

10

12

17

19

24

26

Βασικά σύνολα

$$N = \{0, 1, 2, 3, \dots\} \quad \text{φυσικοί}$$

$$Z = \{\dots, -2, -1, 0, 1, 2, 3, \dots\} \quad \text{Ακέραιος.}$$

$$Q = \left\{ -3, 00, -2, 000, \frac{1}{2}, 2, \frac{5}{2}, \dots \right\}.$$

Αρρητοι. είναι οι αριθμοί που
δεν μπορούν να γραφούν ως κλάσμα.

π.χ $\pi = 3,14.$

$$R = \{ \text{ρητοί} + \text{αρρητοι} \}, \quad \text{πλοζυματικοί.}$$

Υπάρχουν δύο τρόποι να περιγραφεί
ένα σύνολο.

1. Με αναγραφή.

$$A = \{ 1, 2, 3, 4, 5 \}.$$

2. Με περιγραφή.

$$A = \{ x \in \mathbb{N} \mid 1 \leq x \leq 5 \}.$$

Παραδείγματα

$$A = \{x \in \mathbb{Z} \mid x \text{ άρτιος}, 1 < x < 12\}.$$

$$A = \{2, 4, 6, 8, 10\}.$$

$$B = \{-5, -3, -1, 1, 3\}.$$

$$B = \{x \in \mathbb{Z} \mid x \text{ περιττός}, -5 \leq x \leq 3\}.$$

Το $A \subseteq B$ αν κάθε στοιχείο

του A ανήκει και στο B .

n.x

$$A = \{1, 2, 3\}$$

$$A \subseteq B$$

$$B = \{0, 1, 2, 3, 4\}$$

Αρα $A = B$

αν $A \subseteq B$ και $B \subseteq A$

ιδιότητες

i) $A \subseteq A$

ii) $A \subseteq B$ και $B \subseteq \Gamma$ τότε $A \subseteq \Gamma$

iii) $A \subseteq B$ και $B \subseteq A$ τότε $A = B$.

$$A = \{0, 1, 2\} \quad A = \{0, 1, 2\}$$

κενο σωδο



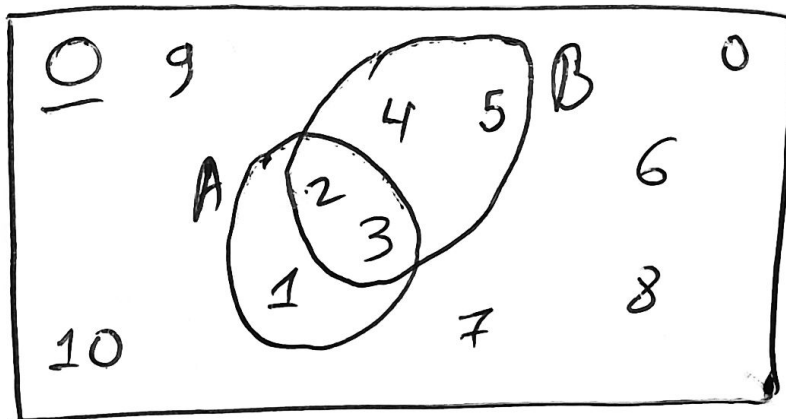
Είναι το σωδο που δεν έχει
τιποτα μέσα.

Διαγραμμα Venn

$$O = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

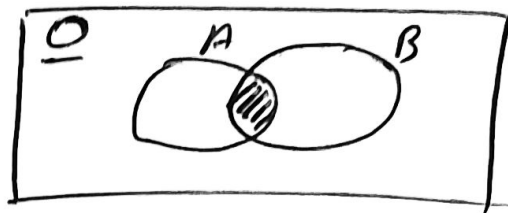
$$A = \{1, 2, 3\}$$

$$B = \{2, 3, 4, 5\}$$



Τομή συνόλων

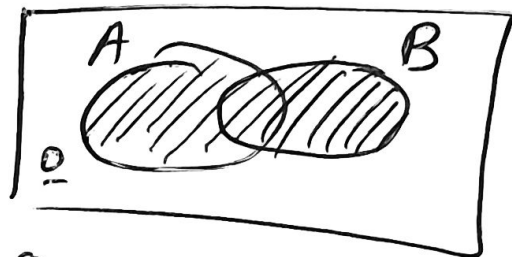
Είναι τα κοινά.



$$A \cap B = \{x \in \Omega \mid x \in A \text{ και } x \in B\}.$$

Ενωση συνόλων

$$A \cup B = \{x \in \Omega \mid x \in A \text{ ή } x \in B\}.$$



Έχει όλα τα στοιχεία του Ω που ανήκουν

Συμπληρωμα του A

είτε στο A

είτε στο B.

$$A' = \{x \in \Omega \mid x \notin A\}.$$

Έχει μέσα όλα τα στοιχεία του

Ω που δεν ανήκουν στο A.

$$\underline{O} = \{x \in \mathbb{N} \mid x < 10\},$$

$$A = \{x \in \underline{O} \mid (x^2 - 4)(x^2 - 9) = 0\},$$

$$B = \{x \in \underline{O} \mid (x^2 + 5x + 6)x^{2024} = 0\},$$

Λύση

$$\underline{O} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

$$A = \{2, 3\}$$

$$B = \{0\}.$$

$$(x^2 - 4)(x^2 - 9) = 0,$$

$$(x-2)(x+2)(x-3)(x+3) = 0$$

$$x-2=0 \quad \vee \quad x+2=0 \quad \vee \quad x-3=0 \quad \vee \quad x+3=0,$$

$$x=2$$

$$x=-2$$

$$x=3$$

$$x=-3$$

$$(x^2 + 5x + 6)x^{2024} = 0$$

$$x^2 + 5x + 6 = 0 \quad \vee \quad x^{2024} = 0$$

$$x = -2$$

$$x = -3$$

$$x = 0$$

$$\alpha \cdot B = 0 \Rightarrow \alpha = 0 \quad \vee \quad B = 0.$$

A^c

$$A = \{2, 3\}$$

$$B = \{0\}$$

$$A \cap B = \emptyset$$

$$A \cup B = \{2, 3, 0\}$$

$$A' = \{0, 1, 4, 5, 6, 7, 8, 9\}$$

$$B' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$(A \cap B)' = \underline{0}$$

Άσκηση

Έστω $\underline{O} = \{x \in \mathbb{Z} / x \text{ διαίρεται από } 20, |x| \leq 9\}$

και $A = \{x \in \underline{O} / (x^2 - 2x - 3)(x - 2) = 0\}$.

και $B = \{x \in \underline{O} / -3 < x < 4\}$.

α) Βρίξτε \underline{O}, A, B .

β) Βρίξτε $A \cup B, A \cap B, A', B'$

γ) Βρίξτε $(A \cup B)'$ και $A' \cap (A \cap B)'$

Αυστη

$$\textcircled{a) \quad \underline{O} = \{-1, 1, 2, -2, 4, -4, 5, -5\}}$$

Διαφορικά του 20

$$\pm 1 \pm 2 \pm 4 \pm 5 \pm 10 \pm 20,$$

$$A = \{-1, 2\}$$

$$(x^2 - 2x - 3)(x - 2) = 0$$

$$x^2 - 2x - 3 = 0$$

n

$$x - 2 = 0$$

$$\Delta = B^2 - 4a\gamma$$

$$x = 2$$

$$\Delta = (-2)^2 - 4 \cdot 1 \cdot (-3)$$

$$\Delta = 4 + 12 = 16$$

$$x = \frac{-B \pm \sqrt{\Delta}}{2a} = \frac{2 \pm 4}{2} \begin{cases} 3 \\ -1 \end{cases}$$

$$B = \{-2, -1, 1, 2\}$$

$$A \cup B = \{-1, 1, -2, 2, 4, -4, 5, -5\}$$

$$A = \{-1, 2\}$$

$$B = \{-2, -1, 1, 2\}$$

$$(B) \cdot A \cup B = \{-2, -1, 1, 2\}$$

$$A \cap B = \{-1, 2\}$$

$$A' = \{1, -2, 4, -4, 5, -5\}$$

$$B' = \{4, -4, 5, -5\}$$

$$(D) \cdot (A \cup B)' = B'$$

$$A' \cap (A \cap B)' = \{1, -2, 4, -4, 5, -5\}$$

$$\bullet (A \cap B)' = \{1, -2, 4, -4, 5, -5\}$$

Άσκηση

$$\text{Έστω } \Omega = \{1, 2, 3, 4, 5, 6\}.$$

$$A = \{1, 2, 3, 4\}.$$

$$B = \{3, 4, 6\}.$$

$$\text{Βρες το } (A' \cap B)' \cap (B' \cup A).$$

Προσευ αρχικά να βρω.

$$A' = \{5, 6\}$$

$$A' \cap B = \{6\}$$

$$(A' \cap B)' = \{1, 2, 3, 4, 5\}$$

$$B' = \{1, 2, 5\}$$

$$B' \cup A = \{1, 2, 5, 3, 4\}$$

$$\text{Άρα } (A' \cap B)' \cap (B' \cup A) = \{1, 2, 3, 4, 5\}.$$

ΕΡΩΤΗΣΕΙΣ ΚΑΤΑΝΟΗΣΗΣ

- I. 1. Στους παρακάτω πίνακες να συμπληρώσετε με το σύμβολο "✓" εκείνα τα τετραγώνια των οποίων ο αντίστοιχος αριθμός ανήκει στο αντίστοιχο σύνολο.
 2. Πώς ονομάζονται οι αριθμοί για τους οποίους έχουν συμπληρωθεί τα τετραγώνια μόνο της τελευταίας γραμμής;
 3. Να χρησιμοποιήσετε τα διαγράμματα του Venn για να παραστήσετε τις διαδοχικές σχέσεις εγκλεισμού των συνόλων \mathbb{N} , \mathbb{Z} , \mathbb{Q} και \mathbb{R} και να τοποθετήσετε μέσα αυτά τους αριθμούς αυτούς.

	-3,5	0	$\sqrt{10}$	$-\frac{13}{5}$	π	$2,\bar{3}$	$\frac{20}{5}$	$\sqrt{100}$	-5
$\in \mathbb{N}$		✓					✓	✓	
$\in \mathbb{Z}$		✓					✓	✓	✓
$\in \mathbb{Q}$	✓	✓		✓		✓	✓	✓	✓
$\in \mathbb{R}$	✓	✓	✓	✓	✓	✓	✓	✓	✓

II. Σε καθεμιά από τις παρακάτω ερωτήσεις να συμπληρώσετε τις ισότητες.

1. Αν $A = \{x \in \mathbb{N} | x \text{ διαιρέτης του } 16\}$ και $B = \{x \in \mathbb{N} | x \text{ διαιρέτης του } 24\}$, τότε:

α) $A \cup B = \{1, 2, 3, 4, 6, 8, 12, 16, 24\}$ β) $A \cap B = \{1, 2, 4, 8\}$

2. Ας θεωρήσουμε ως βασικό σύνολο το σύνολο Ω των γραμμάτων του ελληνικού αλφαβήτου και τα υποσύνολά του

$$A = \{x \in \Omega | x \text{ φωνήεν}\} \quad \text{και} \quad B = \{x \in \Omega | x \text{ σύμφωνο}\}.$$

Τότε:

α) $A \cup B = \{\emptyset\}$ β) $A \cap B = \{\emptyset\}$ γ) $A' = \{B\}$ δ) $B' = \{A\}$

III. Σε καθεμιά από τις παρακάτω ερωτήσεις να βάλετε σε κύκλο τις σωστές αντήσεις.

1. Έστω δύο σύνολα A και B. Τότε:

α) $A \subseteq A \cap B$ β) $B \subseteq A \cap B$ γ) $A \cap B \subseteq A$ δ) $A \cap B \subseteq B$

2. Έστω δύο σύνολα A και B. Τότε:

α) $A \subseteq A \cup B$ β) $A \cup B \subseteq B$ γ) $A \cup B \subseteq A$ δ) $B \subseteq A \cup B$

IV. Σε καθεμιά από τις παρακάτω ερωτήσεις να συμπληρώσετε τις ισότητες.

1. Έστω Ω ένα βασικό σύνολο, \emptyset το κενό σύνολο και $A \subseteq \Omega$. Τότε:

α) $\emptyset' = \Omega$ β) $\Omega' = \emptyset$ γ) $(A')' = A$

2. Έστω $A \subseteq B$. Τότε

α) $A \cap B = A$ β) $A \cup B = B$

② Σε2 27

⑤2 $1 - (-7+3) + (-2-3) =$
 $1 + 7 - 3 - 2 - 3 = 0$

③

⑦ $-2 \cdot (-3) \cdot (-5) = +6 \cdot (-5) = -30$

⑥2 $9 - 3(-1)(-5) = \text{~~9 + 15~~} 9 + 15 = -6$

⑧ $-1 - (-2)(3-8) = -1 + 2(3-8) = -1 + 6 - 16 = -11$

(11)

$$x+y=3$$

$$B = (x-3)(y-1) - x(y+2) =$$

$$= \cancel{xy} - x - 3y + 3 - \cancel{xy} - 2x =$$

$$= -3x - 3y + 3 =$$

$$= -3(x+y) + 3 =$$

$$= -3 \cdot 3 + 3$$

$$= -9 + 3 = \underline{\underline{-6}}$$

$$\textcircled{13} \textcircled{B} B = 1 - 2(a-1) - x(3y-0) + \beta(-x-2) =$$

$$\alpha, \beta \text{ αναδίωξι} \Rightarrow \boxed{\alpha + \beta = 0}$$

$$x, y \text{ αντιστροφή} \Rightarrow \textcircled{xy = 1.}$$

$$B = 1 - 2a + 2 - 3xy + \beta x - \beta x - 2\beta =$$

$$= \cancel{1 - 2a + 2} 3 - 2(\alpha + \beta) - 3 =$$

$$= 3 - 3 = 0$$

$$\textcircled{5} \textcircled{B} \quad B = 2\alpha - 5\beta - \gamma =$$

$$= 2 \cdot (-3) - 5(-2) - (-1) =$$

$$= -6 + 10 + 1 = 5$$

$$\alpha = -3$$

$$\beta = -2$$

$$\gamma = -1$$

$$\textcircled{8} \quad \Delta = 1 - 2\alpha\beta - \alpha\beta\gamma =$$

$$= 1 - 2(-3)(-2) - (-3)(-2)(-1) =$$

$$= 1 - 12 + 6 = -5$$

9

$$A = 1 - 2(a - b) + 3(2a - 3b) - a$$

$$a = -\frac{1}{2}$$

$$b = -\frac{2}{3}$$

$$A = 1 - 2a + 2b + 6a - 9b - a$$

$$A = 1 + 3a - 7b$$

$$A = 1 + 3 \cdot \left(-\frac{1}{2}\right) - 7 \cdot \left(-\frac{2}{3}\right)$$

$$A = 1 - \frac{3}{2} + \frac{14}{3}$$

EKN=6

$$A = \frac{1}{6} - \frac{3}{2} + \frac{14}{3}$$

$$A = \frac{1}{6} - \frac{9}{6} + \frac{28}{6}$$

$$A = \frac{20}{6}$$

$$A = \frac{10}{3}$$

4

$$(B) 5 - 5 \cdot (-3 + 5) - (7 - 8)(-5 + 3) =$$

$$= 5 + 15 - 25 + 35 - 21 - 40 + 24 = 79 - 86 = -7$$

altern

$$\frac{\text{altern}}{5 - 5 \cdot (2) - (-1)(-2) =$$

$$= 5 - 10 - 2 = -5 - 2 = -7$$

$$(8) 1 - 2(7 - 8) - 3 [5 + 4 \cdot (7 - 2 \cdot 5)] =$$

$$= 1 - 14 + 16 - 3 [5 + 4 \cdot (7 - 10)] =$$

$$= 1 - 20 - 3 [5 - 12] =$$

$$= 1 - 20 - 15 + 36 = 37 - 35 = 2$$

8

$$\begin{aligned} \textcircled{B} \quad & -2x + 3(1-x) - 2(-x+3) = \\ & = -\cancel{2x} + 3 - \underline{3x} + \underline{2x} - 6 = \\ & = -3x - 3 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad & 1 - 3(-2x+1) - (6x-3) = \\ & = 1 + \cancel{6x} - \cancel{3} - \underline{6x} + \underline{3} = \\ & = 1 \end{aligned}$$

$$\textcircled{6} \textcircled{B} -\overset{\textcircled{4}}{\frac{3}{4}} + \overset{\textcircled{2}}{\frac{1}{2}} = \frac{-3+2}{4} = -\frac{1}{4}$$

$$\textcircled{8} \overset{\textcircled{3}}{\frac{2}{1}} - \overset{\textcircled{4}}{\frac{7}{3}} = \frac{6-7}{3} = -\frac{1}{3}$$

$$\textcircled{20} -\overset{\textcircled{4}}{\frac{7}{5}} + \overset{\textcircled{5}}{\frac{1}{1}} = \frac{-7+5}{5} = -\frac{2}{5}$$

$$\textcircled{7} \textcircled{B} \frac{3}{5} : \left(-\frac{2}{7}\right) = \frac{3}{5} \cdot \left(-\frac{7}{2}\right) = -\frac{21}{10}$$

$$\textcircled{8} \frac{3}{2} : (-2) = \frac{3}{2} \cdot \left(-\frac{1}{2}\right) = -\frac{3}{4}$$

$$\begin{aligned} \textcircled{20} \frac{2}{3} - 5 \cdot \frac{1}{2} - 1 &= \overset{\textcircled{12}}{\frac{2}{3}} - \overset{\textcircled{3}}{\frac{5}{2}} - \overset{\textcircled{6}}{\frac{1}{1}} = \frac{4-15-6}{6} \\ &= \frac{-17}{6} = -\frac{17}{6} \end{aligned}$$

12

$$\alpha + \beta = -3$$

$$\textcircled{B} \quad A = \frac{\overset{6}{\alpha}}{\underset{1}{6}} + \frac{\overset{1}{\beta}}{6} - \frac{\overset{3}{\alpha}}{2} + \frac{\overset{2}{\beta-1}}{3} =$$

$$= \frac{6\alpha}{6} + \frac{\beta}{6} - \frac{3\alpha}{6} + \frac{2(\beta-1)}{6} =$$

$$= \frac{6\alpha + \beta - 3\alpha + 2(\beta-1)}{6} =$$

$$= \frac{6\alpha + \beta - 3\alpha + 2\beta - 2}{6} =$$

$$= \frac{(3\alpha + 3\beta) - 2}{6} =$$

$$= \frac{3(\alpha + \beta) - 2}{6} =$$

$$= \frac{3 \cdot (-3) - 2}{6} =$$

$$= \frac{-9 - 2}{6} =$$

$$= \left(-\frac{11}{6} \right)$$

10

$$k = \frac{1}{x} + \frac{x}{x-2}$$

α) Για να έχει νόημα ραδιόφωνο η παρασταση πρέπει $x \neq 0$ και $x \neq 2$.

$$\beta), \quad \frac{1}{x} + \frac{x}{x-2}$$

$$k = \frac{x-2}{x(x-2)} + \frac{x^2}{x(x-2)} = \frac{x^2+x-2}{x(x-2)}$$

γ) Για $x = \frac{3}{2}$.

$$k = \frac{1}{\frac{3}{2}} + \frac{\frac{3}{2}}{\frac{3}{2} - \frac{1}{2}} = \frac{1}{\frac{3}{2}} + \frac{\frac{3}{2}}{-\frac{1}{2}} = \frac{2}{3} + \frac{9}{-4} = \frac{2}{3} - \frac{9}{4} = \frac{8}{12} - \frac{27}{12} = -\frac{19}{12}$$
$$= -\frac{19}{12}$$

Εποραιο Μαθημα

Τριτη 5:30-6:30,

Σελ. 18-19

- ①
- ②
- ③
- ④
- ⑤
- ⑥
- ⑦
- ⑧
- ⑪

7

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}.$$

$$A = \{1, 2, 3, 4, 5\}.$$

$$B = \{3, 4, 5, 6, 7\}.$$

(a) $(A \cup B)' = \{8\}.$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}.$$

$$A' = \{6, 7, 8\}.$$

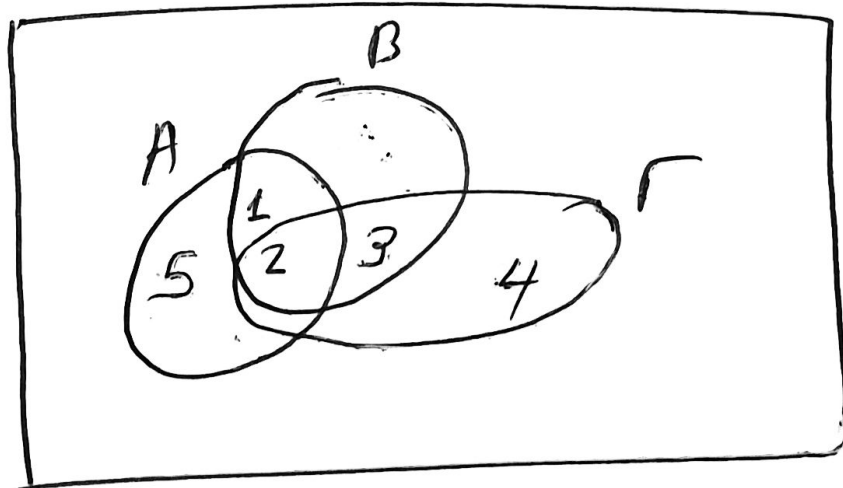
$$A' \cap B' = \{8\}.$$

$$B' = \{1, 2, 8\}$$

Αρα

$$(A \cup B)' = A' \cap B'$$

8



Άσκηση

$$a^3 + b^3 + \gamma^3 = 3ab\gamma$$

$$a + b + \gamma = 0$$

Να λύσει η εξίσωση.

$$(2x-1)^3 + (5x-8)^3 + (9-7x)^3 = 0$$

$$\cancel{2x-1} + \cancel{5x-8} + 9 - \cancel{7x} =$$

$$= 0$$

$$3(2x-1)(5x-8)(9-7x) = 0$$

$$2x-1=0$$

$$x = \frac{1}{2}$$

$$\text{ή } 5x-8=0$$

$$5x=8$$

$$x = \frac{8}{5}$$

$$\text{ή } 9-7x=0$$

$$x = \frac{9}{7}$$

1

Σελ 52

Σχολια

$$A = \left[(x^2 y^3)^{-2} (x y^3)^4 \right] : \left(\frac{x^3}{y^{-1}} \right)^{-3}$$

(b) vdo $A = x^9 y^9$

$$A = \left(\underbrace{x^{-4} y^{-6} x^4 y^{12}} \right) : \left(\frac{x^{-9}}{y^3} \right)$$

$$A = \left(x^{-4+4} y^{-6+12} \right) : \left(\frac{x^{-9}}{y^3} \right)$$

$$A = \left(x^0 y^6 \right) \cdot \frac{y^3}{x^{-9}}$$

$$A = y^6 \cdot \frac{y^3}{x^{-9}}$$

$$A = \frac{y^6 y^3}{x^{-9}} = \frac{y^9}{x^{-9}} = y^9 x^9$$

$$A = x^9 y^9$$

Επορω Μαδύρι

Περνιτσ 5:30 - 7.

Σελ. 27-28-29.

(4) αγ

(5) αγ

(8) αγ.

(11) α

(12) α

(13) οι

(18)

Σχολικό

Σελ 52

$$1. A = \left[(x^2 y^3)^{-2} (x y^3)^4 \right] : \left(\frac{x^3}{y^{-1}} \right)^{-3}$$

i) Νδσ $A = x^9 y^9$

$$A = \left[\underbrace{(x^{-4} y^{-6} x^4 y^{12})} \right] : \left(\frac{y^{-1}}{x^3} \right)^3$$

$$A = \left(x^{-4+4} y^{-6+12} \right) : \frac{y^{-3}}{x^9}$$

$$A = x^0 y^6 \cdot \frac{x^9}{y^{-3}}$$

$$A = 1 \cdot y^6 \cdot \frac{x^9}{y^{-3}} = \frac{y^6 x^9}{y^{-3}} = y^{6-(-3)} x^9$$

$$A = y^9 x^9$$

$$11). \text{ Av } x = 2010 \quad \text{kon} \quad y = \frac{1}{2010}$$

$$A = x^9 y^9$$

$$A = (xy)^9$$

$$A = \left(\cancel{2010} \cdot \frac{1}{\cancel{2010}} \right)^9$$

$$A = 1^9 = \underline{\underline{1}}$$

$$2. \quad A = \left[(xy^{-1})^2 : (x^3y^7)^{-1} \right]^2 \quad \begin{array}{l} x = 0,4 \\ y = -2,5 \end{array}$$

$$A = \left(x^2 y^{-2} : x^{-3} y^{-7} \right)^2$$

$$A = \left[x^{2-(-3)} y^{-2-(-7)} \right]^2$$

$$A = \left(x^5 y^5 \right)^2 = x^{10} y^{10} = (xy)^{10}$$

$$A = \left[(0,4)(-2,5) \right]^{10} = 1^{10} = 1$$

$$3. \quad i) \quad 1001^2 - 999^2 = (1001-999)(1001+999)$$

$$2 \cdot 2000 = 4000$$

$$\boxed{a^2 - b^2 = (a-b)(a+b)}$$

$$ii). \quad 99 \cdot 101 = (100-1)(100+1) = 100^2 - 1^2 = 10000 - 1 = 9999$$

$$iii). \quad \frac{7,23^2 - (4,23)^2}{11,46} = \frac{(7,23-4,23)(7,23+4,23)}{11,46}$$

$$= \frac{3 \cdot 11,46}{11,46} = 3$$

$$4. \text{ i) Nds } (a+B)^2 - (a-B)^2 = 4aB$$

$$a^2 + 2aB + B^2 - (a^2 - 2aB + B^2) = 4aB$$

$$\cancel{a^2} + 2aB + \cancel{B^2} - \cancel{a^2} + 2aB - \cancel{B^2} = 4aB$$

$$(a+B)^2 = a^2 + 2aB + B^2$$

$$(a-B)^2 = a^2 - 2aB + B^2$$

$$4aB = 4aB$$

$$0 = 0 \quad \checkmark$$

$$\text{ii). } \left(\frac{999}{1000} + \frac{1000}{999} \right)^2 - \left(\frac{999}{1000} - \frac{1000}{999} \right)^2 =$$

$$\text{ca. } a = \frac{999}{1000}$$

$$\text{ca. } B = \frac{1000}{999}$$

$$4 \cdot \frac{\cancel{999}}{1000} \frac{\cancel{1000}}{\cancel{999}} = 4$$

5. i) vdo $a^2 - (a-1)(a+1) = 1$

$$a^2 - (a^2 - 1^2) = 1$$

$$\cancel{a^2} - \cancel{a^2} + 1 = 1$$

$$1 = 1 \quad \checkmark$$

ii). $1,3265^2 - 0,3265 \cdot 2,3265$

$\sqrt{a} \cdot a = 1,3265$

$$1,3265^2 - (1,3265 - 1)(1,3265 + 1) = 1$$

$$1,3265^2 - 0,3265 \cdot 2,3265 = 1$$

6. 1ος x

2ος x+1

$$\text{Νδσ } (x+1)^2 - x^2 = x + x + 1$$

$$\cancel{x^2} + 2x + 1 - \cancel{x^2} = 2x + 1$$

$$2x + 1 = 2x + 1$$

$$0 = 0$$

7. $v \in \mathbb{N}$ νδσ $2^v + 2^{v+1} + 2^{v+2}$

οτι αλληλα ας 7.

Πολλη ας 7: $7 \mid k, k \in \mathbb{Z}$.

$$2^v + 2^{v+1} + 2^{v+2} = 2^v + 2^v \cdot 2^1 + 2^v \cdot 2^2 =$$

$$= 2^v + 2 \cdot 2^v + 4 \cdot 2^v =$$

$$= 7 \cdot 2^v \text{ αλληλα ας 7.}$$

Σχολια (Σελ 53)

1. i) $\frac{a^3 - 2a^2 + a}{a^2 - a} = \frac{\cancel{a}(a^2 - 2a + 1)}{\cancel{a}(a-1)} = \frac{(a-1)}{\cancel{a-1}}$

$= a - 1$

ii) $\frac{(a^2 - a) + 2a - 2}{a^2 - 1}$

$= \frac{a(a-1) + 2(a-1)}{(a-1)(a+1)} = \frac{\cancel{(a-1)}(a+2)}{\cancel{(a-1)}(a+1)} = \frac{a+2}{a-1}$

2. i) $\left(a - \frac{1}{a}\right)^2 \cdot \frac{a^3 + a^2}{(a+1)^3} = \left(\frac{a^2 - 1}{a}\right)^2 \cdot \frac{\cancel{a^2}(a+1)}{(a+1)^{\cancel{3}}}$

$= \left(\frac{a^2 - 1}{a}\right)^2 \cdot \frac{a^2}{(a+1)^2} =$

$= \left(\frac{(a-1)(a+1)}{a}\right)^2 \cdot \frac{a^2}{(a+1)^2} = \frac{(a-1)^2 \cancel{(a+1)^2}}{\cancel{a^2}} \cdot \frac{\cancel{a^2}}{\cancel{(a+1)^2}}$

$= (a-1)^2$

$$11). \frac{a^2+a+1}{a+1} \cdot \frac{a^2-1}{a^3-1} =$$

$$= \frac{\cancel{a^2+a+1}}{\cancel{a+1}} \cdot \frac{\cancel{(a-1)}(a+1)}{\cancel{(a-1)}(a^2+a+1)} = \underline{1}$$

$$\boxed{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

$$3. \quad i) (x+y)^2 \cdot (x^{-1} + y^{-1})^{-2} =$$

$$= (x+y)^2 \cdot \left(\frac{1}{x} + \frac{1}{y}\right)^{-2} =$$

$$= (x+y)^2 \cdot \left(\frac{y}{xy} + \frac{x}{xy}\right)^{-2} =$$

$$= (x+y)^2 \cdot \left(\frac{x+y}{xy}\right)^{-2} = (x+y)^2 \cdot \left(\frac{xy}{x+y}\right)^2 =$$

$$= \frac{\cancel{(x+y)^2} \cdot (xy)^2}{\cancel{(x+y)^2}} = x^2 y^2$$

$$11). \frac{x+y}{x-y} \cdot \frac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}} = \frac{x+y}{x-y} \cdot \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} =$$

$$= \frac{x+y}{x-y} \cdot \frac{\frac{y}{xy} - \frac{x}{xy}}{\frac{y^2}{x^2y^2} - \frac{x^2}{x^2y^2}} = \frac{x+y}{x-y} \cdot \frac{\frac{y-x}{xy}}{\frac{y^2-x^2}{x^2y^2}}$$

$$= \frac{x+y}{x-y} \cdot \frac{(y-x)x^2y^2}{xy(y^2-x^2)} =$$

$$= \frac{\cancel{x+y}}{\cancel{x-y}} \cdot \frac{-\cancel{(x-y)}(xy)}{\cancel{xy}(y-x)\cancel{(y+x)}} = \frac{-xy}{y-x}$$

$$4. \text{ Ndo } \left(\frac{x^3+y^3}{x^2-y^2} \right) : \left(\frac{x^2}{x-y} - y \right) = 1.$$

$$\frac{\cancel{(x+y)}(x^2-xy+y^2)}{\cancel{(x-y)}\cancel{(x+y)}} : \left(\frac{x^2}{x-y} - \frac{y(x-y)}{x-y} \right) = 1$$

$$\frac{x^2-xy+y^2}{x-y} : \frac{x^2-yx+y^2}{x-y} = 1 (=)$$

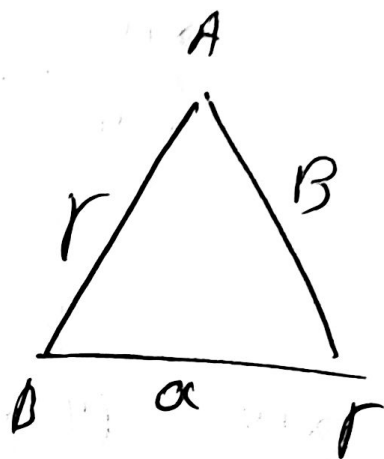
$$\frac{x^2 - xy + y^2}{x - y} \cdot \frac{x - y}{x^2 - xy + y^2} = 1$$

$$1 = 1$$

5. i) $\frac{\alpha}{\beta} = \frac{\beta}{\gamma} = \frac{\gamma}{\alpha}$

υδο $\triangle A\beta\gamma$ ισοσημο

Αρα υδο $\alpha = \beta = \gamma$.



Ισχυα οτι $\frac{\alpha}{\beta} = \frac{\beta}{\gamma} = \frac{\gamma}{\alpha} = k \Rightarrow \alpha = \beta k$

$$\beta = \gamma k$$

$$\gamma = \alpha k$$

$$\gamma = \beta \cdot k \cdot k$$

$$\gamma = \beta \cdot k^2$$

$$\gamma = \gamma k \cdot k^2$$

$$\cancel{\gamma} = \cancel{\gamma} k^3$$

$$\gamma \neq 0$$

$$1 = k^3$$

$$\underline{\underline{k = 1}}$$

Συνεπ

$$\alpha = \beta$$

$$\beta = \gamma$$

$$\alpha = \gamma$$

ισοσημο.

$$\text{ii) Av } \alpha - \beta = \beta - \gamma = \gamma - \alpha = k$$

$$\alpha = \beta + k$$

$$\beta = \gamma + k$$

$$\gamma = \alpha + k$$

Apa

$$\alpha = \beta + k$$

$$\alpha = \gamma + k + k$$

$$\alpha = \gamma + 2k$$

$$0 = \alpha + k + 2k$$

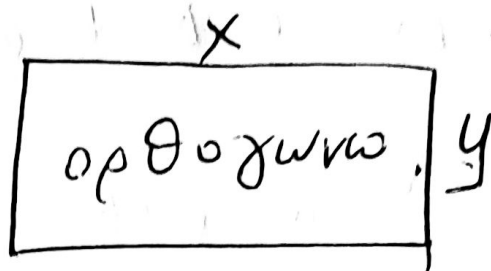
$$0 = \alpha + 3k$$

$$0 = 3k$$

$$\underline{\underline{k=0}}$$

Apa $\alpha = \beta = \gamma$

6. $L = 4a$
 $E = a^2$



Νόμο το ορθογώνιο και τετράγωνο με πλευρά a .

$L = 2x + 2y = 4a \Leftrightarrow x + y = 2a$

↓
 περιφέρεια

οπότε $E = a^2$

$xy = a^2$

$\begin{cases} x + y = 2a & \Rightarrow y = 2a - x \\ xy = a^2 \end{cases}$

$x(2a - x) = a^2$

$2ax - x^2 = a^2$

$0 = x^2 - 2ax + a^2$

$0 = (x - a)^2$

$0 = x - a$

$x = a$

$a + y = 2a$

$y = a$



7

i) Αν α ρητός και β άρρητος
τότε $\alpha + \beta$ άρρητος.

Πρώτα είναι ασαφές αριθμός που μπορεί
να γραφεί ως κλάσμα.

αρα $\alpha = \frac{k}{\lambda}$

- οτω $\alpha + \beta$ είναι ρητός.

Αφού $\alpha + \beta$ είναι ρητός γραφεται
ως αναγόμενο κλάσμα.

$$\alpha + \beta = \frac{\mu}{\nu}$$

$$\frac{k}{\lambda} + \beta = \frac{\mu}{\nu}$$

$$\beta = \frac{\mu}{\nu} - \frac{k}{\lambda}$$

$$\beta = \frac{\mu\lambda}{\nu\lambda} - \frac{k\nu}{\lambda\nu} = \frac{\mu\lambda - k\nu}{\lambda\nu}$$

γιατι β άρρητος

$$\beta = \frac{\mu\lambda - k\nu}{\lambda\nu}$$

Αρα.

Αρα
 $\alpha + \beta$ άρρητος

ii) Αν οι ριζές και Β αλληλ
είναι α·Β αλληλ

Αφού οι ριζές γράφονται ως κλάσματα

$$\alpha = \frac{\kappa}{\lambda}$$

Εστω αΒ είναι ριζές

οπότε $\alpha\beta = \frac{\nu}{\mu}$

$$\frac{\kappa}{\lambda} \beta = \frac{\nu}{\mu}$$

$$\mu \kappa \beta = \lambda \nu$$

$$\beta = \frac{\lambda \nu}{\mu \kappa}$$

Το Β γράφεται
σαν κλάσμα
οπότε είναι
ριζή

Άρα
το αβ αλληλ

Απόδειξη

Δυναμω

1. $(-2)^2 = 2^2 = 4$
 $-2^2 = -4$
 $(-2)^3 = -2^3 = -8$
 $-2^3 = -8$

2. $a^0 = 1$
 $1^v = 1$
 $a^1 = a$

3. $a^{-v} = \frac{1}{a^v}$
 $\left(\frac{a}{b}\right)^{-v} = \left(\frac{b}{a}\right)^v$

4. $a^v \cdot a^k = a^{v+k}$
 $a^v : a^k = a^{v-k}$
 $\frac{a^v}{a^k} = a^{v-k}$

(Idid Basas)

5. $a^v \cdot b^v = (ab)^v$
 $a^v : b^v = \left(\frac{a}{b}\right)^v$
 $\frac{a^v}{b^v} = \left(\frac{a}{b}\right)^v$

6. $(a^v)^k = a^{v \cdot k}$

Σα 35

$$\textcircled{2} \textcircled{B} (-1)^5 - (-7)^2 - (-3)^3 =$$
$$= -1 - 49 + 9 = -50 + 9 = -41$$

$$\textcircled{8} (-4)^3 - 3^4 - (-2)^4 =$$
$$= -64 + 81 - 16 = +81 - 80 = 1$$

↗

$$\textcircled{82} 15 - 5 \cdot 3^2 - 17 \cdot (1 - 9^0) =$$
$$15 - 5 \cdot 9 =$$
$$15 - 45 = -30 \quad \checkmark$$

Σελ 35

$$\textcircled{5} \text{ B) } (2x)^3 \left(\frac{x}{2}\right)^2 = 8x^3 \cdot \frac{x^2}{4} = 2x^5$$

$$\delta) \frac{7^3 \cdot 7^4}{7^5} = \frac{7^7}{7^5} = 7^2 = 49$$

$$\sigma\tau) \left(\frac{3}{5}\right)^{-2} \frac{3^5}{5^2} = \left(\frac{5}{3}\right)^2 \cdot \frac{3^5}{5^2} = \frac{\cancel{5^2}}{3^2} \cdot \frac{3^5}{\cancel{5^2}} = 3^3 = \underline{\underline{27}}$$

$$\textcircled{6} \text{ B) } \frac{5^7 \cdot (-5)^4}{(-5)^9} = - \frac{5^7 \cdot 5^4}{5^9} = - \frac{5^{11}}{5^9} = -5^2$$

$$\textcircled{8} \frac{((-5)^{-3})^2}{(5^{-4})^2} = \frac{(5^{-3})^2}{5^{-8}} = \frac{5^{-6}}{5^{-8}} =$$

$$= 5^2.$$

Σε 2 36

$$\textcircled{7} \textcircled{B} x^2 \cdot (-2x) - 3x(-2x^2) = \cancel{\text{something}}$$
$$= -2x^3 + 6x^3 = 4x^3 \quad \checkmark$$

$$\textcircled{8} -\frac{3}{2}xy^3 \cdot \left(-\frac{4}{9}x^2y\right) =$$

$$= \frac{12}{18}x^3y^4 = \cancel{\frac{2}{3}(xy)^3}$$



Σε 2 37

(18)

$$\textcircled{B} 4^5 \cdot 32^3 = (2^2)^5 \cdot (2^5)^3 = 2^{10} \cdot 2^{15} = 2^{25} \quad \checkmark$$

$$\textcircled{8} \frac{5^{17}}{625} = \frac{5^{17}}{25^2} = \frac{5^{17}}{(5^2)^2} = \frac{5^{17}}{5^4} = 5^{13} \quad \checkmark$$

$$\textcircled{82} \frac{81^5}{27^3} = \frac{(3^4)^5}{(3^3)^3} = \frac{(3^2)^{10}}{3^9} = \frac{3^{20}}{3^9} = 3^{11} \quad \checkmark$$

$$\textcircled{19} \quad 8) \frac{12^{10}}{16^5 \cdot 81^3} = \frac{(3 \cdot 4)^{10}}{(4^2)^5 \cdot (3^4)^3} = \frac{3^{10} \cdot \cancel{4^{10}}}{\cancel{4^{10}} \cdot 3^{12}} =$$

$$= 3^{-2}$$



Σ 2 36

8) B) $(5x^3)^2 - (-2x^2)^3 - 4\left(\frac{x^3}{2}\right)^2 =$

$= 25x^6 - (-2^3x^6) - 4\left(\frac{x^6}{4}\right) =$

$= 5x^6 + 8x^6 - x^6$

~~$= \frac{7x^6 - \frac{4x^6}{16}}{16} = \frac{7x^6 - \frac{x^6}{4}}{4} = \frac{28x^6 - x^6}{4} = \frac{27x^6}{4}$~~

$6x^6$

10) B) $x(2x^3 - 1) = 2x^4 - x$ ✓

8) $5x^2 - x(3x - 2) = 5x^2 - 3x^2 + 2x = 2x^2 + 2x$

52) $x^3 - 3x(x^2 - 3x - 1) = x^3 - 3x^3 + 9x^2 + 3x =$
 $= -2x^3 + 9x^2 + 3x$ ✓

Σελ 37

$x = 2019$

$y = -\frac{1}{2019}$

(15) $A = \frac{(xy^{-2})^3 (x^2y)^{-1}}{(y^{-1})^7 : (-y)}$

~~$A = \frac{x^3 y^{-6} x^{-2} y^{-1}}{y^{-7} : (-y)} \Rightarrow A = \frac{xy^{-7}}{\frac{1}{y^7} \cdot (-\frac{1}{y})} \Rightarrow A = \frac{1 \cdot 1}{y^7 \cdot (-\frac{1}{y})} \Rightarrow$~~

~~$A = \frac{1}{x} \cdot \frac{1}{y^7} \Rightarrow A = \frac{y^7}{x} \cdot \left(\frac{-y}{y^7}\right) \Rightarrow A = \frac{-y}{x} \Rightarrow A = \frac{-(-\frac{1}{2019})}{2019} \Rightarrow$~~

~~$A = \frac{1}{2019} \Rightarrow A = \frac{1}{2019}$~~

$A = \frac{x^3 y^{-6} x^{-2} y^{-1}}{y^{-7} : y^1} = \frac{x^1 y^{-7}}{y^{-8}}$

$= -x^1 y = -xy$

$A = xy$

$A = 2019 \cdot \left(-\frac{1}{2019}\right) = \underline{\underline{-1}}$

Σε2 36

13

$$A = 1 - 3x(2x-1) - (x-1)(2x^2-3) =$$

01

vdo

$$A = -2x^3 - 4x^2 + 6x - 2 =$$

$$= -\cancel{2(x+2)} + \cancel{2(x-3)}$$

$$= -\cancel{2(x^3 + 2x^2 + 3x + 4)}$$

$$= 1 - 6x^2 + 3x - (\cancel{2x^3} - 3x - 2x^2 + 3) =$$

$$= 1 - 6x^2 + 3x - 2x^3 + 3x + 2x^2 - 3 =$$

$$= -2x^3 - 4x^2 + 6x - 2$$

13) Για $x = \left(-\frac{1}{2}\right)^{-1}$ Bpl τω A.

$$x = -\left(\frac{1}{2}\right)^{-1} = -\left(\frac{2}{1}\right)^1 = -2$$

$$x = -2$$

$$-2(-2)^3 - 4(-2)^2 + 6(-2) - 2 =$$

$$= -2(-8) - 4 \cdot 4 + (-12) - 2 =$$

$$= \cancel{16} - \cancel{16} - 12 - 2 =$$

$$= \underline{\underline{-14}}$$

Σε 2. 36

11

$$\textcircled{B} (2x-1)(3x-2) + 5x = 6x^2 - 4x - 3x + 2 + 5x =$$
$$6x^2 - 2x + 2 = \cancel{2(3x^2 - 2x)}$$

✓

$$\textcircled{δ} 3x^3 - x(3x-1)(x-2) = 3x^3 - x(3x^2 - 6x - x + 2) =$$
$$\cancel{3x^3} - \cancel{3x^3} + 6x^2 + \textcircled{0}x^2 - 2x = 7x^2 - 2x$$

2

1. κωλυ/ παραγοντα

$$2. a^2 - b^2 = (a - b)(a + b)$$

$$3. a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

3

1. κωλυ/ παραγοντα

$$2. (a \pm b)^2 = a^2 \pm 2ab + b^2$$

3. παραγοντοποιηση τριωνυμ.

$$\hookrightarrow ax^2 + bx + c = a(x - x_1)(x - x_2)$$

4) 1. κωλυ/ παραγοντα.

2. ορατοποιηση.

$$3. a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$$

4. χωρι/ω σε 3 και 1.

$$n \cdot x \quad x^2 - 2x + 1 - y^2$$

$$(x - 1)^2 - y^2 = (x - 1 - y)(x - 1 + y)$$

4

$$a=2$$

$$B=-3$$

$$x=-2$$

$$B = 3a^2 - 2Bx^4 - B^4$$

$$B = 3(2)^2 - 2(-3)(-2)^4 - (-3)^4$$

$$B = 3 \cdot 4 - (-6) \cdot 16 - 81$$

$$B = 12 - (-6) \cdot 16 - 81$$

$$B = 12 + 96 - 81$$

$$B = 18 - 81$$

$$B = -63$$

$$\text{91235 1022}$$

$$\Delta = B^2 - 4ax$$

$$\Delta = (-3)^2 - 4(2)(-2)$$

$$\Delta = 9 - 8 \cdot (-1)$$

$$\Delta = 9 + 8$$

$$\Delta = 17$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{3 \pm \sqrt{17}}{4}$$

Σ 2 46

$$\textcircled{9} \textcircled{B} x^3 - 3x(2x-1) - (x-1)^3 =$$

$$= x^3 - 6x^2 + 3x - (x^3 + 3x^2 - 3x + 1) =$$

$$= \cancel{x^3} - 6x^2 + 3x - \cancel{x^3} - 3x^2 + 3x - 1 =$$

$$= -9x^2 + 6x - 1 = -\cancel{(9x^2 - 6x + 1)}$$

$$= -\cancel{(3x-1)^2}$$

$$\textcircled{8} x^2 - 2x(1-x) - (2x-1)^2 = x^2 - 2x + 2x^2 - 4x^2 + 4x - 1 =$$

$$= -x^2 + 2x - 1 = -(x^2 - 2x + 1) =$$

$$= -(x-1)^2$$



$$\textcircled{52} 8x(x-1)(x+1) - (2x-1)^3 =$$

$$= 8x(x^2 - 1) - (8x^3 - 12x^2 + 6x - 1) =$$

$$= \cancel{8x^3} - 8x - \cancel{8x^3} + 12x^2 - 6x + 1 =$$

$$= -\cancel{24x^2} - 2x - 1$$

KT ↴

12 Σ 47

$$(B) (a-b)^2 + 4ab = (a+b)^2$$

$$a^2 - 2ab + b^2 + 4ab = a^2 + 2ab + b^2$$

$$\cancel{a^2} - \cancel{2ab} + \cancel{b^2} - 4ab - \cancel{a^2} - \cancel{2ab} - \cancel{b^2} = 0$$

$$0 = 0$$

$$(D) (a+b)^3 - (a-b)^3 - 6a^2b = 2b^3$$

$$(a^3 + 3a^2b + 3ab^2 + b^3) - (a^3 - 3a^2b + 2ab^2 - b^3)$$

$$- 6a^2b = 2b^3$$

$$\cancel{a^3} + 2a^2b + \cancel{2ab^2} + \cancel{b^3} - \cancel{a^3} + 2a^2b - \cancel{2ab^2} + \cancel{b^3}$$

$$- 6a^2b - 2b^3 = 0$$

Ενορχηστρωμένο Μαθημα

Τρίτη 24/9

5:30-6:30

Σελ 35-36-37

(2) α γ ε

(16)

(4) α γ

(8) α γ ε

(5) α γ ε

(19) α β.

(6) α γ

(7) α γ

(10) α γ ε

(11) α γ

12 α β.

(18)

$$(a) 2^7 \cdot 8^5 = 2^7 \cdot (2^3)^5 = 2^7 \cdot 2^{15} = 2^{22}$$

$$(r) 9^5 \cdot 27^3 = (3^2)^5 \cdot (3^3)^3 = 3^{10} \cdot 3^9 = 3^{19}$$

$$(z) \frac{2^{53}}{64} = \frac{2^{53}}{2^6} = 2^{47}$$

(19)

$$(a) \frac{8^5 \cdot 9^6}{6^{13}} = \frac{(2^3)^5 \cdot (3^2)^6}{(2 \cdot 3)^{13}}$$

$$= \frac{2^{15} \cdot 3^{12}}{2^{13} \cdot 3^{13}} = 2^2 \cdot 3^{-1} = 4 \cdot \frac{1}{3} = \frac{4}{3}$$

$$(B) \frac{4^5 \cdot 49^2}{14^5} = \frac{(2^2)^5 \cdot (7^2)^2}{(2 \cdot 7)^5} =$$

$$= \frac{2^{10} \cdot 7^4}{2^5 \cdot 7^5} = 2^5 \cdot 7^{-1} = \frac{32}{7}$$

$$\textcircled{2} \quad \textcircled{g} \quad \left(x + \frac{1}{x}\right)^2 = x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} =$$

$$= x^2 + \frac{2x}{x} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2}$$

$$\textcircled{3} \quad \textcircled{g} \quad (2x-1)^3 = 8x^3 - \cancel{6x^2} + \cancel{6x^2} - (-1)^3 =$$

$$= 8x^3 + 1$$

$$(2x)^3 - 3 \cdot (2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 - 1^3$$

$$8x^3 - 3 \cdot 4x^2 + 6x - 1$$

$$8x^3 - 12x^2 + 6x - 1$$

$$(a-b)^3 = a^3 - 3 \cdot a^2 \cdot b + 3a \cdot b^2 - b^3$$

$$1. \quad (B) \quad (x+7)^2 = x^2 + 2x \cdot 7 + 7^2 = x^2 + 14x + 49 =$$

$$(D) \quad (y-1)^2 = y^2 - 2y \cdot 1 + 1^2 = y^2 - 2y + 1$$

$$(E) \quad (3y-2)^2 = (3y)^2 - 2 \cdot 3y \cdot 2 + 2^2 = 9y^2 - 12y + 4 =$$

$$(G) \quad (x^2+5)^2 = (x^2)^2 + 2x^2 \cdot 5 + 5^2 = x^4 + 10x^2 + 25$$

$$(5) \quad (B) \quad (x+3)(x-3) = x^2 - 3^2 = x^2 - 9$$

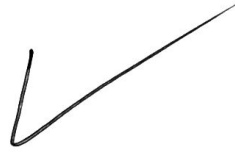
$$(E) \quad (3x^2-5)(3x^2+5) = (3x^2)^2 - 5^2 = 9x^4 - 25 =$$

$$(G) \quad \left(y^2 - \frac{2}{3}\right)\left(y^2 + \frac{2}{3}\right) = (y^2)^2 - \left(\frac{2}{3}\right)^2 = y^4 - \frac{4}{9} =$$

$$\textcircled{22} \textcircled{B} \quad x^3 + (2x-y)^3 + (y-3x)^3 = 3x(2x-y)(y-3x)$$

$$\rightarrow x + 2x - y + y - 3x = 0$$

$$\text{Ans} \quad x^3 + (2x-y)^3 + (y-3x)^3 = 3 \cdot x(2x-y)(y-3x)$$



$$x^3 + y^3 + z^3 = 3xyz$$

$$\text{or} \quad x + y + z = 0$$

$$(13) \quad (d) \quad (x^2-1)(a^2-9) + (3x-a)^2 = (ax-3)^2$$

~~$$(x^2 a^2 - 4x^2 - a^2 + 9) + 9x^2 - a^2 = (ax-3)^2$$~~

~~$$x^2 a^2 - 4x^2 - a^2 + 9 + 9x^2 - a^2 = (ax-3)^2$$~~

~~$$x^2 a^2 - 2a^2 + 9 = (ax-3)^2$$~~

~~$$a^2 x^2 - 2a^2 + 9 = (ax-3)^2$$~~

~~$$a^2 x^2 - 2a^2 + 9 = (ax)^2 + 2ax - 3^2$$~~

~~$$a^2 x^2 - 2a^2 + 9 = a^2 x^2 + 2ax - 9$$~~

$$(x^2 a^2 - 9x^2 - a^2 + 9) + (3x)^2 - 23xa + a^2 = (ax)^2 - 2ax \cdot 3 + 3^2$$

$$x^2 a^2 - 9x^2 - a^2 + 9 + 9x^2 - 6xa + a^2 = a^2 x^2 - 6ax + 9$$

$$a^2 x^2 - 6ax + 9 = a^2 x^2 - 6ax + 9$$

10

$$\begin{aligned} \textcircled{b} \quad & 1 - 2x(-x+1) - (3x-1)^2 = \\ & = 1 + 2x^2 - 2x - (9x^2 - 6x + 1) = \\ & = \cancel{1} + 2x^2 - 2x - 9x^2 + 6x - \cancel{1} = \\ & = \underline{\underline{-7x^2 + 4x}}. \end{aligned}$$

$$\begin{aligned} \textcircled{d} \quad & 1 - 8x^2(x-2) - (2x-1)^3 = \\ & = 1 - 8x^3 + 16x^2 - (8x^3 - 12x^2 + 6x - 1) = \\ & = \cancel{1} - 8x^3 + 16x^2 - 8x^3 + 12x^2 - 6x + 1 = \\ & = \underline{\underline{-16x^3 + 28x^2 - 6x + 2}} \end{aligned}$$

$$\begin{aligned} \textcircled{e} \quad & x^5 - (-x-2)(x-2) - x(-x^2+1)^2 = \\ & = x^5 - (\cancel{-x^2-2x} - x^2 + 2x - 2x + 4) - x[(-x^2)^2 + 2 \cdot (-x^2) + 1] = \\ & = x^5 + x^2 - \cancel{2x} + \cancel{2x} - 4 - x(x^4 - 2x^2 + 1) = \\ & = \cancel{x^5} + x^2 - 4 - \cancel{x^5} + 2x^2 - x = \\ & = \underline{\underline{3x^2 - x - 4}} \end{aligned}$$

$$(13) \quad (b) \quad (a^2-3)^2 - a(a-1)(a-5) = (a-1)(a^3-5) + 4$$

$$\rightarrow a^4 - 6a^2 + 9 - a(a^2 - 5a - a + 5) =$$

$$= \underline{a^4} - \cancel{6a^2} + 9 - \underline{a^3} + \cancel{5a^2} + \cancel{a^2} - 5a =$$

$$= a^4 - a^3 - 5a + 9 = (a-1)(a^3-5) + 4$$

$$(15) \quad (a) \quad \text{Nfo} \quad a(a-2) - (a-1)^2 = -1$$

$$\rightarrow \cancel{a^2} - \cancel{2a} - \cancel{a^2} + \cancel{2a} - 1 = -1$$

$$(B) \quad 2022 \cdot 2020 - 2021^2 =$$

$$\cancel{2020} \cancel{(2022)} = 2020(2022 - 2) - (2021 - 1)^2 =$$

$$= -4$$

16

(a) $(x-1)(x+1)(x^2+1)(x^4+1) = x^8 - 1$.

$\rightarrow 9.11.101.10.000$

$\rightarrow (x^2 - 1) \text{ [scribble]} (x^6 + x^2 + x^4 + 1) =$

$= x^8 + x^4 + x^6 + x^2 - x^6 - x^2 - x^4 - 1 =$

$= x^8 - 1$

$9.11.101.10.000 =$

$= (10-1)(10+1)(10^2+1)(10^4+1) =$

$= 10^8 - 1 = 100,000,000 - 1 = 99,999,999$

13

$$(a) \quad (2x-1)^2 - 3x(x-1) + 3x = (x+1)^2$$

$$4x^2 - 4x + 1 - 3x^2 + 3x + 3x = (x+1)^2$$

$$x^2 + 2x + 1 = x^2 + 2x + 1$$

$$(b) \quad (a^2+1)(x^2+4) = (ax+2)^2 + (x-2a)^2$$

$$ax^2 + 4a^2 + x^2 + 4 = ax^2 + \cancel{4ax} + 4 + x^2 - \cancel{4ax} + 4a^2$$

$$ax^2 + 4a^2 + x^2 + 4 = ax^2 + 4a^2 + x^2 + 4$$

$$\textcircled{4} \quad \text{B/} \quad (x^2 + x + 1)^2 = x^4 + x^2 + 1 + 2x^3 + 2x^2 + 2x$$

$$\textcircled{8} \quad (2a - b - 3\gamma)^2 = (2a + (-b) + (-3\gamma))^2$$

$$= (2a)^2 + (-b)^2 + (-3\gamma)^2 + 2 \cdot 2a(-b) + 2 \cdot 2a(-3\gamma) + 2(-b)(-3\gamma)$$

$$= 4a^2 + b^2 + 9\gamma^2 - 4ab - 12a\gamma + 6b\gamma$$

$$(a + b + \gamma)^2 = a^2 + b^2 + \gamma^2 + 2ab + 2a\gamma + 2b\gamma$$

$$(-x-1)^2 = (x+1)^2$$

$$(-x+1)^2 = (1-x)^2$$

$$(-x-1)^3 = -(x+1)^3$$

$$(-x+1)^3 = (1-x)^3$$

Продолжи

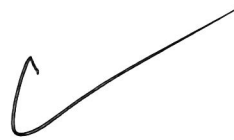
7

$$\begin{aligned} \textcircled{B} \quad (-3x-2)^2 &= \cancel{(-3x-2)} \cdot (3x+2)^2 = \\ &= (9x^2 + 12x + 4) = -9x^2 - 12x + 4 \end{aligned}$$

$$\textcircled{C} \quad (-4x+1)^3 = -(4x-1)^3 =$$

$$= -(64x^3 - 48x^2 + 12x - 1) =$$

$$= -64x^3 + 48x^2 - 12x + 1$$



(23)

$$\text{or } a + b + \gamma = 0$$

$$\text{w/o } \frac{a^2}{b\gamma} + \frac{b^2}{\gamma a} + \frac{\gamma^2}{a b} = 3$$

$$\frac{a^2(a \cdot b \cdot \gamma)}{b\gamma(a \cdot b \cdot \gamma)} + \frac{b^2(a \cdot b \cdot \gamma)}{\gamma a(a \cdot b \cdot \gamma)} + \frac{\gamma^2(a \cdot b \cdot \gamma)}{a b(a \cdot b \cdot \gamma)} = \frac{3(a \cdot b \cdot \gamma)}{(a \cdot b \cdot \gamma)} \Rightarrow$$

\Rightarrow

$$\cancel{a b \gamma} \frac{a^2}{\cancel{b \gamma}} + \frac{b^2}{a \gamma} a b \gamma + a b \gamma \frac{\gamma^2}{a b} = 3 a b \gamma$$

$$a^3 + b^3 + \gamma^3 = 3 a b \gamma$$

γ varilw or $a + b + \gamma = 0$

γ varu antwara,

$$\textcircled{11} \textcircled{a}. (7x-2)^2 - (2x-3)(x-5) =$$

$$\left[(7x)^2 - 2 \cdot 7x \cdot 2 + 2^2 \right] - (2x^2 - 10x - 3x + 15) =$$

$$(49x^2 - 28x + 4) - (2x^2 - 13x + 15) =$$

$$\underline{49}x^2 - \underline{28}x + 4 - \underline{2}x^2 + \underline{13}x - 15 = 47x^2 - 15x - 11$$

$$\textcircled{8} (3x-1)^2 - x(5+2x)(2x-5) =$$

$$(9x^2 - 2 \cdot 3x \cdot 1 + 1^2) - x(10x - 25 + 4x^2 - 10x) =$$

$$(9x^2 - 6x + 1) - x(-25 + 4x^2) = 9x^2 - \underline{6}x + 1 + \underline{25}x - 4x^3 =$$

~~9x^2 - 6x + 1 + 25x - 4x^3~~

$$9x^2 - 19x + 1 - 4x^3$$

$$\textcircled{E} x^3 - (x-1)^3 - 3x(x-2) =$$

$$x^3 - (x^3 - 3x^2 + 3x - 1) - 3x^2 + 6x =$$

$$\cancel{x^3} - \cancel{x^3} + 3x^2 - 3x^2 - 1 + 3x - 3x^2 + 6x =$$

$$3x - 6x^2 - 1 + 6x$$

$$\textcircled{52} \quad -10x^2 + 2(x+3)^3 - (2x-1)^2 =$$

$$-10x^2 + 2(x^3 + 3 \cdot 3^2x + 3 \cdot 3x^2 + 3^3) - [(2x)^2 - 2 \cdot 2x \cdot 1 + 1^2] =$$

$$-10x^2 + 2(x^3 + 27x + 9x^2 + 27) - (4x^2 - 4x + 1) =$$

$$-10x^2 + 2x^3 + 2 \cdot 27x + 2 \cdot 9x^2 + 2 \cdot 27 - 4x^2 + 4x - 1 =$$

$$\underline{-10x^2} + 2x^3 + \underline{54x} + \underline{18x^2} + \underline{54} - \underline{4x^2} + \underline{4x} - \underline{1} =$$

$$4x^2 + 2x^3 + 58x + 53$$

$$\textcircled{17} \textcircled{a} \left(a + \frac{1}{a}\right)^2 - \left(a - \frac{1}{a}\right)^2 = 4.$$

$$\left(a^2 + 2a\frac{1}{a} + \left(\frac{1}{a}\right)^2\right)^2 - \left(a^2 - 2a\frac{1}{a} + \left(\frac{1}{a}\right)^2\right)^2 = 4$$

$$\left(a^2 + 2 + \frac{1}{a^2}\right) - \left(a^2 - 2 + \frac{1}{a^2}\right) = 4$$

$$\cancel{a^2} + 2 + \cancel{\frac{1}{a^2}} - \cancel{a^2} + 2 - \cancel{\frac{1}{a^2}} = 4$$

$$4 = 4 \quad \checkmark$$

$$\textcircled{B} \left(\frac{999}{1000} + \frac{1000}{999}\right)^2 - \left(\frac{999}{1000} - \frac{1000}{999}\right)^2 =$$

~~$$= \left(\frac{999}{1000} + \frac{1000}{999}\right) + \left(\frac{999}{1000} - \frac{1000}{999}\right)$$~~

$$a = \frac{999}{1000}$$

$$\left(\frac{999}{1000} + \frac{1}{\frac{999}{1000}}\right)^2 - \left(\frac{999}{1000} - \frac{1}{\frac{999}{1000}}\right)^2 = 4$$

$$\left(\frac{999}{1000} + \frac{1000}{999}\right)^2 - \left(\frac{999}{1000} - \frac{1000}{999}\right)^2 = 4.$$

Επορω Μαθημα

Σελ 47

(9) α γ ε

(10) α γ ε

(12) α γ ε

(3) α, β, δ.

(4) α .

(7) α γ

(8) α γ

Οδηγός Παραγοντοποίησης

1. Όταν έχω δύο όρους τότε

i) κοινός παραγοντάς

ii) Διαφορά τετραγώνων $a^2 - b^2 = (a-b)(a+b)$

iii) Αθροισμα ή διαφορά κούβων

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

2. Όταν έχω τρεις όρους.

i) κοινός παραγοντάς.

ii). $a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$

iii) Παραγοντοποίηση τριωνύμου.

$$ax^2 + bx + c = a(x-x_1)(x-x_2)$$

3. Όταν έχω τέσσερις όρους τότε

i) κοινός παραγοντάς.

ii) ομαδοποίηση.

$$a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$$

iv) Σωδεική παραγοντοποίηση $(3+1)$

1. (B) $7x - 7y = 7(x - y)$

(D) $3x - 12 = 3(x - 4)$

(52) $9x - 6 = 3(3x - 2)$

(4) $10x - 2 = 2(5x - 1)$

2. (B) $x^2 - 2xy = x(x - 2y)$

(D) $2x^2 + 6x = 2x(x + 3)$

(52) $14x^3 - 21x^2 = 7x^2(2x - 3)$

3. (B) $x^2 - x = x(x - 1)$

(E) $6x^2 + 3x = 3x(2x + 1)$

(52) $6x^4 + 2x^2 = 2x^2(3x^2 + 1)$

8. (B) $x^2 - 4 = x^2 - 2^2 = (x - 2)(x + 2)$

(E) $4x^2 - 49 = (2x)^2 - 7^2 = (2x - 7)(2x + 7)$

(52) $49y^2 - 16 = (7y)^2 - 4^2 = (7y - 4)(7y + 4)$

$$9. \textcircled{d} x^4 - 81 = (x^2)^2 - 9^2 = (x^2 - 9)(x^2 + 9) \\ = (x-3)(x+3)(x^2 + 9)$$

$$\textcircled{e} (5x-1)^2 - 9 = (5x-1)^2 - 3^2 = (5x-1-3)(5x-1+3) \\ (5x-4)(5x+2)$$

$$10. \textcircled{g} x^4 - 16x^2 = x^2(x^2 - 16) = x^2(x-4)(x+4)$$

$$\textcircled{h} 3x^4 - 12x^2 = 3x^2(x^2 - 4) = 3x^2(x-2)(x+2)$$

$$12. \textcircled{b} \frac{11,36^2 - 7,52^2}{3,84} = \frac{(11,36 - 7,52)(11,36 + 7,52)}{3,84} \\ = \frac{3,84 \cdot 18,88}{3,84} = 18,88$$

$$13. \textcircled{b} x^3 - 1 = x^3 - 1^3 = (x-1)(x^2 + x + 1)$$

$$\textcircled{g} (2x-1)^3 - x^3 = (2x-1-x)((2x-1)^2 + (2x-1)x + x^2)$$

$$= (x-1)(4x^2 - 4x + 1 + 2x^2 - x + x^2)$$

$$= (x-1)(7x^2 - 5x + 1)$$

15. (B) $y^2 - 10y + 25 = (y - 5)^2$

(52) $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

16. (8) $25x^2 - 10x + 1 = (5x - 1)^2$

(52) $16x^4 - 8x^2 + 1 = (4x^2 - 1)^2 =$
 $= [(2x - 1)(2x + 1)]^2 =$
 $= (2x - 1)^2 (2x + 1)^2$

19. (a) $3x^2 + x - 2 = 3\left(x - \frac{2}{3}\right)(x + 1) = (3x - 2)(x + 1)$

$\Delta = B^2 - 4ac$
 $\Delta = 1^2 - 4 \cdot 3 \cdot (-2)$
 $\Delta = 1 + 24 = 25$

$x = \frac{-B \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 5}{6}$
 $\begin{cases} \frac{2}{3} \\ -1 \end{cases}$

(8) $x^2 - x - 2 = (x - 2)(x + 1)$

$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-2) = 9$

$x = \frac{1 \pm 3}{2}$
 $\begin{cases} 2 \\ -1 \end{cases}$

(52) $x^2 + x + 1$
 $\Delta < 0$ discriminant
 не раскладывается

$$24. \textcircled{B} (x+1)^2 + x^2 - 1 = 0 \Rightarrow$$

$$(x+1)^2 + (x-1)(x+1) = 0 \Rightarrow$$

$$(x+1)(x+1+x-1) = 0$$

$$(x+1)2x = 0$$

$$x+1=0 \quad \vee \quad 2x=0$$

$$x = -1$$

$$x = 0$$

$$\textcircled{8} x(x-2)^2 = x^2 - 4x + 4 \Rightarrow$$

$$x(x-2)^2 - (x^2 - 4x + 4) = 0 \Rightarrow x(x-2)^2 - x^2 - 4x + 4 = 0$$

$$x(x-2)^2 - (x-2)^2 = 0 \Rightarrow (x-2)^2(x-1) = 0$$

$$(x-2)^2 = 0$$

$$\vee \quad x-1 = 0$$

$$x = 2$$

$$x = 1$$

$$5. \textcircled{B} \quad (x-2)^2 + 3x - 6 = (x-2)^2 + 3(x-2) = \\ = (x-2)(x-2+3) = (x-2)(x+1)$$

$$\textcircled{D} \quad (x-3)^2 - 2x + 6 = \cancel{(x-3)^2} - 2(x-3) = \\ = (x-3)(x-3-2) = (x-3)(x-5)$$

$$\textcircled{E} \quad (3x-2)^2 - 3x + 2 = (3x-2)^2 - (3x-2) = \\ = (3x-2)(3x-2-1) = (3x-2)(3x-3)$$

$$\textcircled{H} \quad [-(2\lambda-1)]^2 + 4(1-2\lambda) = (2\lambda-1)^2 - 4(2\lambda-1) = \\ = (2\lambda-1)(2\lambda-1-4) = (2\lambda-1)(2\lambda-5)$$

$$20. \textcircled{C} \quad x^2 - x - 6 = \cancel{(x-2)(x+3)}$$

$$\Delta = b^2 - 4ac = 1 + 24 = 25$$

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm \sqrt{25}}{2} = \begin{cases} x_1 = \frac{-1+5}{2} = \frac{4}{2} = 2 \\ x_2 = \frac{-1-5}{2} = \frac{-6}{2} = -3 \end{cases}$$

$$25. \textcircled{B} \frac{3x^2 - x}{6x - 2} = \frac{x(3x-1)}{2(3x-1)} = \frac{x}{2}$$

$$\textcircled{D} \frac{3x - x^2}{x^2 - 9} = \frac{x(3-x)}{(x-3)(x+3)} \quad \text{[scribble]} \quad \frac{-x(x-3)}{(x-3)(x+3)}$$

$$= \frac{-x}{x+3}$$

$$\textcircled{E} \frac{1-9x^2}{6x^2-2x} = \frac{(1+3x)(1-3x)}{2x(3x-1)} = \frac{(1+3x)(1-3x)}{-2x(1-3x)}$$

$$29. \quad y = \frac{2x^2 - 5x + 3}{4x^2 - 9}$$

(α) Βρίττα τα x ώστε να ορίεται
η αντίστοιχη παράσταση.

• (α) πρέπει: $4x^2 - 9 \neq 0$

~~$(2x-3)(2x+3) \neq 0$~~

Άρα: $2x - 3 \neq 0$ και $2x + 3 \neq 0$
 $2x \neq 3$ και $2x \neq -3$
 $x \neq \frac{3}{2}$ και $x \neq -\frac{3}{2}$

$x \in \mathbb{R} \neq \frac{3}{2}, -\frac{3}{2}$

(β) Απλοποιούμε

$$\frac{2x^2 - 5x + 3}{4x^2 - 9} = \frac{2x^2 - 2x - 3x + 3}{(2x - 3)(2x + 3)} =$$

$$= \frac{2x(x - 1) - 3(x - 1)}{(2x - 3)(2x + 3)} = \frac{(x - 1)\cancel{(2x - 3)}}{\cancel{(2x - 3)}(2x + 3)} = \frac{x - 1}{2x + 3}$$

$$26. \textcircled{b} \frac{x^2-9}{x^2-6x+9} = \frac{\cancel{(x-3)}(x+3)}{(x-3)^2} = \frac{x+3}{x-3}$$

$$\textcircled{f} \frac{x^3+1}{x^2+x} = \frac{\cancel{(x+1)}(x^2-x+1)}{x\cancel{(x+1)}} = \frac{x^2-x+1}{x}$$

$$\textcircled{20} \frac{x^2-2x+1}{x^3-3x^2+3x-1} = \frac{(x-1)^2}{(x-1)^3} = \frac{1}{x-1}$$

$$32. \textcircled{a} \text{ Nds } \frac{a^3 + b^3}{a + b} - ab = (a - b)^2$$

$$\frac{(a+b)(a^2 - ab + b^2)}{a+b} - ab = (a-b)^2 \Rightarrow$$

$$a^2 - ab + b^2 - ab = (a-b)^2 \Rightarrow a^2 - 2ab + b^2 = (a-b)^2 =$$

$$(a-b)^2 = (a-b)^2$$

$$\textcircled{B} A = \frac{1002^3 + 8}{1004} - 2004 =$$

$$\frac{1002^3 + 2^3}{1004} - 2004 = \frac{(1002 + 2)(1002^2 - 1002 \cdot 2 + 2^2)}{1004} - 2004$$

$$= 1002^2 - 2004 + 2^2 - 2004 = 1002^2 - 2 \cdot 2004 + 2^2 =$$

$$= (1002 - 2)^2$$

$$18. \textcircled{\beta} x^3 - 6x^2 + 12x - 8 = \\ = (x - 2)^3$$

$$23. \textcircled{\gamma} x^2 - 1 = 0$$

$$x^2 - 1^2 = 0$$

$$(x-1)(x+1) = 0$$

$$\begin{array}{l} x-1=0 \quad \vee \quad x+1=0 \\ \underline{x=1} \quad \quad \quad \underline{x=-1} \end{array}$$

$$\textcircled{\delta} x^3 = x$$

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

$$x[(x-1)(x+1)] = 0$$

$$x(x-1)(x+1) = 0$$

$$\begin{array}{l} \vee \quad \underline{x=0} \\ \vee \quad \underline{x=1} \\ \vee \quad \underline{x=-1} \end{array}$$

$$\textcircled{\epsilon} x^3 - x^2 - x + 1 = 0$$

$$x^2(x-1) - (x-1) = 0$$

$$(x^2-1)(x-1) = 0$$

$$(x-1)(x+1)(x-1) = 0$$

$$(x-1)^2(x+1) = 0$$

$$\rightarrow \vee \quad (x-1)^2 = 0 \Rightarrow \underline{x=1} \text{ sind}$$

$$\vee \quad \underline{x=-1}$$

$$34. \quad \frac{\alpha + \beta}{\beta} = 4 \quad \text{and} \quad \frac{\gamma}{\delta - \gamma} = \frac{1}{4}$$

$$\textcircled{a} \text{ vdo } \alpha = 3\beta \quad \text{and} \quad \delta = 5\gamma$$

$$\frac{\alpha + \beta}{\beta} = \frac{4}{1} \Rightarrow 4\beta = \alpha + \beta \Rightarrow \boxed{\alpha = 3\beta}$$

$$\frac{\gamma}{\delta - \gamma} = \frac{1}{4} \Rightarrow 4\gamma = \delta - \gamma \Rightarrow \boxed{\delta = 5\gamma}$$

$$\textcircled{b} \quad \Pi = \frac{\alpha\gamma + \beta\gamma}{\beta\delta - \beta\gamma} \Rightarrow \Pi = \frac{3\beta\gamma + \beta\gamma}{5\beta\gamma - \beta\gamma} \Rightarrow \Pi = \frac{4\beta\gamma}{4\beta\gamma} \Rightarrow$$

$$\Rightarrow \Pi = 1$$

$$27. \textcircled{B} \frac{x^3 - 2x^2 + x}{x^2 - 1} = \frac{x(x^2 - 2x + 1)}{(x-1)(x+1)} =$$

$$\frac{x \cancel{(x-1)}^2}{\cancel{(x-1)}(x+1)} = \frac{x(x-1)}{x+1}$$

$$\textcircled{4} \frac{x-1}{x^2-x+1} \cdot \frac{x^3+1}{x^2-1} = \frac{x-1}{(x-1)^2} \cdot \frac{(x+1)(x^2-x+1)}{(x-1)(x+1)}$$

$$\frac{\cancel{(x-1)} \cancel{(x+1)} (x^2-x+1)^2}{\cancel{(x-1)}^2 \cancel{(x-1)}(x+1)} = \frac{x^2-x+1^2}{(x-1)(x+1)}$$

$$4. \quad (B) \quad x(2x-1) - 3(2x-1) =$$

$$= \underline{(x-3)(2x-1)}$$

$$(5) \quad (x-2)^2 + 3(2-x)(x+1) =$$

$$= (x-2)^2 - 3(x-2)(x+1) =$$

$$= (x-2) \sqrt{(x-2) - 3(x+1)} =$$

$$= \underline{(x-2) \sqrt{(x-2) - (3x+3)}} = (x-2)(x-2-3x+3) = \underline{(x-2)(1-2x)}$$

$$(20) \quad x(2x-1) - 2x + 1 =$$

$$= x(2x-1) - (2x-1) =$$

$$= \underline{(x-1)(2x-1)}$$

$$20. \quad (8) \quad x^2 - 8x + 15 =$$

$$\Delta = b^2 - 4ac = (-8)^2 - 4 \cdot 1 \cdot 15 = 64 - 60 = \underline{4}$$

$$\Delta > 0 \rightarrow x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow x_{1,2} = \frac{8 \pm \sqrt{4}}{2 \cdot 1} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{8 \pm 2}{2} \rightarrow x_1 = \frac{8+2}{2} = \frac{10}{2} = \underline{5}$$

$$x_2 = \frac{8-2}{2} = \frac{6}{2} = \underline{3}$$

$$= \underline{(x-5)(x-3)}$$

$$11. \textcircled{B} \quad 4x^3 - 8x^2 - x + 2 = 4x^3 - x - 8x^2 + 2 =$$

$$x(4x^2 - 1) - 2(4x^2 - 1) = (4x^2 - 1)(x - 2)$$

$$= (2x - 1)(2x + 1)(x - 2)$$

$$\textcircled{D} \quad x^3 - 2x^2 + (2x - 1)(2 - x) =$$

$$x^2(x - 2) + (2x - 1)(2 - x) = x^2(x - 2) + (2x - 1)(x - 2) =$$

$$= (x - 2) [x^2 + (2x - 1)] = (x - 2)(x^2 - 2x + 1)$$

$$= (x - 2)(x - 1)^2$$

$$17. \textcircled{B} \quad 2x^3 - 12x^2 + 18x = \\ 2x(x^2 - 6x + 9) = \\ 2x(x-3)^2$$

$$\textcircled{E} \quad x^2 - y^2 + 6y - 9 =$$

$$= x^2 - (y^2 - 6y + 9)$$

$$= x^2 - (y-3)^2 = (x-y+3)(x+y-3).$$

$$\textcircled{52} \quad (x^2+1)^2 - 4x^2 = \sqrt{(x^2+1)-2x} \sqrt{(x^2+1)+2x}$$

$$= (x-2)^2 (x+2)^2$$

$$\begin{aligned}
 6. \quad \textcircled{B} \quad & \underbrace{ax - ay} - \underbrace{Bx + By} = \\
 & = a(x-y) - B(x-y) = \\
 & = (x-y)(a-B)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{D} \quad & \underbrace{ax - ay - x + y} = \\
 & = x(a-1) - y(a+1) = \\
 & = (a-1)(x-y)
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \textcircled{B} \quad & \underbrace{x^3 - x^2 + x - 1} = \\
 & = x(x^2+1) - 1(x^2+1) \\
 & = (x^2+1)(x-1)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{D} \quad & \overline{xy} - x - y + 1 = \\
 & = x(y-1) - (y-1) \\
 & = (y-1)(x-1)
 \end{aligned}$$

$$38. \quad x^2 - y^2 - 2x = -1.$$

$$\text{vdb } y = x - 1 \quad \vee' \quad y = -x + 1$$

$$x^2 - y^2 - 2x + 1 = 0$$

~~$$x^2 - y^2 - 2x + 1 = 0$$~~

$$x^2 - 2x + 1 - y^2 = 0$$

$$(x - 1)^2 - y^2 = 0$$

$$(x - 1 - y)(x - 1 + y) = 0$$

$$x - 1 - y = 0$$

$$x - 1 = y$$

$$\vee' \quad x - 1 + y = 0$$

$$y = 1 - x$$

$$27. \textcircled{20} \frac{(x+1)^3 - 1}{x} =$$

$$= \frac{(x+1-1)((x+1)^2 + (x+1) \cdot 1 + 1^2)}{x} =$$

$$= \frac{\cancel{x} \cdot (x^2 + 2x + 1 + x + 1 + 1)}{\cancel{x}} =$$

$$= x^2 + 3x + 3$$

$$= \Delta < 0$$

$$\textcircled{5} \left(x - \frac{1}{x}\right)^2 \frac{x^3 - x^2}{(x-1)^3} =$$

$$= \left(\frac{x^2}{x} - \frac{1}{x}\right)^2 \frac{x^2(x-1)}{(x-1)^3} =$$

$$= \left(\frac{x^2 - 1}{x}\right)^2 \frac{x^2(x-1)}{(x-1)^3} =$$

$$= \frac{\cancel{x^2} \cdot \cancel{(x-1)}^2 (x+1)^2}{\cancel{x^2}} \frac{\cancel{x^2} \cdot \cancel{(x-1)}^2}{\cancel{(x-1)}^2}$$

$$= (x+1)^2$$

$$40. \textcircled{a} (3x-1)^3 + (x-2)^3 + (3-4x)^3 = 0$$

$$\rightarrow 3x-1 + x-2 + 3-4x = 0$$

$$\rightarrow 3(3x-1)(x-2)(3-4x) = 0$$

$$\begin{array}{l} \therefore 3x-1=0 \quad ; \quad x-2=0 \quad ; \quad 3-4x=0 \\ \frac{3x}{3} = \frac{1}{3} \quad \quad \quad \underline{\underline{x=2}} \quad \quad \quad \frac{4x}{4} = \frac{3}{4} \\ \underline{\underline{x = \frac{1}{3}}} \quad \quad \quad \quad \quad \quad \quad \quad \quad \underline{\underline{x = \frac{3}{4}}} \end{array}$$

$$37. \quad \forall x+y = xy+1 \quad \text{vdo } x=1 \wedge y=1.$$

~~$$x+y-xy-1=0$$~~

~~$$x$$~~

$$xy - x - y + 1 = 0$$

$$x(y-1) - (y-1) = 0$$

$$\Rightarrow (y-1)(x-1) = 0$$

$$\downarrow$$

$$y-1=0 \quad \wedge \quad x-1=0$$

$$(y=1)$$

$$(x=1)$$

$$40. \quad (B) \quad (3x-1)^3 = x^3 + (2x-1)^3$$

$$\Rightarrow (3x-1)^3 - x^3 - (2x-1)^3 = 0$$

$$(3x-1)^3 + (-x)^3 + (1-2x)^3 = 0$$

$$\rightarrow 3x-1 - x - 2x+1 = 0$$

$$3(3x-1)(-x)(1-2x) = 0$$

$$3x-1=0 \Rightarrow$$

$$\Rightarrow \frac{3x}{3} = \frac{1}{3} \Rightarrow$$

$$\Rightarrow \boxed{x = \frac{1}{3}}$$

∨

$$-x=0 \Rightarrow \text{or } 1-2x=0 \Rightarrow$$

$$\Rightarrow \boxed{x=0}$$

$$\Rightarrow \frac{-2x}{-2} = \frac{-1}{-2} \Rightarrow$$

~~3x-1=0~~

$$\Rightarrow \boxed{x = \frac{1}{2}}$$

$$40. \textcircled{y}. \quad 27(x-2)^3 + 8x^3 = (5x-6)^3$$

$$27(x-2)^3 + 8x^3 - (5x-6)^3 = 0$$

~~$$27(x-2)^3 + 8x^3 + (-5x+6)^3 = 0$$~~

~~$$(27x - 5)$$~~

$$(3(x-2))^3 + (2x)^3 + (6-5x)^3 = 0$$

$$3(x-2) + 2x + 6-5x = 0$$

~~$$3x - 6 + 2x + 6 - 5x = 0$$~~

$$0 = 0.$$

$$\text{den } 3 [3(x-2)] 2x (6-5x) = 0$$

$$= 6x(3x-6)(6-5x) = 0$$

$$= 18x(x-2)(6-5x) = 0$$

$$\checkmark \underline{x=0} \quad \checkmark \underline{x=2} \quad \checkmark \frac{5x=6}{5}$$

$$\underline{\underline{x = \frac{6}{5}}}$$

$$39. (a) (a-b)^2 \cdot (a^{-1}-b^{-1})^{-2} =$$

$$(a-b)^2 \cdot \left(\frac{1}{a} - \frac{1}{b}\right)^{-2} = (a-b)^2 \cdot \frac{1}{\left(\frac{1}{a} - \frac{1}{b}\right)^2} =$$

$$\frac{(a-b)^2}{\left(\frac{1}{a} - \frac{1}{b}\right)^2} = \frac{(a-b)^2}{\left(\frac{b}{ab} - \frac{a}{ab}\right)^2} = \frac{(a-b)^2}{\left(\frac{b-a}{ab}\right)^2} = \frac{(a-b)^2}{\frac{b^2-a^2}{(ab)^2}} = \frac{(a-b)^2 \cdot a^2 b^2}{(b-a)(b+a)}$$

$$= \frac{(a-b) \cancel{a^2 b^2}}{-\cancel{(a-b)}(b+a)}$$

$$(b) \left(\frac{a+b}{a-b}\right)^{-1} \cdot \frac{a^{-2}-b^{-2}}{a^{-1}-b^{-1}} = \left(\frac{a-b}{a+b}\right) \cdot \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\frac{1}{a} - \frac{1}{b}}$$

$$= \frac{a-b}{a+b} \cdot \frac{\frac{b^2}{a^2 b^2} - \frac{a^2}{a^2 b^2}}{\frac{b}{ab} - \frac{a}{ab}}$$

$$= \frac{a-b}{a+b} \cdot \frac{\frac{b^2-a^2}{a^2 b^2}}{\frac{b-a}{ab}} =$$

$$= \frac{a-b}{\cancel{a+b}} \cdot \frac{\cancel{(b-a)}(b+a) \cancel{ab}}{a^2 b^2 \cancel{(b-a)}} = \frac{a-b}{ab}$$

$$41. \textcircled{f} (x^2 - x)^2 - 8(x^2 - x) + 12 =$$

$$A = b^2 - 4ac = 64 - 48 = 16$$

$$x_{1,2} = \frac{8 \pm \sqrt{16}}{2} = \begin{cases} x_1 = \frac{8}{2} = 4 \\ x_2 = \frac{4}{2} = 2 \end{cases}$$

$$= (x^2 - x - 6)(x^2 - x - 2)$$

$$\text{DETW } x^2 - x = \epsilon$$

$$t^2 - 8t + 12 = (t - 6)(t + 2)$$

$$= (x^2 - x - 6)(x^2 - x + 2)$$

$$= (x - 3)(x + 2)(x - 2)(x + 1)$$

$$33. \quad \frac{\alpha}{\beta} = -\frac{3}{2} \quad (\Rightarrow) \quad \boxed{\alpha = -\frac{3}{2}\beta}$$

$$\begin{aligned} \textcircled{a} \quad \frac{6\alpha - 3\beta}{5\beta} &= \frac{6 \cdot \left(-\frac{3}{2}\beta\right) - 3\beta}{5\beta} = \frac{-9\beta - 3\beta}{5\beta} \\ &= \frac{-12\cancel{\beta}}{5\cancel{\beta}} = -\frac{12}{5} \end{aligned}$$

$$\textcircled{b} \quad \frac{5\alpha}{3\alpha + \beta} = \frac{5 \left(-\frac{3}{2}\beta\right)}{3 \left(-\frac{3}{2}\beta\right) + \beta} = \frac{-\frac{15}{2}\beta}{-\frac{9}{2}\beta + \beta} = \frac{-\frac{15}{2}\beta}{-\frac{9}{2}\beta + \frac{2\beta}{2}} =$$

$$\begin{aligned} \textcircled{c} \quad \frac{3\alpha - 5\beta}{2\alpha - \beta} &= \frac{3 \left(-\frac{3}{2}\beta\right) - 5\beta}{2 \left(-\frac{3}{2}\beta\right) - \beta} = \frac{-\frac{9}{2}\beta - 5\beta}{-2\beta - \beta} = \\ &= \frac{-\frac{9}{2}\beta - 5\beta}{-3\beta} \end{aligned}$$

Επορω Μαθημα

④ $\frac{\Sigma \epsilon \tau \sigma \omicron}{\alpha \gamma \epsilon}$

⑤ α γ ε τ

⑥ α γ

⑦ α γ

⑧ α β γ σ ζ

⑩ α β δ ε

⑪ α γ

⑬ α γ

⑮ α γ δ ε

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