $x_1 < x_2$

(+)

$f \nearrow$

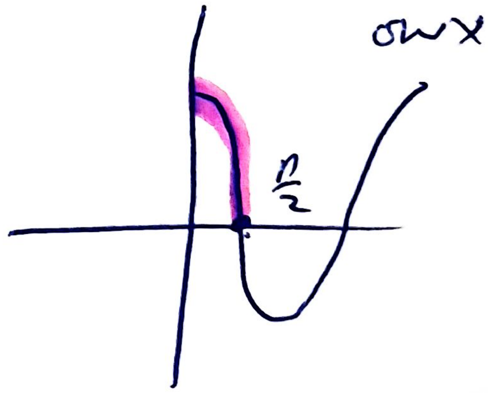
$\Rightarrow f$ 1-1,

$$\eta\mu x = \sigma\upsilon\nu x$$

$$\frac{\eta\mu x}{\sigma\upsilon\nu x} = 1$$

$$\varepsilon\varphi x = 1$$

$$x = \frac{\pi}{4}.$$



Επόμενο Μάθημα

1. Επανάληψη τις λυμένες ασκήσεις του Μπάρλα.

   Σελ. 66-67

   5 9 10 12 16.

2. **Ασκήσεις**

   Σελ 93

   2 α β δ

   4 α β γ

   5

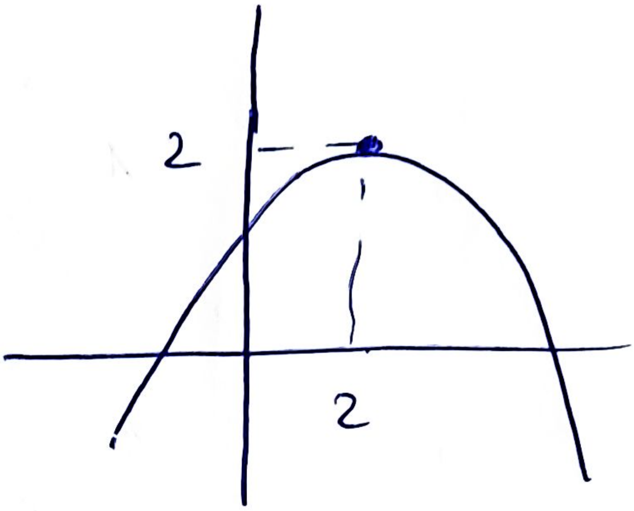
   6.

   8 α

   9 α

Σελ 94

11



(α) $f(x^6+4)=f(x^4+4)$.

- $x^6 \geq 0 \Rightarrow x^6+4 \geq 4$
- $x^4 \geq 0 \Rightarrow x^4+4 \geq 4$

$\Big\}$ $\forall x>2$ η $f \downarrow$ άρα 1-1.

$x^6+4 = x^4+4$

$x^6 = x^4 \Rightarrow x^6 - x^4 = 0$

$x^4(x^2-1)=0$

$x^4=0$ ή $x^2-1=0$

$x=0$ $\quad x=1$ $\quad x=-1$

(β). $f(ημx) = f(\sqrt{3}\, συνx)$.

- $ημx \in [-1,1]$
- $-1 \leq συνx \leq 1 \Rightarrow -\sqrt{3} \leq \sqrt{3}\, συνx \leq \sqrt{3}$

$\sqrt{3}\, συνx \in [-\sqrt{3}, \sqrt{3}]$

$\Big\}$ $\forall x<2$ η $f \nearrow$

$f(\eta\mu x) = f(\sqrt{3}\,\sigma\upsilon\nu x)$

f 1-1

$\eta\mu x = \sqrt{3}\,\sigma\upsilon\nu x$

$\frac{\eta\mu x}{\sigma\upsilon\nu x} = \sqrt{3}$ $\Leftrightarrow \varepsilon\varphi x = \sqrt{3}$

$\varepsilon\varphi x = \varepsilon\varphi \frac{\pi}{3}$

$x = \kappa\pi \pm \frac{\pi}{3}$

Τσεκ

Εστω $\sigma\upsilon\nu x = 0$

Τοτε $\eta\mu x = \sqrt{3} \cdot 0$

$\eta\mu x = 0$

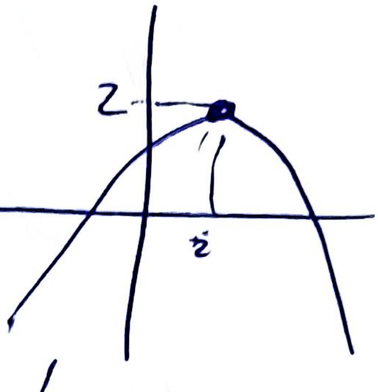
ομως αυτο ειναι ατοπο.

γιατι $\eta\mu^2 x + \sigma\upsilon\nu^2 x = 1$

$0^2 + 0^2 \neq 1$

Αρα $\sigma\upsilon\nu x \neq 0$.

(γ) $f(f(x)) = 2$



Αφού το $(2,2)$ ολ. μέγιστο

τότε $f(2)=2$ μόνον!

$f(x)=2$

$\underline{\underline{x=2}}$

⑦ Σελ 107

⑧ $f(x) = \dfrac{2e^x - 1}{e^x + 1}$, $D_f = \mathbb{R}$.

Εστω

$f(x_1) = f(x_2)$

$$\frac{2e^{x_1} - 1}{e^{x_1} + 1} = \frac{2e^{x_2} - 1}{e^{x_2} + 1}$$

$$(2e^{x_1} - 1)(e^{x_2} + 1) = (2e^{x_2} - 1)(e^{x_1} + 1).$$

$$\cancel{2e^{x_1}e^{x_2}} + 2e^{x_1} - e^{x_2} \cancel{-1} = \cancel{2e^{x_2}e^{x_1}} + 2e^{x_2} - e^{x_1} \cancel{-1}$$

$$3e^{x_1} = 3e^{x_2}$$

$$e^{x_1} = e^{x_2}$$

$$x_1 = x_2$$

Άρα f 1-1 άρα αντιστρέφεται.

$\boxed{\text{Θέτω } y=f(x)}$

$$y=\frac{2e^x-1}{e^x+1}.$$

$$y(e^x+1)=2e^x-1 \iff ye^x+y=2e^x-1$$

$$ye^x-2e^x=-1-y$$

$$e^x(y-2)=-1-y$$

$$e^x=\frac{-1-y}{y-2} \qquad y\neq 2$$

$$x=\ln\frac{-1-y}{y-2} \iff x=\ln\left(\frac{1+y}{2-y}\right)$$

$\boxed{\text{Θέτω } x=f^{-1}(y)}$

πρέπει $\frac{1+y}{2-y}\geq 0$

$$f^{-1}(y)=\ln\left(\frac{1+y}{2-y}\right)$$

ή

$$f^{-1}(x)=\ln\left(\frac{1+x}{2-x}\right)$$

y		-1		2	
$1+y$	$-$	0	$+$		$+$
$2-y$	$+$		$+$	0	$-$
~	$-$		$+$		$-$

$y\in[-1,2)$.

$$D_{f^{-1}}=[-1,2)$$

(3) $f(x)=\sqrt{1-\ln x}$

πρεπει $1-\ln x \geq 0$ και $x>0$

$1 \geq \ln x$

$e^{1} \geq e^{\ln x}$ $\qquad D_f=(0,e]$.

$e \geq x$

• $x_1<x_2 \Rightarrow \ln x_1 < \ln x_2 \Rightarrow -\ln x_1 > -\ln x_2$

$1-\ln x_1 > 1-\ln x_2$

$\sqrt{1-\ln x_1} > \sqrt{1-\ln x_2}$

$f(x_1) > f(x_2)$

$f \downarrow$

f γν. μονοτονη

f [illegible] 1-1

θετω $y=f(x)$

$y=\sqrt{1-\ln x}$ $\quad y \geq 0$

$y^2=1-\ln x$

$\ln x = 1-y^2$

$e^{\ln x}=e^{1-y^2}$ $\qquad$ θετω $x=f^{-1}(y)$

$x=e^{1-y^2}$ ή $f^{-1}(y)=e^{1-y^2}$

$f^{-1}(x)=e^{1-x^2}$

$D_{f^{-1}}=[0,+\infty)$.

Τελος

$x \in (0, e]$

δηλαδη $\quad 0 < x \leq e$

$0 < e^{1-y^2} \leq e$

$0 < e^{1-y^2}$ και $e^{1-y^2} \leq e$

$y \in R \qquad \cancel{1} - y^2 \leq \cancel{1}$

$-y^2 \leq 0$

$y \in R$

Δεν πηρα κατι!

52 $f(x) = x - \ln(1+e^x)$

πρεπει $1+e^x > 0$ που ειναι

αρα $D_f = \mathbb{R}$.

$$f(x) = \ln e^x - \ln(1+e^x) = \ln\left(\frac{e^x}{1+e^x}\right)$$

$\underline{f(x_1) = f(x_2)}$

$$\ln\left(\frac{e^{x_1}}{1+e^{x_1}}\right) = \ln\left(\frac{e^{x_2}}{1+e^{x_2}}\right)$$

$$\frac{e^{x_1}}{1+e^{x_1}} = \frac{e^{x_2}}{1+e^{x_2}}$$

$$e^{x_1}(1+e^{x_2}) = e^{x_2}(1+e^{x_1})$$

$$e^{x_1} + \cancel{e^{x_1}e^{x_2}} = e^{x_2} + \cancel{e^{x}e^{x_2}}$$

$$e^{x_1} = e^{x_2}$$ f ει 1-1

$$x_1 = x_2$$

Θετω $f(x)=y \iff y=\ln\frac{e^x}{e^x+1}$

$$e^y=\frac{e^x}{e^x+1} \iff e^y(e^x+1)=e^x$$

$$e^ye^x+e^y=e^x$$

$$e^ye^x-e^x=-e^y$$

$$e^x(e^y-1)=-e^y$$

$$e^x=\frac{-e^y}{e^y-1}$$ πρπ $e^y-1\neq 0$

$e^y\neq 1$

$\boxed{y\neq 0}$

$$e^x=\frac{e^y}{1-e^y}$$

$$x=\ln\left(\frac{e^y}{1-e^y}\right)$$ πρπ $\frac{\overset{(+)}{e^y}}{1-e^y}>0$

$\Rightarrow 1-e^y>0$

Θετω $x=f^{-1}(y)$

$1>e^y$

$$f^{-1}(y)=\ln\frac{e^y}{1-e^y}$$

$\boxed{0>y}$

$$f^{-1}(x)=\ln\left(\frac{e^x}{1-e^x}\right)$$

$D_{f^{-1}}=(-\infty,0)$.

8 Δίνεται $f(x) = (x-2)^2 + 3, \ x \geq 2$

α) $x_1 < x_2 \Rightarrow x_1 - 2 < x_2 - 2$

$\Leftrightarrow (x_1-2)^2 < (x_2-2)^2$

(Αφού $x > 2$ το $x - 2 > 0$)

$$\underbrace{(x_1-2)^2 + 3}_{f(x_1)} < \underbrace{(x_2-2)^2 + 3}_{f(x_2)}$$

Άρα $f \nearrow$

Β' τρόπος

Έστω $f(x_1) = f(x_2)$

$$(x_1-2)^2 + \cancel{3} = (x_2-2)^2 + \cancel{3}$$

$$(x_1-2)^2 = (x_2-2)^2$$

$$|x_1 - 2| = |x_2 - 2|$$ αφού $x \geq 2$, $x - 2 \geq 0$

$$x_1 - \cancel{2} = x_2 - \cancel{2}$$

$$\underline{\underline{x_1 = x_2}}$$ Άρα f 1-1.

Θέτω $y=f(x)$

$y=(x-2)^2+3$ $\quad y>0$

$y-3=(x-2)^2$ $\quad y-3\geq 0 \Rightarrow y\geq 3$

$\sqrt{y-3}^{\,2}=(x-2)^2$

$|\sqrt{y-3}|=|x-2|$ αφού $x\geq 2$ το $x-2\geq 0$

$\sqrt{y-3}=x-2$

$x=\sqrt{y-3}+2$

$f^{-1}(y)=\sqrt{y-3}+2$

ή

$f^{-1}(x)=\sqrt{x-3}+2$. $\quad D_{f^{-1}}=[3,+\infty)$.

Τέλος

$x\geq 2 \Rightarrow \sqrt{y-3}\cancel{+2}\geq \cancel{2}$

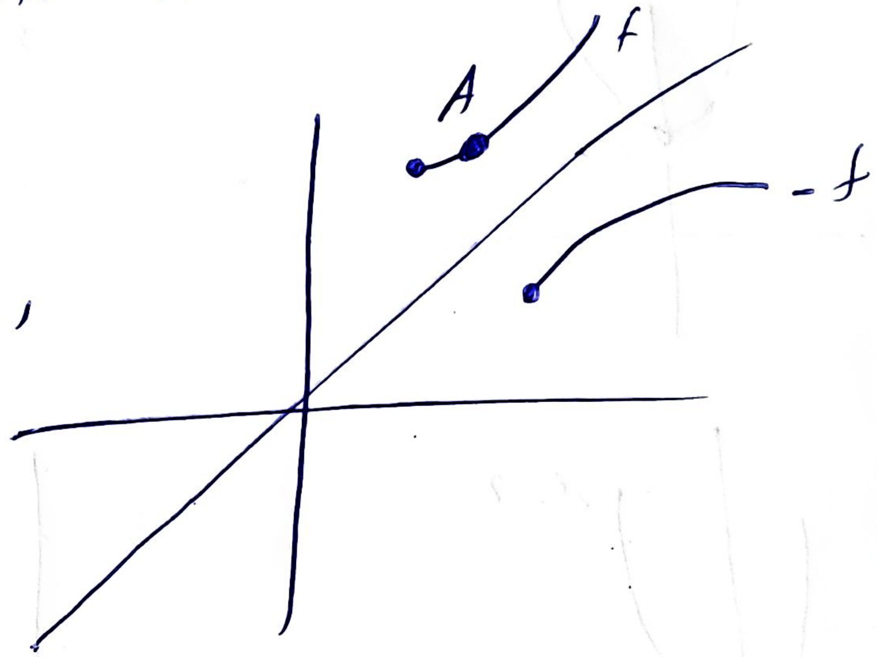
$\sqrt{y-3}\geq 0$ που ισχύει!

(γ) Προσοχη

Αφου C_f και $C_{f^{-1}}$ ειναι

συμμετρικά ως προς $y=x$

τοτε Αν $f(a)=B$ το $f^{-1}(B)=a$



$A(x, f(x)) \in C_f$

$B(f(x), x) \in C_{f^{-1}}$

$$d(A,B)=\sqrt{(f(x)-x)^2+(x-f(x))^2}=\sqrt{(x-f(x))^2+(x-f(x))^2}$$

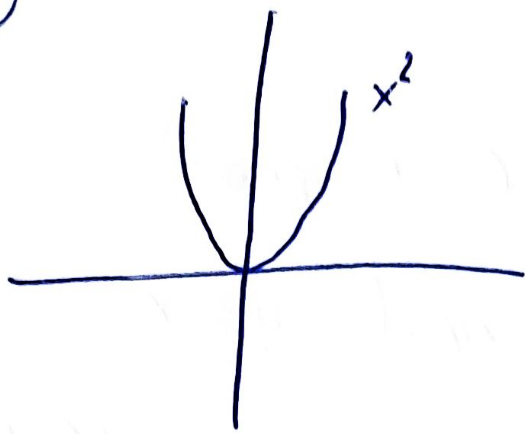
$$d(A,B)=\sqrt{2\cdot(x-f(x))^2}=\sqrt{2}\sqrt{(x-f(x))^2}$$

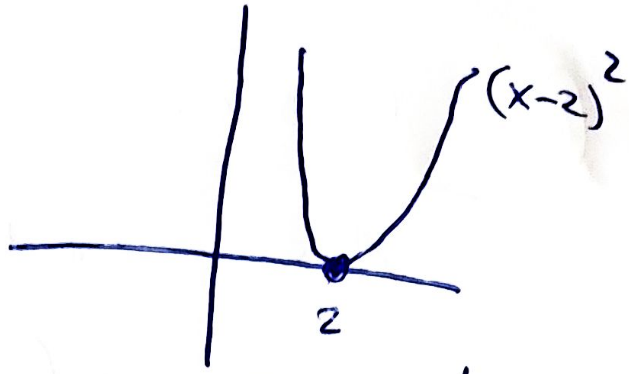
$$=\sqrt{2}\,|x-f(x)|=\sqrt{2}\,(f(x)-x).$$

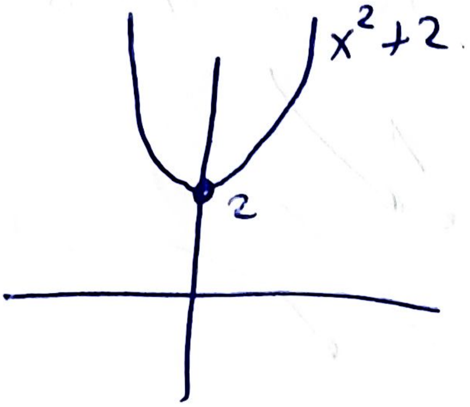
$$d(x)=\sqrt{2}\,(f(x)-x)=\sqrt{2}\,((x-2)^2+3-x)=$$

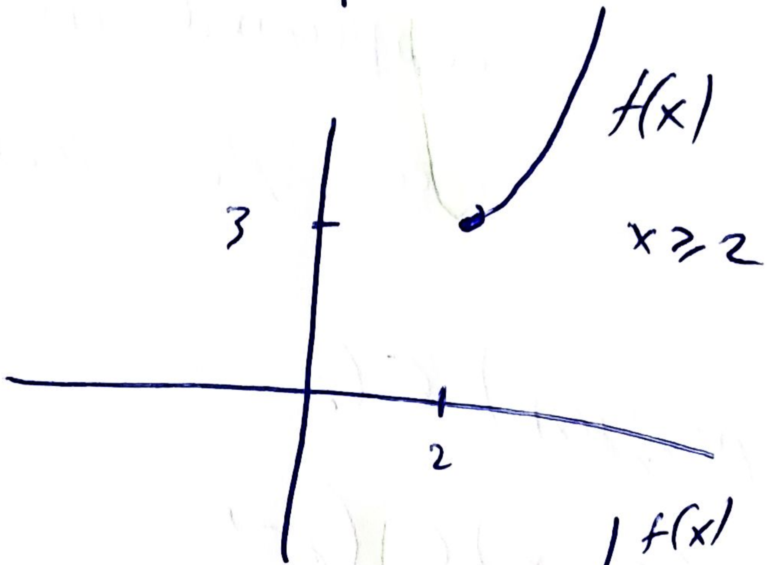
$$=\sqrt{2}\,(x^2-4x+4+3-x)=\sqrt{2}\,(x^2-5x+7).$$

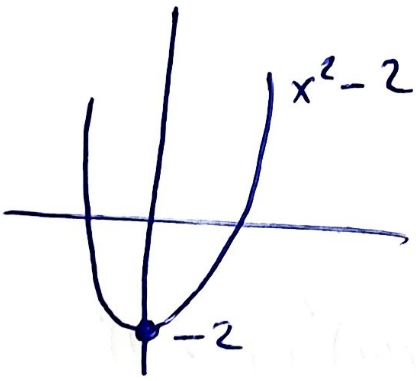
(B)

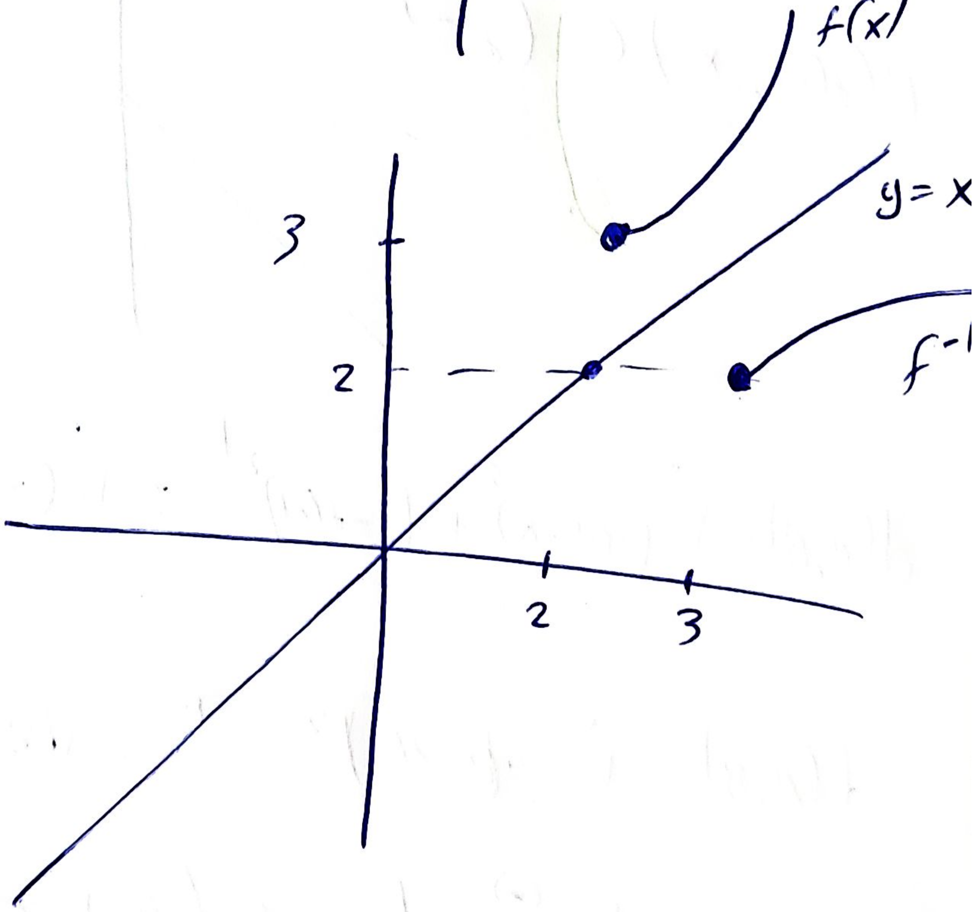












$d(x) = \sqrt{2}\left(x^2 - 5x + 7\right).$

Παραβολή με κορυφή

$$K\left(-\frac{B}{2a}, -\frac{\Delta}{4a}\right)$$

$$K\left(-\frac{5}{2}, -\frac{-3}{4}\right)$$

$$K\left(-\frac{5}{2}, \frac{3}{4}\right).$$

Ελάχιστη απόσταση

$$\frac{3\sqrt{2}}{4}.$$

(20) Σελ 109

$f(x) = e^{x-1} + 2x - 3$. $D_f = \mathbb{R}$.

(α). $x_1 < x_2 \Rightarrow x_1 - 1 < x_2 - 1 \Rightarrow e^{x_1-1} < e^{x_2-1}$ (+)

$x_1 < x_2 \Rightarrow 2x_1 < 2x_2 \Rightarrow 2x_1 - 3 < 2x_2 - 3$

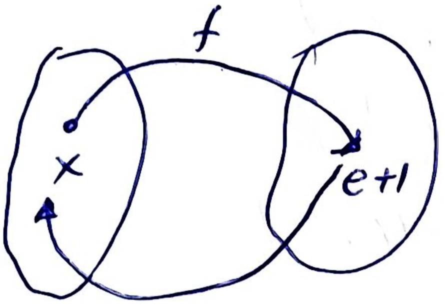
$f(x_1) < f(x_2)$

$f \nearrow$

$\Rightarrow f$ 1-1

άρα αντιστρ

(β) Ψάχνω το $f^{-1}(e+1)$.



$f(x) = e+1$

$f^{-1}(e+1) = x$

$f(x) = e+1$

$f(x) = f(2)$

f 1-1

$x = 2$

$f^{-1}(e+1) = 2$

8) i) $f(1+f^{-1}(x+1))=0$

$f(1+f^{-1}(x+1))=f(1)$

f 1-1

$\cancel{1}+f^{-1}(x+1)=\cancel{1}$

$f^{-1}(x+1)=0$

Αφού f συνάρτηση

$f(f^{-1}(x+1))=f(0)$

Ισότητα

Αν $x_1=x_2$
τότε $f(x_1)=f(x_2)$
αφού f συνάρτηση

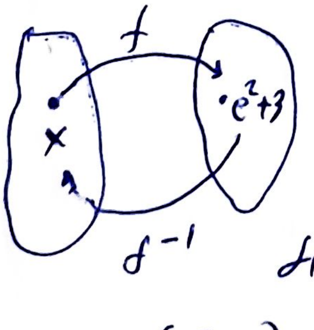
$x+1=f(0)$

$x+1=e^{-1}-3$

$\boxed{x=\frac{1}{e}-4.}$

ii) $f(x^2+x)+f^{-1}(e^2+3)>e+4.$

$f(x^2+x)+3>e+4$



$f(x)=e^2+3$

$f^{-1}(e^2+3)=x$

$f(x^2+x)>e+1$

$f(x^2+x)>f(2)$

$f\nearrow$

$x^2+x>2$

$x^2+x-2>0$

$x\in(-\infty,-2)\cup(1,+\infty)$

x	-2	1
x^2+x-2	$+\ 0\ -$	$0\ +$

Επομενο Μαθημα

Σελ 93

5 6 Δημητρης

4 5 6

7 α β γ ε η

17 18 19

$\left.\begin{array}{l} \\ \\ \\ \end{array}\right\}$ 01 01 Z

Τσεκαρω τετραδια !!

Θεωρια

1 5 10 9.

Σελ. 93

(5) Εστω $f(x) = x^5 + 2x^3 - 3$

(α) Εστω $x_1 < x_2 \Rightarrow x_1^5 < x_2^5$ (+)

Εστω $x_1 < x_2 \Rightarrow 2x_1^3 < 2x_2^3 \Rightarrow 2x_1^3 - 3 < 2x_2^3 - 3$.

$f(x_1) < f(x_2)$

$f \nearrow$ άρα f 1-1

(β) $x^5 + 2x^3 = 3$

$$\underbrace{x^5 + 2x^3 - 3}_{f(x)} = 0$$

$f(x) = 0$

$f(x) = f(1)$

f 1-1

$x = 1$

(γ) Αν $\lambda^5-\mu^5=2\mu^3-2\lambda^3$

νδο $\lambda=\mu$.

$$\underbrace{\lambda^5+2\lambda^3-3}_{f(\lambda)} = \underbrace{\mu^5+2\mu^3-3}_{f(\mu)}$$

f 1-1

$\lambda=\mu$.

(δ) $(3x-2)^5+2(3x-2)^3=3$.

$(3x-2)^5+2(3x-2)^3-3=0$

$f(3x-2)=f(1)$

f 1-1

$3x-2=1$

$3x=3$

$\underline{\underline{x=1}}$

Σελ 107

5 $f(x) = \ln x + 1$ $D_f = (0, +\infty)$

α) $x_1 < x_2 \Rightarrow \underbrace{\ln x_1 + 1}_{f(x_1)} < \underbrace{\ln x_2 + 1}_{f(x_2)}$

$f \nearrow$ άρα 1-1

β) Αφού η f 1-1 αντιστρέφεται.

$f(x) = y \Leftrightarrow y = \ln x + 1 \Leftrightarrow y - 1 = \ln x$

$e^{y-1} = x$

$$\boxed{f^{-1}(x) = e^{x-1} \quad x \in R}$$

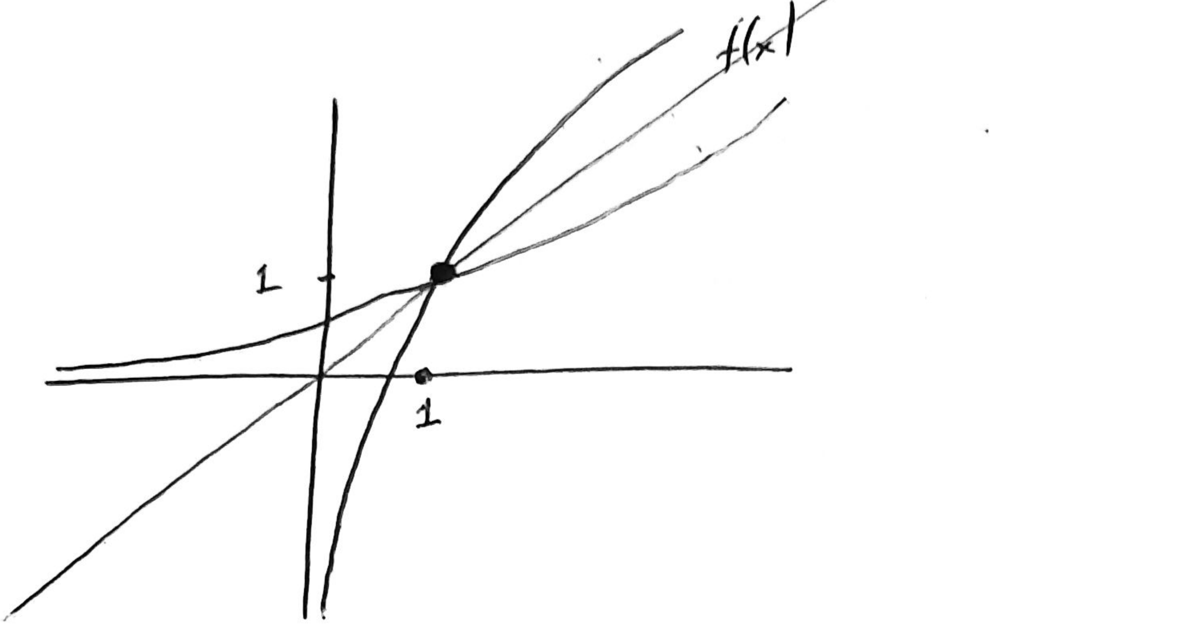
Τέλος

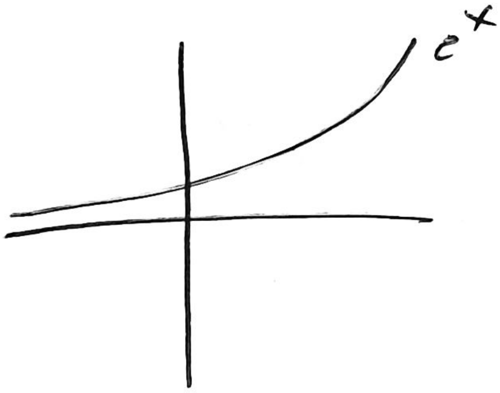
$x > 0 \Rightarrow e^{y-1} > 0$ που ισχύει!

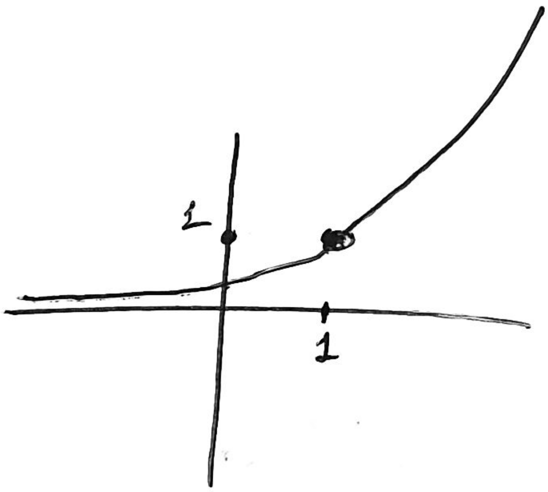
(γ)

$f(x) = \ln x + 1 \quad , \quad x > 0$

$f^{-1}(x) = e^{x-1} \quad , \quad x \in R.$







(6) $f(x) = 2 + \sqrt{x-1}$ $\qquad D_f = [1, +\infty)$.

$$x_1 < x_2 \Rightarrow \underbrace{2 + \sqrt{x_1 - 1}}_{f(x_1)} < \underbrace{2 + \sqrt{x_2 - 2}}_{f(x_2)}$$

$$f \nearrow$$

$$f(x) = y \Rightarrow y = 2 + \sqrt{x-1} \qquad y > 0$$

$$y - 2 = \sqrt{x-1} \qquad y - 2 \geq 0$$

$$(y-2)^2 = x - 1 \qquad y \geq 2$$

$$x = (y-2)^2 + 1$$

$$\boxed{f^{-1}(x) = (x-2)^2 + 1 \quad x \geq 2}$$

[illegible]

$$x \geq 1 \Rightarrow (y-2)^2 + \cancel{1} \geq \cancel{1} \Rightarrow (y-2)^2 \geq 0$$

[illegible]

7

(a) $f(x) = 1 - \sqrt{1+e^x}$ ορι $1+e^x \geq 0$

που ισχ

$D_f = \mathbb{R}$.

• Έστω $x_1 < x_2 \Rightarrow 1 - \sqrt{1+e^{x_1}} < 1 - \sqrt{1+e^{x_2}}$

$f(x_1) < f(x_2)$

$f \nearrow$ άρα 1-1

$y = 1 - \sqrt{1+e^x}$

$\sqrt{1+e^x} = 1 - y$ $\qquad 1 - y \geq 0 \Rightarrow y \leq 1$

$1 + e^x = (1-y)^2$ $\qquad$ πρέπει

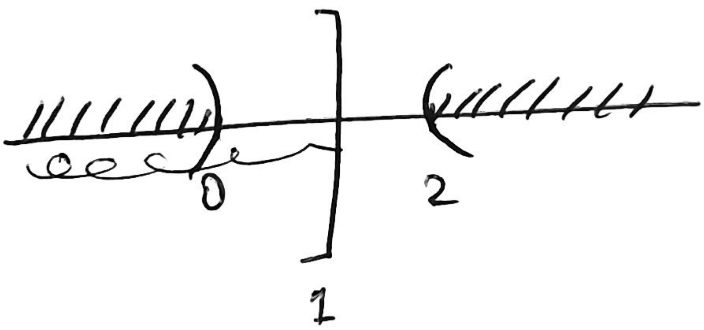
$e^x = (1-y)^2 - 1$ $\qquad (1-y)^2 - 1 > 0$

$(1-y)^2 > 1$

$(1-y)^2 > 1^2$

$|1-y| > |1|$

$|1-y| > 1$



$1-y > 1$ ή $1-y < -1$

$0 > y$ ή $2 < y$

$y \in (-\infty, 0) \cup (2, +\infty)$

$D_{f^{-1}} = (-\infty, 0)$.

(B) $f(x) = 1 - \ln(1+e^x)$ πρέπει $1+e^x > 0$ που ισχύει

$D_f = \mathbb{R}$

Γνωστή $f \downarrow$ άρα 1-1

$y = 1 - \ln(1+e^x)$

$\ln(1+e^x) = 1 - y$

$1+e^x = e^{1-y}$

$e^x = e^{1-y} - 1$

$e^{1-y} - 1 > 0$

$e^{1-y} > 1$

$1-y > 0$

$1 > y$

$x = \ln(e^{1-y} - 1)$

$f^{-1}(x) = \ln(e^{1-x} - 1)$

$x > 1$

ⓡ $f(x) = \dfrac{2x-1}{x-2}$ $\qquad D_f = \mathbb{R} - \{2\}$.

Εστω

$f(x_1) = f(x_2) \Rightarrow \dfrac{2x_1-1}{x_1-2} = \dfrac{2x_2-1}{x_2-2}$

$\Leftrightarrow (2x_1-1)(x_2-2) = (2x_2-1)(x_1-2)$

$\cancel{2x_1x_2} - 4x_1 - x_2 \cancel{+2} = \cancel{2x_1x_2} - 4x_2 - x_1 \cancel{+2}$

$3x_2 = 3x_1$

$x_1 = x_2$ $\qquad$ f 1-1,

$y = \dfrac{2x-1}{x-2}$

$y(x-2) = 2x-1$

$yx - 2y = 2x - 1$

$yx - 2x = 2y - 1$

$x(y-2) = 2y-1$,

$x = \dfrac{2y-1}{y-2}$, $y \neq 2$

$f^{-1}(x) = \dfrac{2x-1}{x-2}$, $x \neq 2$.

Τελος

$x \neq 2 \Rightarrow \dfrac{2y-1}{y-2} \neq 2$

$\cancel{2y} - 1 \neq \cancel{2y} - 4 \Rightarrow -1 \neq -4$

που ισχύει!

(ε) $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ πρπ $\frac{1+x}{1-x} > 0$

x		-1		1	
$1+x$	$-$	0	$+$		$+$
$1-x$	$+$		$+$	0	$-$
$\frac{1+x}{1-x}$	$-$	‖	$+$	‖	$-$

$D_f = (-1,1)$.

Ευκολα αν $f(x_1) = f(x_2) \Leftrightarrow \ldots \Leftrightarrow x_1 = x_2$

$$y = \ln\left(\frac{1+x}{1-x}\right)$$

$$e^y = \frac{1+x}{1-x} \Leftrightarrow e^y(1-x) = 1+x$$

$$e^y - xe^y = 1+x$$

$$e^y - 1 = x + xe^y$$

$$e^y - 1 = x(1+e^y)$$

$$f^{-1}(x) = \frac{e^x-1}{e^x+1} \qquad x = \frac{e^y-1}{e^y+1} \qquad e^y+1 \neq 0$$

που ισχύει

Τέλος

$$-1 < x < 1$$

$$-1 < \frac{e^y - 1}{e^y + 1} < 1$$

$$-e^y - 1 < e^y - 1 < e^y + 1$$

$-e^y - \cancel{1} < e^y \cancel{-1}$ και $\cancel{e^y} - 1 < \cancel{e^y} + 1$

$0 < 2e^y$ που ισχυει..

$-1 < 1$ που ισχυει!

$$D_{f^{-1}} = \mathbb{R}.$$

(n) $f(x) = \dfrac{\ln x}{\ln x - 1}$.

$\boxed{x > 0}$

και $\ln x - 1 \neq 0$.

$\ln x \neq 1$

$\boxed{x \neq e}$

$D_f = (0, e) \cup (e, +\infty)$.

Έστω αν $f(x_1) = f(x_2) \Leftrightarrow \dots \Rightarrow x_1 = x_2$

$$y = \frac{\ln x}{\ln x - 1}$$

$$y(\ln x - 1) = \ln x$$

$$y \ln x - y = \ln x$$

$$y \ln x - \ln x = y$$

$$\ln x (y - 1) = y \qquad \underline{\underline{y \neq 1}}$$

$$\ln x = \frac{y}{y-1}$$

$$x = e^{\frac{y}{y-1}}$$

$$f^{-1}(x) = e^{\frac{x}{x-1}}$$

$x \neq 1$

Τελο'

$0<x<e$ ή $x>e$

$0<e^{\frac{y}{y-1}}<e$

$0<e^{\frac{y}{y-1}}$ ή $e^{\frac{y}{y-1}}<e^1$

$\frac{y}{y-1}<1$

$\frac{y}{y-1}-1<0$

$\frac{\overset{+}{1}}{y-1}<0$

$y-1<0$

$y<1$

$e^{\frac{y}{y-1}}>e^1$

$\frac{y}{y-1}>1$

$\frac{y}{y-1}-1>0$

$\frac{y}{y-1}-\frac{y-1}{y-1}>0$

$\frac{y-y+1}{y-1}>0$

$\frac{\overset{+}{1}}{y-1}>0$

$y-1>0$

$y>1$

$D_{f^{-1}}=\mathbb{R}-\{1\}$.

Σελ 109

17 $f(x) = e^x + x$

(α) $f \nearrow$ άρα 1-1 άρα αντιστρέψιμη.

(β) $f^{-1}(x) = 0$

$f\left(f^{-1}(x)\right) = f(0)$

$x = f(0)$

$\underline{\underline{x = 1}}$.

Έστω $f^{-1}(x) > 0 \overset{f \nearrow}{\Longrightarrow} f\left(f^{-1}(x)\right) > f(0)$

$x > 1$

Έστω $f^{-1}(x) < 0 \Longrightarrow x < 1$

x		1	
$f^{-1}(x)$	$-$	0	$+$

18 (α) Αφου f γν. μονοτονη

$\Rightarrow f \nearrow$ ή $f \searrow$

Γνωριζω οτι $1 < 3$ και $\underset{2}{\underset{\parallel}{f(1)}} > \underset{-2}{\underset{\parallel}{f(3)}}$

Αρα $f \searrow$ αρα αντιστρ.

(β) $f(-2+f^{-1}(x+2)) = 2$

$$f(-2+f^{-1}(x+2)) = f(1)$$

$$f: 1-1$$

$$-2+f^{-1}(x+2) = 1$$

$$f^{-1}(x+2) = 3$$

$$f(f^{-1}(x+2)) = f(3)$$

$$x+2 = -2$$

$$\underline{\underline{x = -4}}$$

(γ) $f^{-1}(f(e^x-1)-4) < 3$

$f\downarrow$

$$f\left(f^{-1}(f(e^x-1)-4)\right) > f(3)$$

$$f(e^x-1)-4 > -2$$

$$f(e^x-1) > 2$$

$$f(e^x-1) > f(1)$$

$f\downarrow$

$$e^x-1 < 1$$

$$e^x < 2$$

$$x < \ln 2$$

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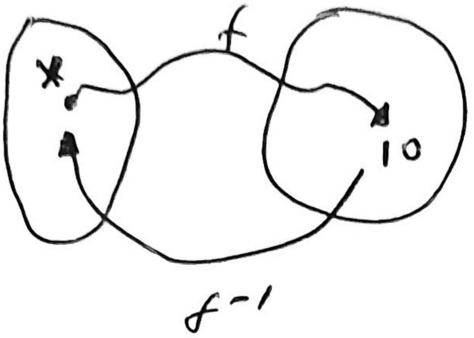
$f(x) = x^3 + x$

(α) $f \nearrow \Rightarrow f: 1\text{-}1$

(β) Βρδ $f^{-1}(10)$.



$f(x) = 10 \iff f^{-1}(10) = x$

$f(x) = f(2)$

$f: 1\text{-}1$

$\underline{\underline{x = 2}} \longrightarrow f^{-1}(10) = 2$

(γ) i) $f\left(1 + f^{-1}(x^2 - 3x)\right) = 0$

$$f\left(1 + f^{-1}(x^2 - 3x)\right) = f(0)$$

$f: 1\text{-}1$

$$1 + f^{-1}(x^2 - 3x) = 0$$

$$f^{-1}(x^2 - 3x) = -1$$

$f\left(f^{-1}(x^2 - 3x)\right) = f(-1) \Rightarrow x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$x = 1$, $x = 2$

11). $f^{-1}\left(f(x^2-1)+4f^{-1}(-10) \right) < 1$ B1

$f \nearrow$

$f\left(f^{-1}\left(f(x^2-1)+4f^{-1}(-10) \right)\right) < f(1)$

$f(x^2-1)+4f^{-1}(-10) < 2.$

$\boxed{\text{Αφού } f(-2)=-10 \iff f^{-1}(-10)=-2.}$

$f(x^2-1)+4(-2) < 2$

$f(x^2-1) < 8+2$

$f(x^2-1) < 10$

$f(x^2-1) < f(2)$ $\qquad x \in (-\sqrt{3}, \sqrt{3}).$

$f \nearrow$

$x^2-1 < 2 \iff x^2 < 3$

Σελ 108

13

- $f:\mathbb{R}\to\mathbb{R}$
- $f^3(x)+f(x)-x=0 \quad \forall x\in\mathbb{R}.$

$f(\mathbb{R})=\mathbb{R}.$ $\longrightarrow$ $\underline{f^3(x)+f(x)=x}.$

(α) Έστω $f(x_1)=f(x_2) \Rightarrow f^3(x_1)=f^3(x_2)$

Έστω $f(x_1)=f(x_2)$ $\quad$ (+)

$$\underbrace{f^3(x_1)+f(x_1)}_{x_1}=\underbrace{f^3(x_2)+f(x_2)}_{x_2}$$

$$x_1=x_2$$

(β) Θέτω $f(x)=y$ και $x=f^{-1}(y)$

$y^3+y-f^{-1}(y)=0$ $\qquad$ $D_{f^{-1}}=\mathbb{R}.$

$f^{-1}(y)=y^3+y$ $\qquad$ $\underline{f^{-1}(x)=x^3+x}$

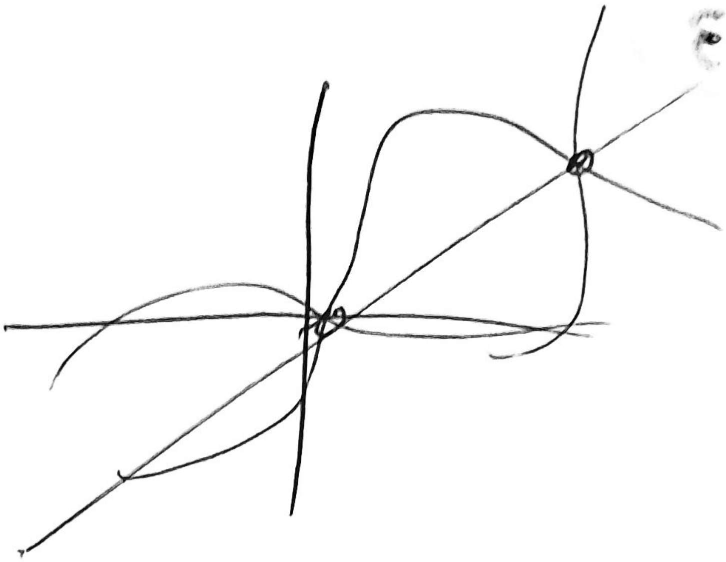
(γ) $f(x) = x$

$f^{-1}(x) = x$

$x^3 + \cancel{x} = \cancel{x}$

$x^3 = 0$

$x = 0$



Ισοδυναμ

$f(x) = x$

$f^{-1}(x) = x$

Ισοδυναμ

$f(x) = x$

$f^{-1}(x) = x$

$f(x) = f^{-1}(x)$

$f \nearrow$

④ $D_f = (-2,3) \cup (3,7]$

$\{ T_f = (-1,5]$

(B) i) $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2} f(x) = -1$.

ii). $\left.\begin{array}{l} \lim\limits_{x \to 1^-} f(x) = 1 \\ \lim\limits_{x \to 1^+} f(x) = 2 \end{array}\right\}$ Το $\lim\limits_{x \to 1} f(x)$ δεν υπάρχει.

iii). $\left.\begin{array}{l} \lim\limits_{x \to 3^-} f(x) = 3 \\ \lim\limits_{x \to 3^+} f(x) = 3 \end{array}\right\}$ $\lim\limits_{x \to 3} f(x) = 3$.

iv). $\left.\begin{array}{l} \lim\limits_{x \to 5^-} f(x) = 2 \\ \lim\limits_{x \to 5^+} f(x) = 4 \end{array}\right\}$ Το όριο δεν υπάρχει.

v) $\lim\limits_{x \to 7} f(x) = 3$.

Σελ 148

7 $\lim\limits_{x \to 1} f(x) = 4$. και $\lim\limits_{x \to 1} g(x) = -1$.

(α) $\lim\limits_{x \to 1} \left(f(x) + g(x) \right) = \lim\limits_{x \to 1} f(x) + \lim\limits_{x \to 1} g(x) =$

$= 4 + (-1) = 4 - 1 = 3$.

(β) $\lim\limits_{x \to 1} \left(2f(x) - g(x) \right) = 2 \cdot 4 - (-1) = 8 + 1 = 9$

(γ) $\lim\limits_{x \to 1} \left(f(x)\, g(x) + g^2(x) \right) = 4 \cdot (-1) + (-1)^2$

$= -4 + 1 = -3$

8 (γ) $\lim_{x \to 1} \left(\frac{3x-2}{x+1} \right) = \frac{3 \cdot 1 - 2}{1+1} = \frac{1}{2}$

9 (στ) $\lim_{x \to -\frac{1}{2}} \frac{2x^2+x}{4x^2-1} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x \to -\frac{1}{2}} \frac{x\cancel{(2x+1)}}{(2x-1)\cancel{(2x+1)}}$

$= \lim_{x \to -\frac{1}{2}} \frac{x}{2x-1} = \frac{-\frac{1}{2}}{-1-1} = \frac{-\frac{1}{2}}{-2} = \frac{1}{4}.$

(γ) $\lim_{x \to 5} \left(\frac{2x-10}{x^2-5x} \right) \overset{\left(\frac{0}{0}\right)}{=\!=} \lim_{x \to 5} \frac{2\cancel{(x-5)}}{x\cancel{(x-5)}} =$

$= \lim_{x \to 5} \frac{2}{x} = \frac{2}{5}.$

10 (γ) $\lim_{x \to 1} \left(\frac{x^2-2x+1}{x^2-x} \right) \overset{\left(\frac{0}{0}\right)}{=\!=}$

$= \lim_{x \to 1} \frac{(x-1)^{\cancel{2}}}{x\cancel{(x-1)}} = \lim_{x \to 1} \frac{x-1}{x} = \frac{0}{1} = 0.$

11

(β) $\lim_{x\to -1} \frac{x^3+1}{x^3+x+2} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}(x^2-x+2)}$

$a^3 \pm B^3 = (a \pm B)(a^2 \mp aB + B^2)$ $\qquad = \frac{1+1+1}{1+1+2} = \frac{3}{4}$

1	0	1	2	-1
	-1	1	-2	
1	-1	2	0	

(γ) $\lim_{x\to 3} \frac{x^4-81}{x^3-27} = \lim_{x\to 3} \frac{(x^2)^2-9^2}{x^3-3^3}$

$= \lim_{x\to 3} \frac{(x^2-9)(x^2+9)}{(x-3)(x^2+3x+9)} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)(x^2+9)}{\cancel{(x-3)}(x^2+3x+9)}$

$= \frac{6\cdot 18}{27} = \frac{6\cdot \cancel{3}\cdot 6}{\cancel{3}\cdot 9} = \frac{2\cdot \cancel{3}\cdot 6}{\cancel{3}\cdot 3}$

$= 4.$

12 (B) $\lim_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$

$$= \lim_{x \to 1} \left(\frac{1}{1-x} - \frac{3}{(1-x)(1+x+x^2)} \right)$$

$$= \lim_{x \to 1} \frac{1+x+x^2-3}{(1-x)(1+x+x^2)}$$

$$= \lim_{x \to 1} \frac{x^2+x-2}{-(x-1)(1+x+x^2)}$$

$$= \lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{-\cancel{(x-1)}(1+x+x^2)}$$

$$= \frac{3}{-3} = -1$$

13 (1) $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{x+2}-2}$ $\left(\dfrac{0}{0}\right)$

$$= \lim_{x \to 2} \frac{(x-2)(\sqrt{x+2}+2)}{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}$$

$$= \lim_{x \to 2} \frac{(x-2)(\sqrt{x+2}+2)}{\sqrt{x+2}^{\,2}-2^2}$$

$$= \lim_{x \to 2} \frac{(x-2)(\sqrt{x+2}+2)}{x+2-4}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(\sqrt{x+2}+2)}{\cancel{x-2}} = 4.$$

14

(γ) $$\lim_{x\to -2} \frac{2\sqrt{3+x} - \sqrt{3x^2-8}}{x^2+x-2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to -2} \frac{\left(2\sqrt{3+x} - \sqrt{3x^2-8}\right)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}{(x+2)(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \lim_{x\to -2} \frac{\left(2\sqrt{3+x}\right)^2 - \left(\sqrt{3x^2-8}\right)^2}{(x+2)(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \lim_{x\to -2} \frac{4(3+x) - (3x^2-8)}{(x+2)(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \lim_{x\to -2} \frac{12+4x-3x^2+8}{(x+2)(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \lim_{x\to -2} \frac{-3x^2+4x+20}{(x+2)(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \lim_{x \to -2} \frac{-3\left(x - \frac{10}{3}\right)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \lim_{x \to -2} \frac{-3\left(x - \frac{10}{3}\right)}{(x-1)\left(2\sqrt{3+x} + \sqrt{3x^2-8}\right)}$$

$$= \frac{\cancel{-3}\left(-2 - \frac{10}{3}\right)}{\cancel{-3}\cdot(2+2)} = \frac{-\frac{16}{3}}{4}$$

$$= -\frac{16}{12} = -\frac{4}{3}.$$

14

5) $$\lim_{x\to 1} \frac{\sqrt{3x-2} - 2x+1}{x^4 - 1} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{\sqrt{3x-2} - (2x-1)}{(x^2)^2 - 1^2}$$

$$= \lim_{x\to 1} \frac{\left[\sqrt{3x-2} - (2x-1)\right]\left[\sqrt{3x-2} + (2x-1)\right]}{(x^2-1)(x^2+1)\left(\sqrt{3x-2} + 2x - 1\right)}$$

$$= \lim_{x\to 1} \frac{\cancel{\sqrt{3x-2}}^{\cancel{2}} - (2x-1)^2}{(x-1)(x+1)\left(\sqrt{3x-2} + 2x - 1\right)}$$

$$= \lim_{x\to 1} \frac{3x-2 - (4x^2 - 4x + 1)}{(x-1)(x+1)\left(\sqrt{3x-2} + 2x - 1\right)}$$

$$= \lim_{x\to 1} \frac{-4x^2 + 7x - 3}{(x-1)(x+1)\left(\sqrt{3x-2} + 2x - 1\right)} =$$

$$= \lim_{x \to 1} \frac{-4\cancel{(x-1)}\left(x - \frac{3}{4}\right)}{\cancel{(x-1)}(x+1)\left(\sqrt{3x-2} + 2x - 1\right)}$$

$$= \frac{-4\left(1 - \frac{3}{4}\right)}{2 \cdot (1+1)} = \frac{-4+3}{4} = \frac{-1}{4}.$$

19

(B) $\lim_{x \to -1} \dfrac{\overset{+}{|x^3-3x-1|}+x}{\underset{-}{|x^3+5x+4|}-2} =$

$= \lim_{x \to -1} \dfrac{x^3-3x-1+x}{-x^3-5x-4-2} =$

$= \lim_{x \to -1} \dfrac{x^3-2x-1}{-x^3-5x-6} = \lim_{x\to -1} \dfrac{\cancel{(x+1)}(x^2-x-1)}{\cancel{(x+1)}(-x^2+x-6)}$

$= \dfrac{1}{-6}$

$$\begin{array}{cccc|c} 1 & 0 & -2 & -1 & -1 \\ \downarrow & -1 & 1 & +1 & \\ 1 & -1 & -1 & 0 & \end{array}$$

$(x+1)(x^2-x-1)$

$$\begin{array}{cccc|c} -1 & 0 & -5 & -6 & -1 \\ \downarrow & 1 & -1 & 6 & \\ -1 & 1 & -6 & 0 & \end{array}$$

$(x+1)(-x^2+x-6)$

20 (B) $\lim_{x \to -1} \dfrac{|x^2-1| - x^2+x+2}{\sqrt{x+2}-1}$.

Το όριο ~~δεν~~ υπάρχει!

x	-1	1
x^2-1	$+ \quad 0 \quad -$	$0 \quad +$

$\to \lim_{x\to -1^-} \dfrac{|x^2-1| - x^2+x+2}{\sqrt{x+2}-1} = \lim_{x\to -1^-} \dfrac{\cancel{x^2}-1-\cancel{x^2}+x+2}{\sqrt{x+2}-1}$

$\lim_{x\to -1^-} = \dfrac{x+1}{\sqrt{x+2}-1} = \lim_{x\to -1^-} \dfrac{(x+1)(\sqrt{x+2}+1)}{(x+2-1)}$

$= \lim_{x\to -1^-} \dfrac{\cancel{(x+1)}(\sqrt{x+2}+1)}{\cancel{x+1}} = 2.$

$\to \lim_{x\to -1^+} \dfrac{|x^2-1| - x^2+x+2}{\sqrt{x+2}-1} = \lim_{x\to -1^+} \dfrac{-x^2+1-x^2+x+2}{\sqrt{x+2}-1}$

$= \dfrac{-2x^2+x+3}{\sqrt{x+2}-1} = \dfrac{-2(x+1)(x-\frac{3}{2})(\sqrt{x+2}+1)}{x+2-1} = \dfrac{-2\cancel{(x+1)}(x-\frac{3}{2})(\sqrt{x+2}+1)}{\cancel{x+1}}$

$= -2(x-\frac{3}{2})(\sqrt{x+2}+1) = -2(-1-\frac{3}{2})(\sqrt{-1+2}+1) = -2+2+1 = 1$

Επομενα Μαθηματα

Δευτερα 1/7 11:30-1.

Δευτερα 8/7 11:30-1

Τεταρτη 10/7 8:30-10

Παρασκευη 12/7 11-1

Σαββατο 13/7 9-10:30.

⋮

Σαββατο 20/7 διωρο.

Επομαω Μαθημα

Δευτερα 11:30 - 1

Σελ 111

26

27

28

29

30

31

Ασκησεις ορίων

Σελ 147

2

3

8 α β

9 α β δ ε

10 α β

11 α

12 α

13 α β δ ε στ ζ η θ

14 α β.

19 α γ

20 α γ.